The Chinese University of Hong Kong
Department of Mechanical and Automation Engineering

Candidacy Examination
2020-21

May 14, 2021 (Friday)
14:30- 17:00
ERB 804 and via ZOOM Meeting

Time Allowed: Two and a Half Hours

1. This is a closed book/closed-note examination.
2. Answer **THREE** questions only.
3. Each question carries equal marks. (Full mark is 30.)
4. **Remember to write your student number and question number clearly on the front page of your answer sheets.**

DO NOT TURN OVER THE PAGE

UNTIL INSTRUCTED TO DO SO

Department of Mechanical and Automation Engineering
Candidacy Examination (Written)

May 14, 2021 (Friday) **14:30 – 17:00**

< Remember to write your <u>student number, course code and page number</u> clearly on <u>each page</u> of answer sheets. >

Question 1 (Advanced Robotics) – ENGG 5402

1. Kinematics and dynamics: Consider a SCARA robot shown in Fig. 1.
    a. (2%) Find out its *classic* Denavit-Hartenberg parameters table.
    b. (2%) Compute its direct kinematics.
    c. (2%) Solve its inverse kinematics.
    d. (2%) Compute its Jacobian.
    e. (2%) Find out its dynamic modal

For simplicity: link 3 and 4 can be considered as a unique rigid body (link) which translates along axis z_2 of the prismatic joint 3 and rotates about axis z_3 of the revolute joint 4. Let m_{l_3} denote the mass of this complete rigid body (link). It is also assumed that the centre of mass of this link is located on axis z_3. Use I_{l_4} for inertia tensor of this link relative to the centre of mass. Also assume that the motors of joint 3 and 4 have negligible mass and inertia. Students are also free to choose the symbols they prefer to help with the presentation.

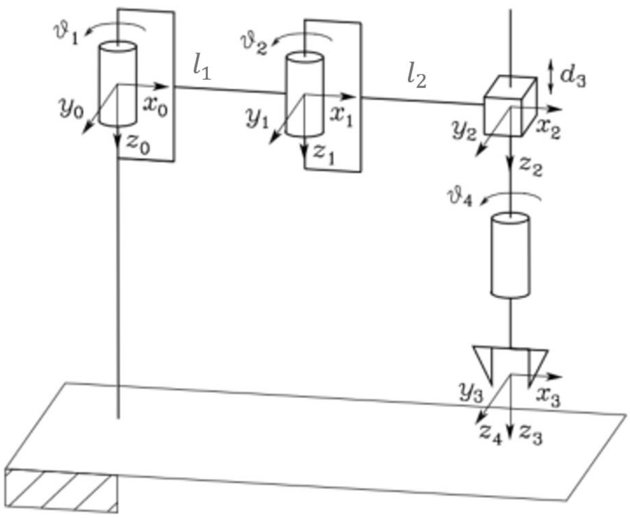


Fig. 1. SCARA manipulator

Note: for students who prefer to use *modified* Denavit-Hartenberg, they can redraw the figure with proper symbols but with clear explanation, and work out the same five sub-questions.

End of Question 1 (Advanced Robotics) – ENGG 5402

ENGG 5403 STUDENT'S NAME: ____________

LINEAR SYSTEM THEORY AND DESIGN STUDENT'S ID: ____________

QUALIFYING EXAMINATION

May 14, 2021

Question	Points	Score
Linear Systems Theory	10	
Total:	10	

Duration: 50 minutes.

Write your student number clearly on the front page of your answer book.
This is an closed-book examination with an A4 formula sheet.
Answer all questions.
Put all your answers on the answer book provided.
Question papers should be returned after the examination.
You are allowed to use a calculator from the university approved list of calculators.

Even if you are unable to prove the answer of one part, you may use that answer in other parts. Always justify your answers.

Question 1 [10 points]: Linear Systems Theory

Consider a linear system characterized by

$$\dot{x} = Ax + Bu, \quad y = Cx + Du$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$ are the state, input and output variables, respectively.

(a) [2 points] What are the essential structural properties associated with a general LTI system? In general, why does a nonminimum phase system give a worse control performance compared to a minimum phase one?

(b) [2 points] Let $T \in \mathbb{R}^{n\times n}$ be any nonsingular matrix. Let $\tilde{A} = T^{-1}AT$ and $\tilde{B} = T^{-1}B$. Show that $(\tilde{A}, \tilde{B})$ is controllable if and only if (A, B) is controllable.

(c) [2 points] Assume the rank of the controllability matrix of (A, B) is equal to r. It can be showed (in an ENGG 5405 homework assignment problem) that there exists a nonsingular transformation T such that

$$\tilde{A} = T^{-1}AT = \begin{bmatrix} A_{cc} & A_{c\bar{c}} \\ 0 & A_{\bar{c}\bar{c}} \end{bmatrix}, \quad \tilde{B} = T^{-1}B = \begin{bmatrix} B_c \\ 0 \end{bmatrix},$$

where $A_{cc} \in \mathbb{R}^{r\times r}$, $A_{c\bar{c}} \in \mathbb{R}^{r\times(n-r)}$, $A_{\bar{c}\bar{c}} \in \mathbb{R}^{(n-r)\times(n-r)}$ and $B_c \in \mathbb{R}^{r\times m}$. Show that

(i) (A_{cc}, B_c) is completely controllable, and

(ii) $A_{\bar{c}\bar{c}}$ contains all the uncontrollable modes of (A, B).

(d) [2 points] Given a system with

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},$$

construct an nonsingular transformation T such that the transformed system is in the so-called controllability canonical form, i.e.,

$$\tilde{A} = T^{-1}AT = \begin{bmatrix} A_{cc} & A_{c\bar{c}} \\ 0 & A_{\bar{c}\bar{c}} \end{bmatrix}, \quad \tilde{B} = T^{-1}B = \begin{bmatrix} B_c \\ 0 \end{bmatrix},$$

where (A_{cc}, B_c) is completely controllable. Show step by step the detailed construction procedure.

(e) [1 point] Why and how are the gain margin and phase margin used to measure the robustness of the overall control system in the standard unity negative feedback setting?

(f) [1 point] Why does the LQR control have an guarantee of at least 60° phase margin and infinite gain margin? Does the LQG control have such properties? What is the LTR control technique for?

End of Question 2 (Linear System Theory and Design) – ENGG 5403

Question 3 (Micromachining and Microelectromechanical Systems) – ENGG 5404

1. You are tasked to develop a new a silicon-based atomic force microscope (AFM) tip via the "bulk silicon micromachining" process. Fig. 1 presents the conceptual design.

(a) List your proposed fabrication process as well as the involved processing materials, chemicals,

(b) Generate mask drawings for each step in (a).

(c) Based on your process, estimate the sharpness of the fabricated tip, i.e., tip radius. Explain how the tip radius and aspect ratio can determine the resolution of an AFM.

(d) Will the AFM tip suffer from fatigue? If so, how to address it?

(e) Explain the operation principle of an AFM.

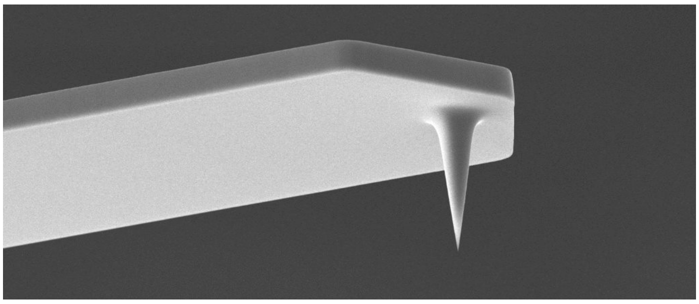


Fig. 1. Tip of an AFM

End or Question 3 (Micromachining and Microelectromechanical Systems) – ENGG 5404

Consider the single-input, single output system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \vdots &= \vdots \\ \dot{x}_{n-1} &= x_n \\ \dot{x}_n &= \alpha(x) + \beta(x)u \\ y &= x_1 \end{aligned}$$

where $x = (x_1, \cdots, x_{n-1}, x_n)^T$, $\alpha(x)$ and $\beta(x)$ are continuous for all $x \in R^n$, and are not known exactly. Assume there exist known functions $\hat{\alpha}(x)$ and $F(x)$ such that, for all x, $|\alpha(x) - \hat{\alpha}(x)| \leq F(x)$. Also, assume there exist two known positive real numbers b_{min} and b_{max} such that

$$b_{min} \leq \beta(x) \leq b_{max} \quad \text{for all } x \tag{1}$$

(a) Define the estimate $\hat{\beta}$ of $\beta(x)$ by $\hat{\beta} = (b_{min} b_{max})^{1/2}$ and let $b = (b_{max}/b_{min})^{1/2}$. Show that

$$b^{-1} \leq \frac{\hat{\beta}}{\beta(x)} \leq b \quad \text{for all } x$$

(b) Show that

$$\left| \frac{\hat{\beta}}{\beta(x)} - 1 \right| \leq b - 1 \tag{2}$$

(c) Given any smooth time function $y_d(t)$, $t \geq 0$, let $e = y - y_d$ and $s = e^{(n-1)} + \alpha_1 e^{(n-2)} + \cdots + \alpha_{n-1} e$ for some real number $\alpha_1, \ \cdots, \ \alpha_{n-1}$. Show that, for any $\eta > 0$, $u = \hat{\beta}^{-1}[\hat{u} - \phi(x) sgn(s)]$, where $\hat{u} = -\hat{\alpha}(x) + y_d^{(n)} - e^{(n)} - \alpha_1 e^{(n-1)} - \cdots - \alpha_{n-1}\dot{e}$ and $\phi(x) \geq b(F(x) + \eta) + (b-1)|\hat{u}|$ for all x, is such that

$$\frac{1}{2}\frac{d}{dt} s^2 \leq -\eta |s|$$

End of Question 4 (Nonlinear Control Systems) – MAEG 5070

Question 5 (Smart Materials and Structures) – MAEG 5080

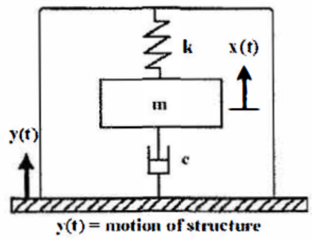
An accelerometer can be modeled as shown in the figure. The motion of the accelerometer mass relative to the base y(t) is denoted as z(t).

a. Derive the equation of motion for the relative motion z(t) in terms of base excitation y(t).

b. Assume harmonic base motion, $y(t) = Y\cos\omega t$, the steady-state solution of the relative motion is $z(t) = Z\cos(\omega t - \varphi)$. Show the relative displacement ratio is

$$\frac{Z}{Y} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \quad \text{where } r = \frac{\omega}{\omega_n},\ \omega_n = \sqrt{\frac{k}{m}},\ \text{and } \zeta = \frac{c}{2m\omega_n}$$

c. For $0 < \zeta < \frac{1}{\sqrt{2}}$, find the frequency ratio r at which the amplitude ratio in part (b) attains the maximum value. Also find the value of the maximum amplitude.



End of Question 5 (Smart Materials and Structures) – MAEG 5080

Question 6 (Topics in Robotics) – MAEG 5090

a) Write the pseudocode for the inverse dynamics for an *N* link serial robot. State the input and output of the algorithm and state its computational complexity.

b) Write the pseudocode for the Recursive Newton Euler algorithm (RNEA) to compute the forward dynamics in order to determine $\ddot{\mathbf{q}}$. State the input and output of the algorithm and state its computational complexity.

c) For the ZMP walking problem in the form

$$\mathbf{x}_{k+1} = A\mathbf{x}_k + B\mathbf{u}_k$$
$$\mathbf{y}_k = C\mathbf{x}_k + D\mathbf{u}_k$$

where $\mathbf{y}_k = \begin{bmatrix} p_x & p_y \end{bmatrix}^T$ is the ZMP position in *x* and *y* directions.

i. Define what is $\mathbf{x}_k, \mathbf{u}_k$ and express A, B, C, D.

ii. Express the state space relationship for the entire horizon of *N* time steps at time step *k* in the form

$$\mathbf{X}_k = S\mathbf{x}_k + T\mathbf{U}_k$$
$$\mathbf{Z}_k = U\mathbf{x}_k + V\mathbf{U}_k$$

Define $\mathbf{U}_k, \mathbf{X}_k$ and write the expressions of the S, T, U, V using the state-space form in part i).

iii. Write the MPC in standard quadratic program form:

$$\min_{\mathbf{U}_k} \frac{1}{2}\mathbf{U}_k^T H \mathbf{U}_k + \mathbf{f}^T\mathbf{U}_k$$
$$\text{s.t.} \quad F\mathbf{U}_k \le \mathbf{g}$$

It is sufficient to only use the terms from part ii) and ($\mathbf{Z}_k^{\min}, \mathbf{Z}_k^{\max}$)

d) For a system with the equations of motion $M(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{c}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}$, define a Quadratic Programming optimisation problem with decision variables $\ddot{\mathbf{q}}, \boldsymbol{\tau}$ to solve for the following objectives and constraints:

a. (objective) acceleration level motion tracking task for reference motion $\mathbf{x}_r(t), \dot{\mathbf{x}}_r(t), \ddot{\mathbf{x}}_r(t)$

b. (constraint) constraints on the joint space velocity $\mathbf{q}_{min} \le \mathbf{q} \le \mathbf{q}_{max}$. Hint: write the constraint with respect to $\ddot{\mathbf{q}}$

c. (constraint) constraints on the contact force at task position $\mathbf{x}_c = J_c\dot{\mathbf{q}}$ with friction coefficient μ.

e) For the task sequence below, write the set of tasks using a weight-optimisation approach for a whole-body controller in order to accomplish the task sequence. Note that you do not need to write the mathematical formulation for the tasks, but only to state the tasks, the type of task and the corresponding weighting.

End of Question 6 (Topics in Robotics) – MAEG 5090

Question 7 (Nanomaterials and Nanotechnology: Fundamentals and Applications) – MAEG 5120

Design a nanoscale linear servomotor based on telescoping and inter-layer effect of a multiwalled carbon nanotube (CNT). Explain the assembly process of the device and its working principle. (5%) Design a CNT-based scissors which can be applied for controlled "cutting" of a freestanding nanotube/nanowire. Explain the assembly process of the device and its working principle. (5%) Note: Add sketch if necessary.

End of Question 7 (Nanomaterials and Nanotechnology: Fundamentals and Applications) – MAEG 5120

Question 8 (Computational Mechanics) – MAEG 5130

1. For the 1D elastic problem with the strong form of:

$$\frac{d}{dx}\left(3\frac{du}{dx}\right) + x = 0, \qquad 2 \le x \le 4$$

$$u(4) = 0.05$$

$$\frac{du}{dx}\Big|_{x=2} = 0.5$$

   a. Derive the weak form.
   b. If the domain is discretized by two 2-node linear elements with the same length, approximate the displacement and strain fields.
   c. If the domain is discretized by one 3-node quadratic element with equally spaced nodes, approximate the displacement and strain fields.
   d. Compare the results in b and c with analytical solution. What is your conclusion?

End of Question 8 (Computational Mechanics) – MAEG 5130

Question 9 (Materials Characterization Techniques) – MAEG 5140

Construct ^1H and D (or ^2H) NMR spectra for CH_3CD_3 (D = ^2H). Hint: consider spin-spin coupling; spins for ^1H and D are 1/2 and 1, respectively .

End of Question 9 (Materials Characterization Techniques) – MAEG 5140

Question 10 (Advanced Heat Transfer and Fluid Mechanics) – MAEG 5150

Answer each question concisely. (10 marks)

1. On the E-S graph, draw the schematic to show the spontaneous change of state.

2. Recent research has indicated that freestanding graphene has a higher thermal conductivity than the graphene supported by a substrate. Explain the reason concisely.

3. Given the follow chemical reactions (ignore the backward reactions):

$$CH_4 + OH \xrightarrow{k_1} CH_3 + H_2O \quad \text{(Rxn. 1)}$$

$$CH_3 + OH \xrightarrow{k_2} CH_2O + H_2 \quad \text{(Rxn. 2)}$$

$$CH_2O \xrightarrow{k_3} CO + H_2 \quad \text{(Rxn. 3)}$$

(1) Write out rate expressions for d[CH_3]/dt, d[CH2O]/dtand d[H_2]/dt.

(2) Assuming Rxn. 3 is much faster than Rxn. 3. Which species is at the quasi-steady state?

(3) Simply your expression for d[H_2]/dt, in terms of [CH_3] and [OH].

4. To assess the velocity scale of the smallest eddy, the Kolmogorov velocity, V, can be used. It is related to the kinematic viscosity, υ, and the turbulence dissipation rate, ε. Derive the relationship between Kolmogorov velocity and viscosity and dissipation rate using dimensional analysis.

5. To derive the transport equations for Reynolds stresses, the following term needs to be derived:

$$\overline{u_i{}' \frac{\partial^2 u_j}{\partial x_k \partial x_k} + u_j{}' \frac{\partial^2 u_i}{\partial x_k \partial x_k}}$$

Use the following two terms to express this term.

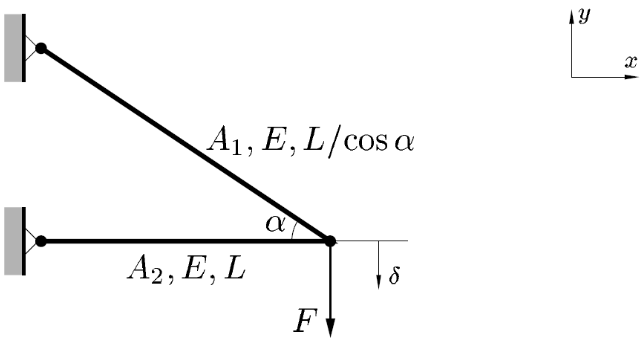
$$\frac{\partial^2 (\overline{u_i{}' u_j{}'})}{\partial x_k \partial x_k}, \quad \overline{\frac{\partial u_i{}'}{\partial x_k} \frac{\partial u_j{}'}{\partial x_k}}$$

End of Question 10 (Advanced Heat Transfer and Fluid Mechanics) – MAEG 5150

Question 11 (Design for Additive Manufacturing) – MAEG 5160

Category: This question relates to the topological optimization of the truss structures.

Main question: Consider the truss structures in the figure below. The bars have lengths according to the figure, and consist of a material with Young's modulus E and density ρ. The force F >0 and the angle α = 30°. We want to find the cross-sectional areas A_1 and A_2 such that the total weight of the whole structure is minimized subject to stress constraints and a constraint on the tip displacement δ.



Constraints to be noted:

1.The stress σ in the truss constraints are $|\sigma_i| \leq \sigma_0, \quad i = 1, 2,$ for a given stress bound $\sigma_0 > 0$.

2.The displacement constraint is $\delta \leq \delta_0,$ in the negative Y direction, where $\delta_0 = \frac{\sigma_0 L}{E}$ is a given bound on the tip displacement (please use hook's law to think how to get this equation).

Sub-questions (the order of the sub-questions also serves as the hint/approach to solving the problem) and marking scheme (total 10 marks):

(1) Free-body diagram and the balance of force equation in X and Y directions (2 marks)
(2) The elongation of two trusses and its relationship to the tip displacement δ (2 marks)
(3) Re-formulate the displacement constraint in terms A_1 and A_2, the force F and the stress σ in the truss (2 marks)
(4) Re-formulate the stress constraint in terms A_1 and A_2, the force F and the stress σ in the truss (1 mark)
(5) Express the total weight as the objective function of the optimization problem (1 mark)
(6) Write down the full expression of the optimization problem (1 mark) and get the result/answer for A_1 and A_2 in terms of the force F and the stress σ (1 mark)

End of Question 11 (Design for Additive Manufacturing) – MAEG 5160