Department of Mechanical and Automation Engineering
Qualifying/Candidacy Examination (Written)

May 29, 2020 (Friday) **14:30 – 17:00** **By ZOOM Meeting**

< Remember to write your <u>student number, course code and page number</u> clearly on <u>each page</u> of answer sheets. >

<u>Question 1 (Advanced Robotics) – ENGG 5402</u>

1. **Dynamics** Consider a 2-DoF Prismatic-Revolute manipulator (PR Robot) shown in Fig. 1. Assume that all joints are frictionless. $q = [d,\ \theta]^T$ are the joint variables of the manipulator, where d is the displacement of the first link and θ is the angular position of the second link . $\tau = [\tau_1\ \tau_2]^T$ are the generalized forces for the manipulator. We also assume that the mass is uniformly distributed in both links and the total mass for both link are equal such that $m_1 = m_2 = m$. Both links have the same length, width, and height such that $L_1 = L_2 = L$ and $w_1 = w_2 = h_1 = h_2 = a$.

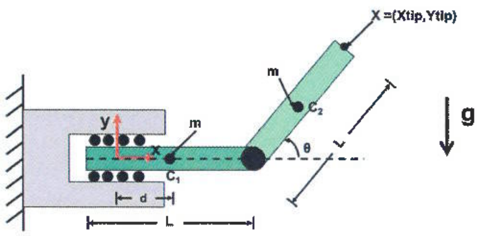

Figure 1: A 2-DoF Prismatic-Revolute planar arm (PR Robot)

(a) (2%) For a common two-link manipulator, the inertial matrix has the form $M = m_1 J_{v1}^T J_{v1} + m_2 J_{v2}^T J_{v2} + J_{\omega 1}^T {}^{C_1}I_1 J_{\omega 1} + J_{\omega_2}^T {}^{C_2}I_2 J_{\omega_2}$ where J_{vi} is the linear Jacobian of the center of mass of link i, $J_{\omega i}$ is the Jacobian related to the angular velocity of link i, and ${}^{C_i}I_i$ is the inertia tensor of link i expressed in frame $\{C_i\}$. Calculate ${}^0J_{v1}$ and ${}^0J_{v2}$.

(b) (1%) Compute ${}^{C_1}J_{\omega 1}$ and ${}^{C_2}J_{\omega 2}$.

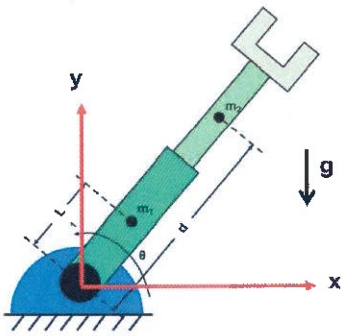

Figure 2: A 2-DoF Revolute-Prismatic planar arm (RP Robot)

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(c) For the RP Robot in Fig. 2, the joint variables are $q = [\theta, d]^T$, while the operational space is chosen as $P = [x, y]^T$. The links for the RP Robot have inertia tensors

$$^{C_1}I_1 = \begin{bmatrix} I_{xx1} & 0 & 0 \\ 0 & I_{yy1} & 0 \\ 0 & 0 & I_{zz1} \end{bmatrix}$$

,

$$^{C_2}I_2 = \begin{bmatrix} I_{xx2} & 0 & 0 \\ 0 & I_{yy2} & 0 \\ 0 & 0 & I_{zz2} \end{bmatrix},$$

and the total mass m_1 and m_2, where $m_1 = m_2 = m$. The center of mass of link 1 is located at a distance L from the joint-1 axis. The center of mass of link 2 is at the "variable" distance d from the joint-1 axis as it can move in and out along the direction of the link 1. $T = [\tau_1, \tau_2]^T$ is the applied torques for each link. The direction of the gravity vector is also indicated in Fig. 2.

i. (2%)Derive the equations of motion using Lagrange method in the following form:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = T$$

Find $M(q)$, $C(q, \dot{q})$, $G(q)$ in terms of system parameters and joint variables. Verify that $\dot{M} - 2C$ of this robot is skew-symmetric.

ii. (2%)Design a computed torque controller for the nonlinear system in (i) such that the resulting close-loop system is decoupled, critically damped, and with natural frequency $\omega = 30$.

(d) (3%) In this question, we want to compare the inertial properties among a 2-DoF Prismatic-Revolute manpiulator (PR Robot, Fig. 1), a 2-DoF Revolute-Prismatic manipulator (RP Robot, Fig. 2), a 2-DoF Prismatic-Prismatic manipulator (PP Robot, Fig. 3), and a 2-DoF Revolute-Revolute manipulator (RR Robot, Fig. 4). For a 2-DoF manipulator, the inertial matrix can be written as

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix},$$

Please answer the following questions without calculating the inertial matrices $M(q)$

i. (1%)Which robot(s) will it be if $M_{12} = M_{21} = 0$? PR Robot or RP Robot or PP robot or RR robot? Please provide corresponding explanation.

ii. (1%)Which robot(s) will it be if $M_{22} = m_2 = m$? PR Robot or RP Robot or PP robot or RR robot? Please provide corresponding explanation.

iii. (1%)Which robot will it be if $M_{11} = m_1 + m_2$? PR Robot or RP Robot? Please provide corresponding explanation.

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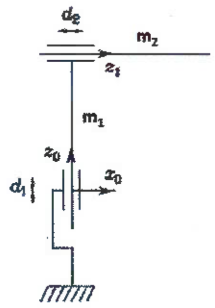


Figure 3: Example of a 2-DoF Planar Prismatic-Prismatic Manipulator (PP Robot)

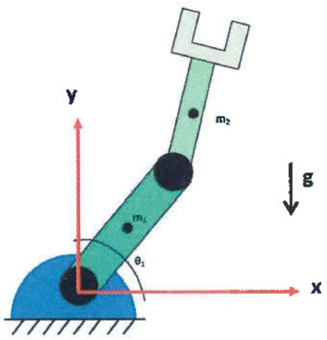


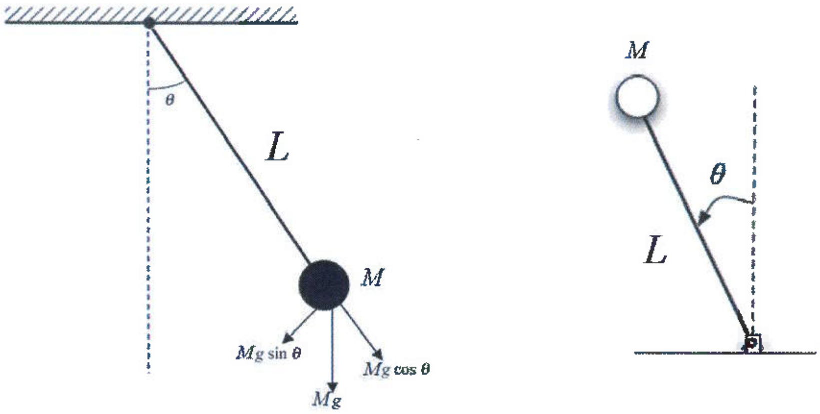
Figure 4: Example of a 2-DoF Planar Revolute-Revolute Manipulator (RR Robot)

End of Question 1 (Advanced Robotics) – ENGG 5402

Question 2 (Linear System Theory and Design) – ENGG 5403

Even if you are unable to prove the answer of one part, you may use that answer in other parts. Always justify your answers.

Consider a pendulum or inverse pendulum system below.



Let the input force applied to the system be $u(t)$ and the output variable be $\theta(t)$. For simplicity, we let $L = g$ and $ML^2 = 1$. It can be showed that the dynamic equation governing the system is nonlinear and is given by

$$\dot{x} = \begin{pmatrix} \dot{\theta} \\ \ddot{\theta} \end{pmatrix} = f(x, u) = \begin{pmatrix} \dot{\theta} \\ -\sin\theta + u \end{pmatrix}$$

and

$$y = \theta$$

(a) [1 point] Show that

$$(u_0, x_0, y_0) = \left(0, \begin{bmatrix} \pi \\ 0 \end{bmatrix}, \pi\right)$$

is one of the two operating points of the system. What is the other operating point?

(b) [2 points] Show that the linearized model of the system around the operating point in Part (a) is given by

$$\begin{pmatrix} \dot{\tilde{\theta}} \\ \ddot{\tilde{\theta}} \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} \tilde{\theta} \\ \dot{\tilde{\theta}} \end{pmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \qquad \tilde{y} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{pmatrix} \tilde{\theta} \\ \dot{\tilde{\theta}} \end{pmatrix},$$

where $\tilde{\theta} = \theta - \pi$ and $\tilde{y} = y - \pi$.

(c) [2 points] Determine the controllability (or stabilizability), observability (or detectability) and the invariant zeros of the linearized system in Part (b).

(d) [2 points] Assume that both θ and $\dot{\theta}$ are available for feedback in the linearized system in Part (b), and

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad R = 1.$$

Find the corresponding LQR control law.

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(e) [1 point] What are the guaranteed gain margin and phase margin of the control system obtained in Part (d)? Why?

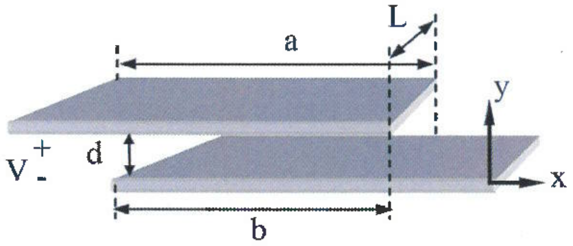
(f) [1 point] Explain what the physical meanings of the gain and phase margins are in control systems, and why they are important.

(g) [1 point] How can one stabilize the system in Part (b) if only $\boldsymbol{\theta}$ is available for feedback?
Note: There is no need to design an actual control law.

End of Question 2 (Linear System Theory and Design) – ENGG 5403

Question 3 (Micromachining and Microelectromechanical Systems) – ENGG 5404

1. A microscale electrostatic actuator can be modeled by two parallel plates. As shown below, two parallel silicon plates separated by ***d*** overlap a distance ***b*** and have width ***L***; they are charged to a voltage ***V***.



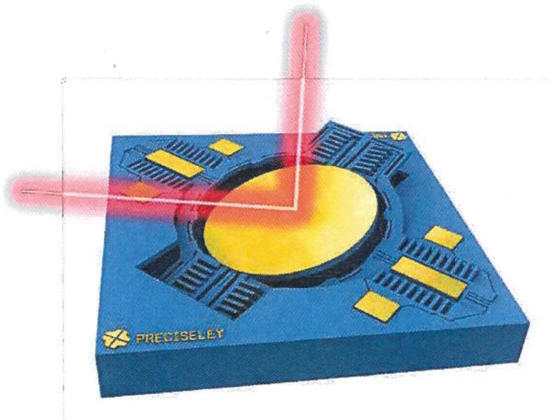
(A) What is the force $f_{\perp}$ [N] attracting the two plates in the y direction?

(B) What is the force $f_{//}$ [N] attracting the two plates in the x direction?

(C) Discuss the pros and cons of the above two types of electrostatic actuators. Which one is better for high-speed operation? Which one is better for long range positioning application? Give reasons to justify your answer.

(D) For the conceptual design of a 2-axis scanning MEMS mirror with a diameter of 500 microns shown below. Explain the actuation principles. If the actuators cannot provide sufficiently large actuation force, how can one generate large scan angle?

(E) Propose processes to microfabricate the 2-axis MEMS mirror shown below.



End or Question 3 (Micromachining and Microelectromechanical Systems) – ENGG 5404

Question 4 (Topics in Computer-Aided Geometric Design) – MAEG 5030

Given a point cloud as a set of points (xi, yi, zi) (i=1, ..., n), which are captured by a RGB-D camera with (xi, yi) being the pixel coordinate and (zi) being the depth information obtained from the RGB-D image. Please describe the steps of approximation this point cloud by a cubic Bezier surface patch. (Hint: (xi, yi) gives a natural parameterization for each point - i.e., (u,v)).

End of Question 4 (Topics in Computer-Aided Geometric Design) – MAEG 5030

Question 5 (Nonlinear Control Systems) – MAEG 5070

1. Assume $V(t)$ is continuously differentiable for $t \geq t_0$ for some t_0, and $V(t)$ has a finite limit as $t \to \infty$, and $\dot{V}$ is uniformly continuous for $t \geq t_0$. Show that $\dot{V}(t) \to 0$ as $t \to \infty$.

Hint: Assume $\dot{V}(t)$ does not approach zero as $t \to \infty$. Then $\exists \varepsilon_0 > 0$ with the property that $\forall T > 0$, $\exists t > T$ such that $|\dot{V}(t)| \geq \varepsilon_0$. Thus, there exists an infinite sequence $t_1, t_2, ...$ such that $t_i \to \infty$ as $i \to \infty$ with the property that $|\dot{V}(t_i)| \geq \varepsilon_0$.

2. Consider the system

$$\dot{x} = f(x) \tag{1}$$

where $x \in R^n$, f is continuous for all $x \in R^n$, and $f(0) = 0$. Let $V(x)$ be a scalar function with continuous first partial derivative such that

◇ for some $l > 0$, the region $\Omega_l \triangleq \{x \mid V(x) < l\}$ is bounded

◇ $\dot{V}(x) \leq 0, \forall x \in \Omega_l$

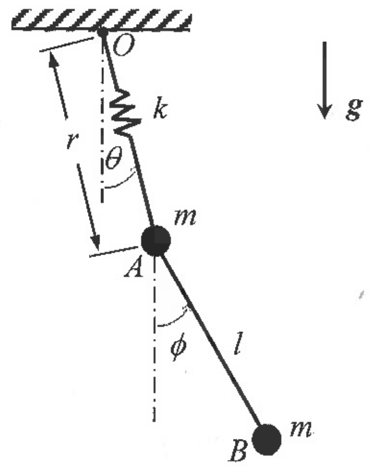
Show that every solution $x(t)$ originating in Ω_l is such that $\dot{V}(x(t))$ tends to zero as t tends to ∞.

Hint: You can use the result of Problem 1.

End of Question 5 (Nonlinear Control Systems) – MAEG 5070

Question 6 (Smart Materials and Structures) – MAEG 5080

Two wheels A and B, each of mass m, are connected by a massless axle of length l. Each wheel is considered to have its mass concentrated as a particle at its hub. The hub of wheel A is attached by a spring of stiffness k and unstressed length r_o to a fixed point O. Using r, θ, and ϕ as generalized coordinates, obtain the differential equations of motion.



End of Question 6 (Smart Materials and Structures) – MAEG 5080

Question 7 (Topics in Robotics) – MAEG 5090

a) Explain the main steps/idea of the Composite Rigid-Body Algorithm (CRBA) and why it is $O(N^3)$ where N is the number of links in the system. You can assume that there's a function $\boldsymbol{\tau} = ID(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$ that solves the inverse dynamics.

b) Explain why the Articulated Body Algorithm (ABA) is $O(N)$?

Hint: the following equations may help you.

$$I_i^A = I_i + \sum_{j \in \mu(i)} \left(I_j^A - I_j^A S_j (S_j^T I_j^A S_j)^{-1} S_j^T I_j^A \right)$$
$$\mathbf{p}_i^A = \mathbf{p}_i + \sum_{j \in \mu(i)} \left(\mathbf{p}_j^a + I_j^A S_j (S_j^T I_j^A S_j)^{-1} (\boldsymbol{\tau}_i - S_j^T \mathbf{p}_j^a) \right)$$
$$\mathbf{p}_j^a = \mathbf{p}_j^A + I_j^A \dot{\mathbf{h}}_j \dot{\mathbf{q}}_j$$
$$\mathbf{a}_i = \mathbf{a}_{\lambda(i)} + \dot{\mathbf{h}}_i \dot{\mathbf{q}}_i + \mathbf{h}_i \ddot{\mathbf{q}}_i$$
$$\mathbf{f}_i^J = I_i^A \mathbf{a}_i + \mathbf{p}_i^A$$
$$\boldsymbol{\tau}_i = \mathbf{h}_i^T \mathbf{f}_i^J = \mathbf{h}_i^T (I_i^A \mathbf{a}_i + \mathbf{p}_i^A)$$

c) For ZMP walking, formulate the equations (state space and constraints) for the Linear Inverted Pendulum Model (LIPM) in the form

$$\mathbf{x}_{k+1} = A\mathbf{x}_k + B\mathbf{u}_k$$
$$\mathbf{y}_k = C\mathbf{x}_k + D\mathbf{u}_k$$

where $\mathbf{y}_k = \begin{bmatrix} p_x & p_y \end{bmatrix}^T$ is the ZMP position in x and y directions. Define $\mathbf{x}_k, \mathbf{u}_k$ and express A, B, C, D. Also state what constraints should be defined for walking.

d) State two advantages of using MPC over model-based reactive control schemes (such as PID).

e) For a system with the equations of motion $M(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{c}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}$, define a Quadratic Programming optimisation problem with decision variables $\ddot{\mathbf{q}}, \boldsymbol{\tau}$ to solve for the following objectives and constraints:

a. (objective) acceleration level motion tracking task for reference motion $\mathbf{x}_r(t), \dot{\mathbf{x}}_r(t), \ddot{\mathbf{x}}_r(t)$

b. (constraint) constraints on the interaction force such that it is within a specified friction cone (define all necessary terms for the constraint with diagrams)

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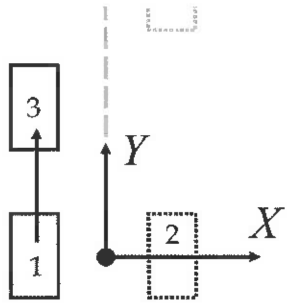
f) For a walking motion in the figure below, where the right foot is on the ground at position #2 while the left foot moves from position #1 to #3, an optimisation-based whole-body control problem will be defined:

i. Describe the types (motion, torque, contact etc.) of tasks (objectives or constraints) in order to produce the walking motion

ii. Describe the order of priorities for the tasks in part ii)



End of Question 7 (Topics in Robotics) – MAEG 5090

Question 8 (Nanomaterials and Nanotechnology: Fundamentals and Applications) – MAEG 5120

1.1 Scaling law is crucial for the design of devices/machines at the small scales. (i) List four forces which scale with the linear dimension (i.e. $\boldsymbol{L}$), $\boldsymbol{L^2}$, $\boldsymbol{L^3}$ and $\boldsymbol{L^4}$, respectively. (ii) Based on Trimmer's vertical bracket notation, we can write:

$$F = \begin{bmatrix} L^1 \\ L^2 \\ L^3 \\ L^4 \end{bmatrix}, \quad L\text{: dimension}$$

Derive the relationship between the acceleration (***a***), time (***t***), power per volume (***P/V***) and the linear dimension ***L*** in rigid-body dynamics. (**5 points**)

1.2 In comparison with Scanning Tunneling Microscopy (STM), list two main advantages of Atomic Force Microscopy (AFM) (**2 points**). List three different applications of AFM other than a microscope for topography at nanoscale (**3 points**).

End of Question 8 (Nanomaterials and Nanotechnology: Fundamentals and Applications) – MAEG 5120

Question 9 (Computational Mechanics) – MAEG 5130

1. For the 1D elastic problem with the strong form of:

$$\frac{d}{dx}\left(2\frac{du}{dx}\right) + 3x = 0, \qquad 1 \le x \le 5$$

$$u(1) = 0.001$$

$$\frac{du}{dx}\Big|_{x=5} = 1$$

   a. Derive the weak form.

   b. If the domain is discretized by 2 2-node linear elements with the same length, approximate the displacement and strain fields.

   c. If the domain is discretized by 1 3-node quadratic element with equally spaced nodes, approximate the displacement and strain fields.

   d. Compare the results in b and c with analytical solution. What is your conclusion?

End of Question 9 (Computational Mechanics) – MAEG 5130

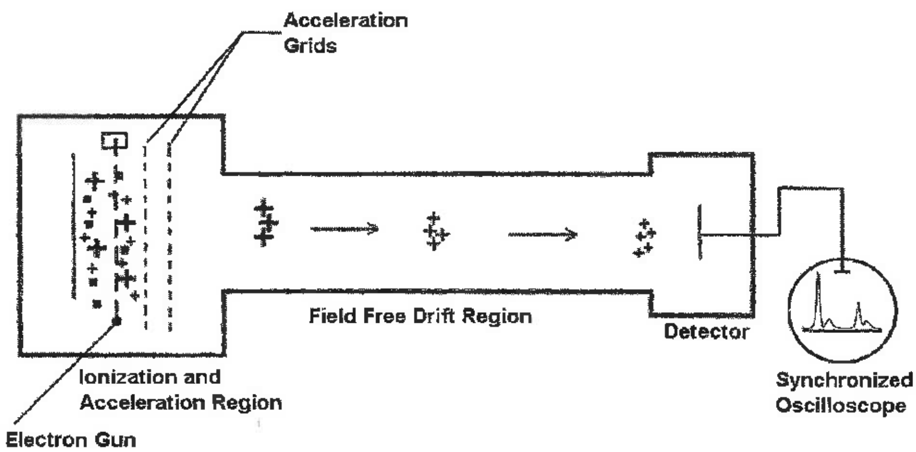
Question 10 (Materials Characterization Techniques) – MAEG 5140

a) The abundances of Br and Cl isotopes are given below:

^{35}Cl	75%	^{37}Cl	25%
^{79}Br	50%	^{81}Br	50%

Construct a mass spectrum for $(ClBr_2)^+$ ions. Make the intensity of the most intense peak 100.

b) A major part of a mass spectrometry system is mass analyzer. (a) Show how a time-of-flight analyzer works (or in other words, how m/z value is measured by measuring the time of an ion travelling through some distance); (b) Assume that the acceleration potential is 300 V, and the free-field flight tube is 2 meters long, calculate the time-of-flight for ions with m/z values of 10, 20, 100, respectively; and (3) Discuss whether the settings in (2) would allow a resolution of 0.1 in m/z if the ion detector has a time resolution of 0.1 μs.

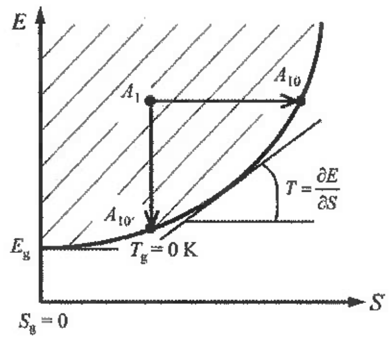


End of Question 10 (Materials Characterization Techniques) – MAEG 5140

Question 11 (Advanced Heat Transfer and Fluid Mechanics) – MAEG 5150

Answer each question concisely.

(1) Explain the physical meaning of the vertical line from A_1 to $A_{10'}$ on the following E-S graph.



(2) Recent research has indicated that freestanding graphene has a higher thermal conductivity than the graphene supported by a substrate. Explain the reason concisely.

(3) The molar analysis of a gas mixture at 25°C, 0.1 MPa is 50% N_2, 25% CO_2, and 25% O_2. Determine the mass fraction and partial pressure of each component. Molecular weights of N_2, CO_2, and O_2 are 28, 44, and 32 kg/kmol, respectively.

(4) Butane (C_4H_{10}) burns completely with 200% of theoretical air. Determine the amount of N_2 in the products, in kmol per kmol of fuel.

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(5) Consider a steady and incompressible flow of a Newtonian fluid with the velocity field $u = -2xy$, $v = y^2 - x^2$, and $w = 0$. The gravitational acceleration is negligible.

(a) Does this flow satisfy conservation of mass?

(b) Find the pressure gradients in the all the dimensions at the point $(x, y, z) = (\rho^{-3}, \rho^{-3}, \rho^{-3})$.

Note: continuity and momentum equations for incompressible flow with constant viscosity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

(6) To assess the time scale of the smallest eddy after the mesh filter, the Kolmogorov time, ι, can be used. It is related to the kinematic viscosity, v, and the turbulence dissipation rate, ε.

Derive the relationship between Kolmogorov time and viscosity and dissipation rate using dimensional analysis.

(7) To derive the transport equations for Reynolds stresses, the following term needs to be derived:

$$\overline{u_i'\frac{\partial u_j}{\partial t} + u_j'\frac{\partial u_i}{\partial t}}$$

Simplify this term.

<u>End of Question 11 (Advanced Heat Transfer and Fluid Mechanics) – MAEG 5150</u>