Department of Mechanical and Automation Engineering
Candidacy Examination (Written)

November 13, 2020 (Friday) 14:30 – 17:00 By ZOOM Meeting

< Remember to write your student number, course code and page number clearly on each page of answer sheets. >

Question 1 (Advanced Robotics) – ENGG 5402

1. **Dynamics**

(a) For the RP Robot shown in Fig. 2, the joint variables are $q = [\theta, d]^T$, while the operational space is chosen as $P = [x, y]^T$. The links for the RP Robot have inertia tensors

$$^{C_1}I_1 = \begin{bmatrix} I_{xx1} & 0 & 0 \\ 0 & I_{yy1} & 0 \\ 0 & 0 & I_{zz1} \end{bmatrix}$$

,

$$^{C_2}I_2 = \begin{bmatrix} I_{xx2} & 0 & 0 \\ 0 & I_{yy2} & 0 \\ 0 & 0 & I_{zz2} \end{bmatrix},$$

and the total mass m_1 and m_2, where $m_1 = m_2 = m$. The center of mass of link 1 is located at a distance L from the joint-1 axis. The center of mass of link 2 is at the "variable" distance d from the joint-1 axis as it can move in and out along the direction of the link 1. $T = [\tau_1, \tau_2]^T$ is the applied torques for each link. The direction of the gravity vector is also indicated in Fig. 1.

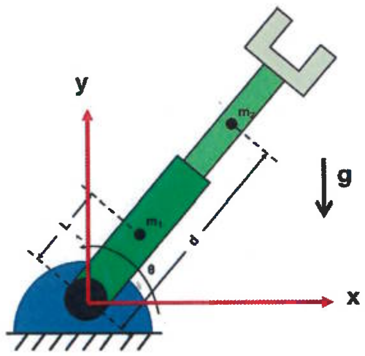


Figure 1: A 2-DoF Revolute-Prismatic planar arm (RP Robot)

(2%)Derive the equations of motion using Lagrange method in the following form:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = T$$

Find $M(q)$, $C(q, \dot{q})$, $G(q)$ in terms of system parameters and joint variables.

(b) (2%) Verify that $\dot{M} - 2C$ of this robot is skew-symmetric.

(c) (2%)Design a computed torque controller for the nonlinear system in (i) such that the resulting close-loop system is decoupled, critically damped, and with natural frequency $\omega = 30$.

(d) (2%)Assume the end-effector position $P = [x, y]^T$ is chosen at the center of the mass of link 2. We would like to express the dynamics of the manipulator with respect to Cartesian variables at the end-effector as below.

$$M_x(q)\ddot{X} + V_x(q, \dot{a}) + G_x(q) = F$$

where F is the external force acting on the end-effector of the robot. Find the Jacobian of the manipulator and derive $M_x(q)$ in terms of system parameters and joint variables.

(e) (2%) We want to estimate the dynamics parameters for this system and we rewrite the dynamics as the following linear parameter estimation equation:

$$T = Y(q, \dot{q}, \ddot{q})\pi,$$

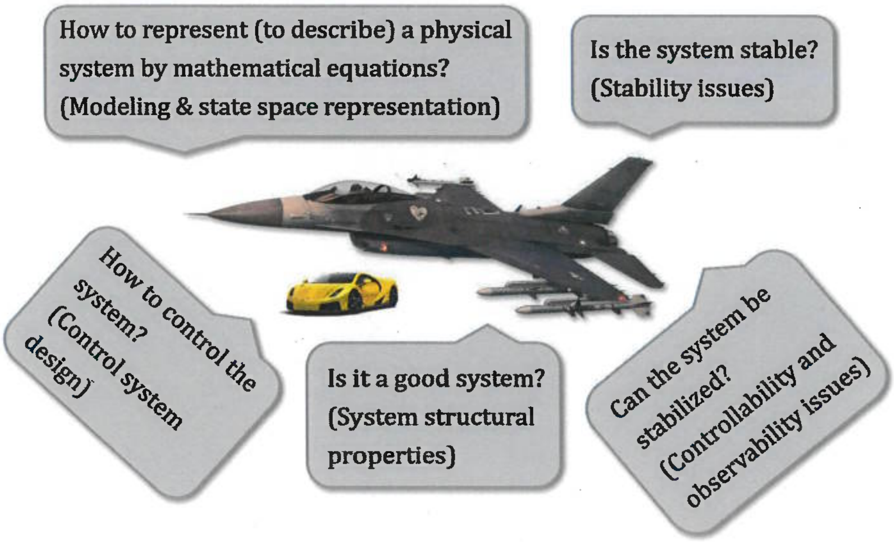
where $\pi \in \mathbb{R}^{7\times1}$ is the parameter vector and $Y(q, \dot{q}, \ddot{q}) \in \mathbb{R}^{2\times7}$ is the regressor matrix. Using the result in part (i), find a set of π and $Y(q, \dot{q}, \ddot{q})$ that will allow us to identify each system paramter $(m_1, m_2, l_1, I_{zz1}, I_{zz2})$.

<u>End of Question 1 (Advanced Robotics) – ENGG 5402</u>

Question 2 (Linear System Theory and Design) – ENGG 5403

Answer all questions. You are allowed to use a calculator from the university approved list of calculators. Even if you are unable to prove the answer of one part, you may use that answer in other parts. Always justify your answers.

The whole course of ENGG 5403 on linear system theory and design is centered around some physical systems and is to address the related issues as illustrated in the figure below.



Based on what you have learned from this course and/or your own understanding from somewhere else, answer the following questions with concise statements, if you agree the stated facts, or with succinct arguments, if you do not agree.

(a) [2 points] For a given physical system, be it mechanical or electrical or something else, how could one describe it by a set of mathematical equations? What kind of equations are commonly used to describe a physical system? What are the key considerations and challenges in dynamical model- ing? Suggest one or two commonly used techniques for dynamical modeling.

(b) [2 points] What is the stability issue of a dynamical system? Is the system stability important? Why or why not? Suggest one or two commonly used techniques for determining the stability of a dynamical system.

(c) [2 points] What is the controllability issue of a dynamical system? Is the system controllability important? Why or why not? Suggest two or more commonly used techniques for determining the controllability of a linear time-invariant system. Why is it said that the concepts of the system observability and controllability are dual?

(d) [2 points] Why is it more important to design a good system than to design a good controller to control a bad system? You might argue this fact if you do not agree. How can one classify a system being good or bad?

(e) [2 points] What are the key considerations when it comes to control a physical system? Suggest two or more commonly used control techniques and comment on their advantages and disadvantages. For a multivariable system, what is the control technique you would recommend to control it? Why?

End of Question 2 (Linear System Theory and Design) – ENGG 5403

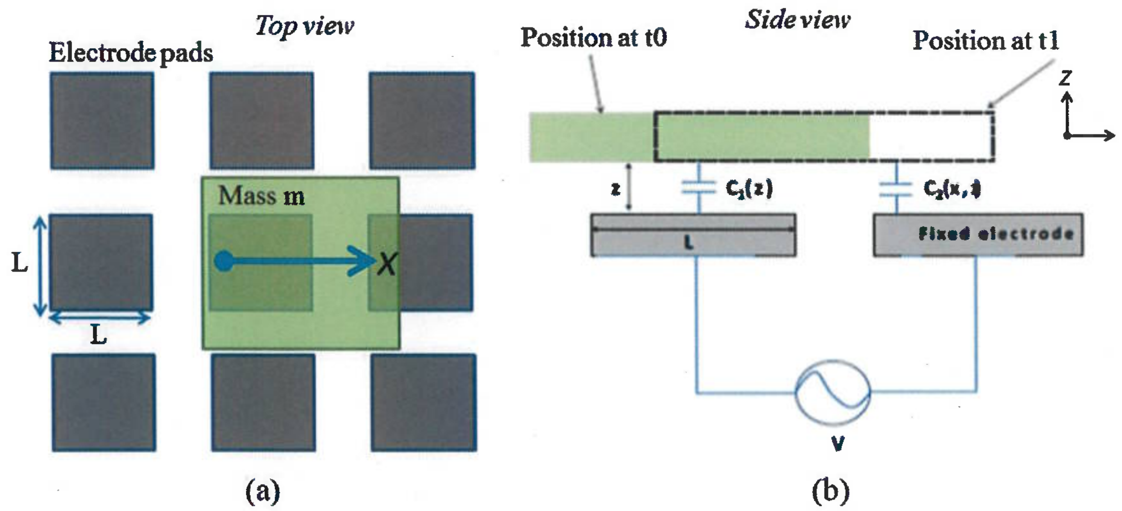
Question 3 (Micromachining and Microelectromechanical Systems) – ENGG 5404

1. A planar electrostatic actuation system drives a levitated mass ***m*** as shown in the figure below.

(a) Derive a mathematical expression for the instantaneous acceleration of the mass. Assume only the capacitance between the mass and one pad will change at any time in the direction of motion, and the gap thickness between the levitated mass and the fixed electrodes does not change.

(b) Design a motion guidance mechanism that will guide mass to freely move in the *x* and *y* directions.

(c) List the microfabrication process to make this electrostatic device.



Schematic of a planar electrostatic actuation system. (a) Top view. (b) Side view.

End of Question 3 (Micromachining and Microelectromechanical Systems) – ENGG 5404

Question 4 (Nonlinear Control Systems) – MAEG 5070

Consider the system $\dot{x} = f(x)$ where $x \in R^n$ and f is continuously differentiable with $f(0) = 0$.

(a) **(5 marks)** Assume all the eigenvalues of the Jacobian matrix J of f at the origin have negative real parts. Find a Lyapunov function $V(x)$ for the system such that the derivative of $V(x)$ along the trajectory of $\dot{x} = f(x)$ is locally negative definite. As a result, the equilibrium at $x = 0$ of this system is asymptotically stable.

(b) **(3 marks)** Give examples to show that if the eigenvalues of the Jacobian matrix of f at the origin have non-positive real parts, and at least one of the eigenvalues of J has zero real part, then the equilibrium at $x = 0$ of this system can be stable, unstable, and asymptotically stable.

(c) **(2 marks)** Show that if at least one of the eigenvalues of J has positive real part, then the equilibrium at $x = 0$ of this system is unstable.

Hint for part (c): For simplicity, you can assume that $n = 2$.

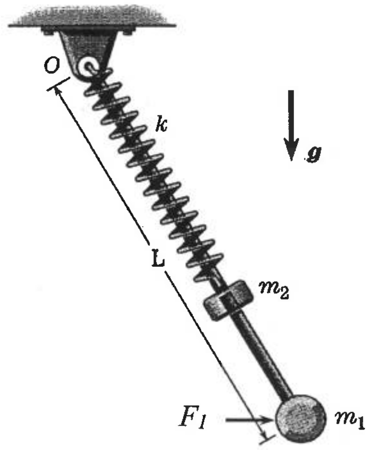
End of Question 4 (Nonlinear Control Systems) – MAEG 5070

Question 5 (Smart Materials and Structures) – MAEG 5080

A simple plane pendulum of mass m_1 and length L is pivoted about point O. A mass m_2 slides frictionlessly along the massless rigid link of the pendulum and is connected to point O by a spring of stiffness k as sketched in the figure. The unstretched length of the spring is l_o ($l_o < L$). A horizontal force $F_1(t)$ is applied to the pendulum.

(a) How many generalized coordinates needed to completely specify the system, why? Write down the constraint equations.

(b) Derive the equations of motion for the system.



End of Question 5 (Smart Materials and Structures) – MAEG 5080

Question 6 (Topics in Robotics) – MAEG 5090

a) Write the pseudocode for the Recursive Newton Euler algorithm (RNEA) to compute the forward dynamics in order to determine $\ddot{\mathbf{q}}$. Express and explain the computational complexity of the main steps. You can assume that there's a function $\boldsymbol{\tau} = ID(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$ that solves the inverse dynamics.

b) For the ZMP walking problem in the form

$$\mathbf{x}_{k+1} = A\mathbf{x}_k + B\mathbf{u}_k$$
$$\mathbf{y}_k = C\mathbf{x}_k + D\mathbf{u}_k$$

where $\mathbf{y}_k = \begin{bmatrix} p_x & p_y \end{bmatrix}^T$ is the ZMP position in x and y directions.

i. Define what is $\mathbf{x}_k, \mathbf{u}_k$ and express A, B, C, D.

ii. Formulate the MPC problem of tracking a reference output trajectory $\mathbf{y}_{ref}[k]$ (weighting Q) while minimising control effort $\mathbf{u}[k]$ (weighting R) for a horizon of N=5 steps.

iii. What is the control effort optimal solution $\mathbf{U}^*$ for the above problem?

iv. What are the purpose of terminal cost and constraints?

c) State two advantages of using MPC over reactive control schemes (such as PID).

d) Show that the null space vector $\mathbf{z} \in \mathbb{R}^n$ in the equation below can be arbitrarily selected and does not affect the relationship $\dot{\mathbf{x}} = J\dot{\mathbf{q}}$

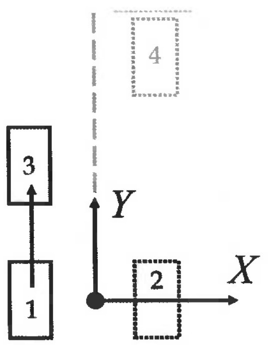
$$\dot{\mathbf{q}} = J^{\#}\dot{\mathbf{x}} + (I - J^{\#}J)\mathbf{z}$$

e) For a walking motion in the figure below, where the right foot is on the ground at position #2 while the left foot moves from position #1 to #3.

i. Describe the types (motion, torque, contact etc.) tasks (objectives/constraints) in order to produce the walking motion

ii. Mathematically define the tasks $T_i(\ddot{\mathbf{q}}, \boldsymbol{\tau}, \mathbf{F}_c)$ from part i) and specify if they are objectives to be minimised or constraints

iii. Describe the order of priorities for the tasks in part ii)



End of Question 6 (Topics in Robotics) – MAEG 5090

Question 7 (Nanomaterials and Nanotechnology: Fundamentals and Applications) – MAEG 5120

1. The following figure show typical dimension ranges of patterned features produced by several light- and/or ink-based printing methods, in which the two-photon polymerization technique is capable of conducting sub-micron and nanomachining for 3D structures less than 1 micron. Explain the two-photon absorption phenomenon with schematic drawing. (4%)

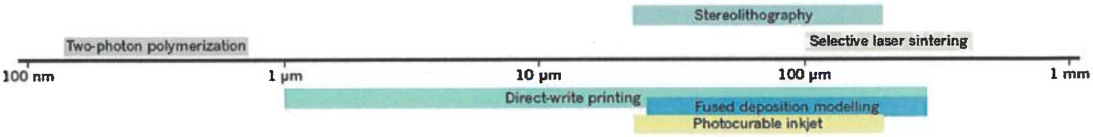
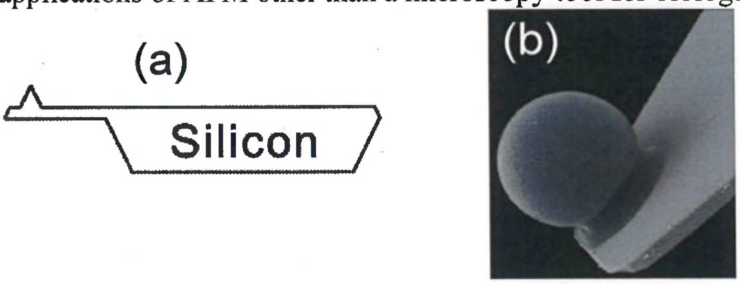


Figure is reprinted from Nature 2016 (doi:10.1038/nature21003).

2. (i) When a non-contact image mode is applied by using an Atomic Force Microscope (AFM), which surface gives you better resolution by AFM scanning: a hydrophilic or hydrophobic surface? Explain the reason briefly. (2%) (ii) Please describe the advance of using the AFM probe with a colloid tip (Fig. b) for mechanical properties measurement of micro-tissue in comparison with the conventional AFM tip (Fig. a). (2%) (iii) list two applications of AFM other than a microscopy tool for biological cells (2%)



End of Question 7 (Nanomaterials and Nanotechnology: Fundamentals and Applications) – MAEG 5120

Question 8 (Computational Mechanics) – MAEG 5130

1. For the 1D elastic problem with the strong form of:

$$\frac{d}{dx}\left(2\frac{du}{dx}\right) + 3x = 0, \qquad 1 \le x \le 5$$

$$u(1) = 0.001$$

$$\frac{du}{dx}|_{x=5} = 1$$

   a. Derive the weak form.
   b. If the domain is discretized by 2 2-node linear elements with the same length, approximate the displacement and strain fields.
   c. If the domain is discretized by 1 3-node quadratic element with equally spaced nodes, approximate the displacement and strain fields.
   d. Compare the results in b and c with analytical solution. What is your conclusion?

End of Question 8 (Computational Mechanics) – MAEG 5130

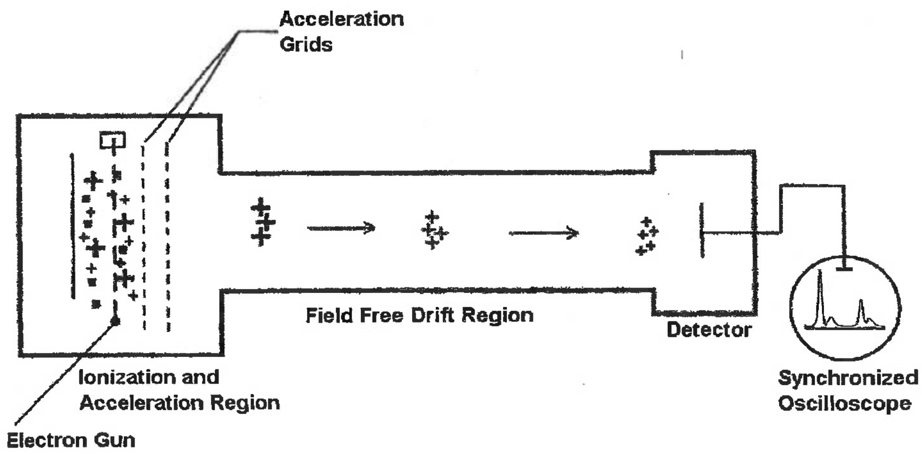
Question 9 (Materials Characterization Techniques) – MAEG 5140

a) The abundances of Br and Cl isotopes are given below:

^{35}Cl	75%	^{37}Cl	25%
^{79}Br	50%	^{81}Br	50%

Construct a mass spectrum for $(ClBr_2)^+$ ions. Make the intensity of the most intense peak 100.

b) A major part of a mass spectrometry system is mass analyzer. (a) Show how a time-of-flight analyzer works (or in other words, how m/z value is measured by measuring the time of an ion travelling through some distance); (b) Assume that the acceleration potential is 300 V, and the free-field flight tube is 2 meters long, calculate the time-of-flight for ions with m/z values of 10, 20, 100, respectively; and (3) Discuss whether the settings in (2) would allow a resolution of 0.1 in m/z if the ion detector has a time resolution of 0.1 μs.



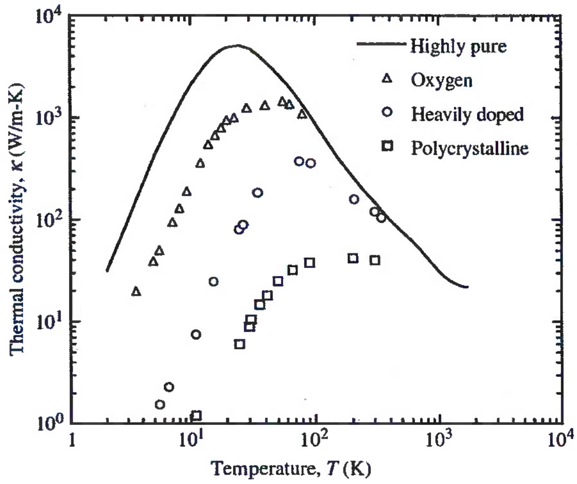
End of Question 9 (Materials Characterization Techniques) – MAEG 5140

Question 10 (Advanced Heat Transfer and Fluid Mechanics) – MAEG 5150

Answer each question concisely. (10 marks)

(1) On the E-S graph, draw the schematic to show the spontaneous change of state.

(2) The following figure shows thermal conductivity of silicon. Explain the temperature dependence trend.



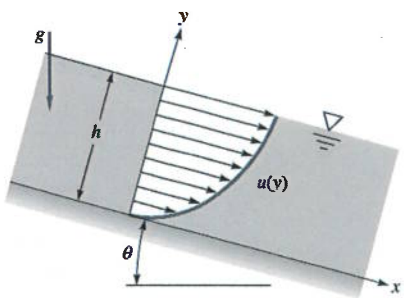
(3) The molar analysis of a gas mixture at 25°C, 0.1 MPa is 60% N_2, 30% CO_2, and 10% O_2. Determine the mass fraction and partial pressure of each component. Molecular weights of N_2, CO_2, and O_2 are 28, 44, and 32 kg/kmol, respectively.

(4) A gas turbine burns octane (C_8H_{18}) completely with 400% of theoretical air. Determine the amount of N_2 in the products, in kmol per kmol of fuel.

(5) A constant-thickness film of viscous liquid flows in laminar motion down a plate inclined at angle θ, as shown in the following figure. The flow is fully developed. The thickness of the film is h. The velocity profile is:

$$u = C \cdot y(2h - y) \qquad v = w = 0$$

Find the constant C in terms of the kinematic viscosity (υ), gravitational acceleration (g), and the angle (θ); and calculate the volume flux Q per unit width in terms of these parameters.



Continuity and momentum equations for incompressible flow with constant viscosity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho g_x-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\rho g_y-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=\rho g_z-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

(6) To assess the scale of the smallest eddy after the mesh filter, the Kolmogorov length, η, can be used. It is related to the kinematic viscosity, υ, and the turbulence dissipation rate, ε. Derive the relationship between Kolmogorov length and viscosity and dissipation rate using dimensional analysis.

(7) To derive the transport equations for Reynolds stresses, the following term needs to be derived:

$$\overline{u_i'\frac{\partial(u_j'u_k')}{\partial x_k}+u_j'\frac{\partial(u_i'u_k')}{\partial x_k}}$$

Use $\overline{u_i'u_j'u_k'}$ and its relevant derivatives to express this term.

<u>End of Question 10 (Advanced Heat Transfer and Fluid Mechanics) – MAEG 5150</u>

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