The Chinese University of Hong Kong
Department of Mechanical and Automation Engineering

Candidacy Examination 2021-22

May 13, 2022 (Friday)
14:30- 17:00
via ZOOM Meeting

Time Allowed: Two and a Half Hours

1. This is a closed book/closed-note examination.

2. Answer **THREE** questions only.

3. Each question carries equal marks. (Full mark is 30.)

4. **Remember to write your student number, course code/question number, and page number clearly on each page of your answer sheets.**

DO NOT TURN OVER THE PAGE

UNTIL INSTRUCTED TO DO SO

< Remember to write your student number, course code/question number and page number clearly on each page of answer sheets. >

Question 1 (Advanced Robotics) – ENGG 5402

1. Analyse the kinematics and dynamics of the SCARA robot shown in Fig. 1.

a. (2%) Find out its *classic/standard* Denavit-Hartenberg parameters table. To simplify the problem, in Fig. 1, all coordinates are already provided.

b. (3%) Compute its direct kinematics. The task considered in this question is the end-effector position (P_x, P_y, P_z).

c. (2%) Solve its inverse kinematics based on the definition in question *b.*

d. (1.5%) Compute its Jacobian based on the definition in question *b* and *c*. Either analytical or geometric Jacobian is ok.

e. (1.5%) Find out its dynamic modal. For simplicity: link 3 and 4 can be considered as a unique rigid body (link) which translates along axis z_2 of the prismatic joint 3 and rotates about axis z_3 of the revolute joint 4. Let m_{l_3} denote the mass of this complete rigid body (link). It is also assumed that the centre of mass of this link is located on axis z_3. Use I_{l_4} for inertia tensor of this link relative to the centre of mass. Also assume that the motors of joint 3 and 4 have negligible mass and inertia. Students are also free to choose the symbols they prefer to help with the presentation. Hint: to compute inertia matrix, Christoffel symbols (for computing Centrifugal and Coriolis forces) of the model.

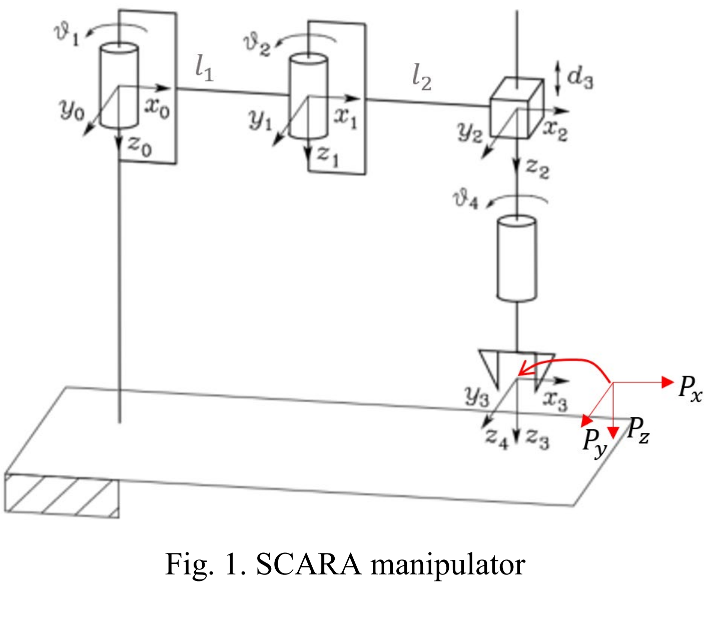


Fig. 1. SCARA manipulator

Note: for students who prefer to use *modified* Denavit-Hartenberg, they can redraw the figure with proper symbols but with clear explanation, and work out the same five sub-questions.

End of Question 1 (Advanced Robotics) – ENGG 5402

Consider a linear time-invariant system characterized by

$$\Sigma : \begin{cases} \dot{x} = A\,x + B\,u + E\,w \\ z = C_2\,x + D_2\,u, \end{cases}$$

where $C_2 = 0_{m\times n}$, $D_2 = I_m$, and where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $w \in \mathbb{R}^l$ and $z \in \mathbb{R}^m$, are the state, control input, disturbance input and controlled output, respectively. Assume that the state variable x is available for feedback, i.e., the measurement output $y = x$, and assume that (A, B) is stabilizable and (A, B, C_2, D_2) has no invariant zeros on the imaginary axis.

(a) [2 points] Show that the subsystem (A, B, C_2, D_2) has a total of n invariant zeros and are given by $\lambda(A)$, i.e., the eigenvalues of A.

(b) [2 points] Show that there exist an $n \times n$ nonsingular transformation T such that

$$\tilde{A} = T^{-1}AT = \begin{bmatrix} A_- & 0 \\ 0 & A_+ \end{bmatrix},$$

where $A_- \in \mathbb{R}^{n_-\times n_-}$ and $A_+ \in \mathbb{R}^{n_+\times n_+}$ are stable and unstable matrices, respectively.
Hint: You can use any mathematical result to answer this question so long as it is properly justified.

(c) [2 points] Let us define a state transformation $x = T\tilde{x}$, where T as given in Part (b). It is easy to verify that the given system Σ can be transformed into the following:

$$\begin{cases} \dot{\tilde{x}} = \begin{bmatrix} A_- & 0 \\ 0 & A_+ \end{bmatrix} \tilde{x} + \begin{bmatrix} B_- \\ B_+ \end{bmatrix} u + \begin{bmatrix} E_- \\ E_+ \end{bmatrix} w \\ z = \begin{bmatrix} 0 & 0 \end{bmatrix} \tilde{x} + I\;\; u, \end{cases}$$

where B_-, B_+, E_- and E_+ are respectively appropriate constant matrices. Show that (A, B) is stabilizable if and only if (A_+, B_+) is controllable.

(d) [3 points] Show that the solution to the corresponding H_2 Riccati equation for the transformed system in Part (c), if existent, can be partitioned as follows:

$$P = \begin{bmatrix} 0 & 0 \\ 0 & P_+ \end{bmatrix}, \quad P_+ > 0.$$

Find the H_2 optimal state feedback control law $u = F\tilde{x}$ for the transformed system in terms of P_+. Show that the resulting closed-loop system has poles at $\lambda(A_-)$ and $\lambda(-A_+)$.

(e) [1 point] Show that $\gamma_2^* = 0$, i.e., the disturbance can be totally rejected, if and only if $E_+ = 0$, i.e., the disturbance is not allowed to enter the unstable invariant zero subspace.

Note: If you are unable to answer the above problems in the general setting, you could use a specific numerical example to verify all the results in Parts (a) to (e). However, the maximum mark you could receive for this examination will be 5 instead of 10.

End of Question 2 (Linear System Theory and Design) – ENGG 5403

Question 3 (Micromachining and Microelectromechanical Systems) – ENGG 5404

1. As shown in the figure below, KOH is used to micro-fabricate a square diaphragm on a (100) silicon wafer (550 μm thick) with a desired membrane thickness of 8 μm and a size of 400 × 400 μm^2. The etch rate of KOH to silicon is 40 microns/hour.

(a) Assuming the wet etching process is uniform, estimate the required etch time.

(b) If the etching finished 2 minutes later than the correct timing calculated in (a), estimate the mechanical property (i.e., membrane stiffness and resonant frequency) variations in percentile. Use the equation of a clamped circular membrane to perform first order analysis, i.e., stiffness **$(k) \sim 192[\pi E h^3/12(1-v^2)]/r^2$**, where E is the modulus; v is the Poisson's ratio; h and r are the thickness and radius of the diaphragm. (E = 140 GPa for silicon)

(c) Draw the photomask for the process. List one suitable masking material for the KOH etching.

(d) If during the photolithography step, the mask is angularly misaligned for 3°, what will be the resulting diaphragm size, thickness. Based on this, estimate the resonant frequency of the membrane.

(e) Since KOH etching is not truly uniform, what additional processing steps will you include to improve the precision of the membrane thickness?

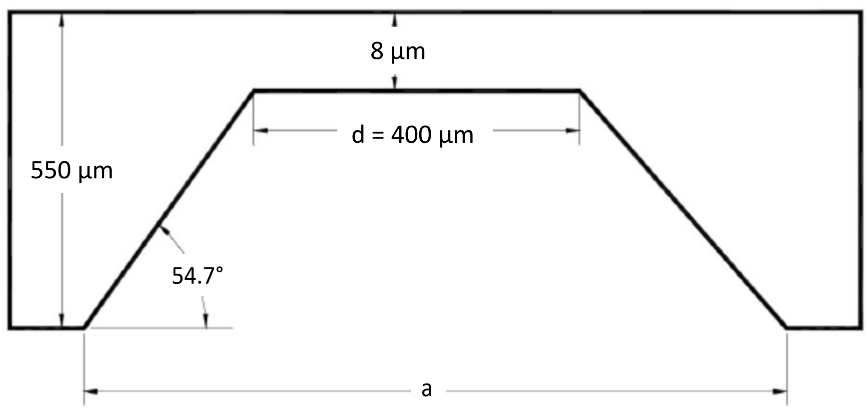


Figure. Cross-section diagram of the square diaphragm microfabricated by KOH.

End or Question 3 (Micromachining and Microelectromechanical Systems) – ENGG 5404

Question 4 (Nonlinear Control Systems) – MAEG 5070

Consider the following system

$$a_0 y^{(n)} + \sum_{i=1}^{m} a_i f_i(x,t) = u \tag{1}$$

where $x = [y\ \dot{y}\ \cdots\ y^{(n-1)}]^T$ is the state vector, f_i, $i = 1, \cdots, m$, are known, continuous and bounded for all x and t, and a_i, $i = 0, 1, \cdots, m$, are unknown constants. Given a desired output $y_d(t)$ which is smooth, let $e = y - y_d$ be the output tracking error. Let

$$s = e^{(n-1)} + \alpha_1 e^{(n-2)} + \cdots + \alpha_{n-1} e \tag{2}$$

where $\alpha_1,\ \cdots,\ \alpha_{n-1}$ are real numbers such that $\Delta(\lambda) = \lambda^{(n-1)} + \alpha_1 \lambda^{(n-2)} + \cdots + \alpha_{n-1}$ is a stable (Hurwitz) polynomial.

(i) Assume the sign of a_0 is known. Let $\hat{a}_i$ be the estimation of a_i, $i = 0, 1, \cdots, m$, and

$$f_0(x,t) = y_d^{(n)} - \alpha_1 e^{(n-1)} - \cdots - \alpha_{n-1}\dot{e} \tag{3}$$

Show that the following control law

$$u = \hat{a}_0 f_0(x,t) - ks + \sum_{i=1}^{m} \hat{a}_i f_i(x,t) \tag{4}$$

is such that

$$a_0 \dot{s} + ks \quad = \quad -\sum_{i=0}^{m} \tilde{a}_i f_i(x,t) \tag{5}$$

where k is a constant of the same sign as a_0 and $\tilde{a}_i = a_i - \hat{a}_i$, $i = 0, 1, \cdots, m$.

(ii) Choose the following adaptation law

$$\dot{\hat{a}}_i = -\gamma_i sgn(a_0)\ s\ f_i,\ \ i = 0, \cdots, m \tag{6}$$

where $\gamma_i > 0$. Let

$$V = \frac{1}{2} sgn(a_0) a_0 s^2 + \frac{1}{2}\sum_{i=0}^{m} \gamma_i^{-1} \tilde{a}_i^2 \tag{7}$$

Show that the derivative of V along the solution of the closed-loop system satisfies

$$\dot{V} = -|k| s^2 \tag{8}$$

(iii) Show that $s(t)$ tends to 0 as t goes to ∞.

(iv) Show that $e(t), \dot{e}(t), \cdots, e^{(n-1)}(t)$ all tend to 0 as t goes to ∞.

Hint: Consider the following system

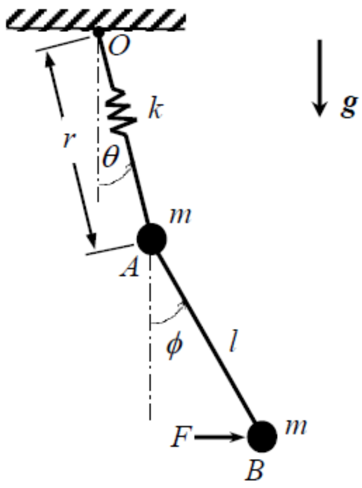
$$\dot{x} = Ax + u(t), \quad t \geq 0 \tag{9}$$

where $x \in \Re^n$, $A \in \Re^{n \times n}$ is Hurwitz, and $u \in \Re^n$ is continuous over $[0, \infty)$. Then, for any $x(0)$, the solution $x(t)$ of (9) is bounded if $u(t)$ is bounded, and $x(t)$ tends to 0 as t goes to ∞ if $u(t)$ tends to 0 as t goes to ∞.

End of Question 4 (Nonlinear Control Systems) – MAEG 5070

Question 5 (Smart Materials and Structures) – MAEG 5080

Two wheels A and B, each of mass m, are connected by a massless axle of length l. Each wheel is considered to have its mass concentrated as a particle at its hub. The hub of wheel A is attached by a spring of stiffness k and unstressed length r_o to a fixed point O. A horizontal force $F(t)$ is applied to wheel B. Using r, θ , and ϕ as generalized coordinates, obtain the differential equations of motion.



End of Question 5 (Smart Materials and Structures) – MAEG 5080

Question 6 (Topics in Robotics) – MAEG 5090

Part A: Modeling of a typical soft rotary origami joint.

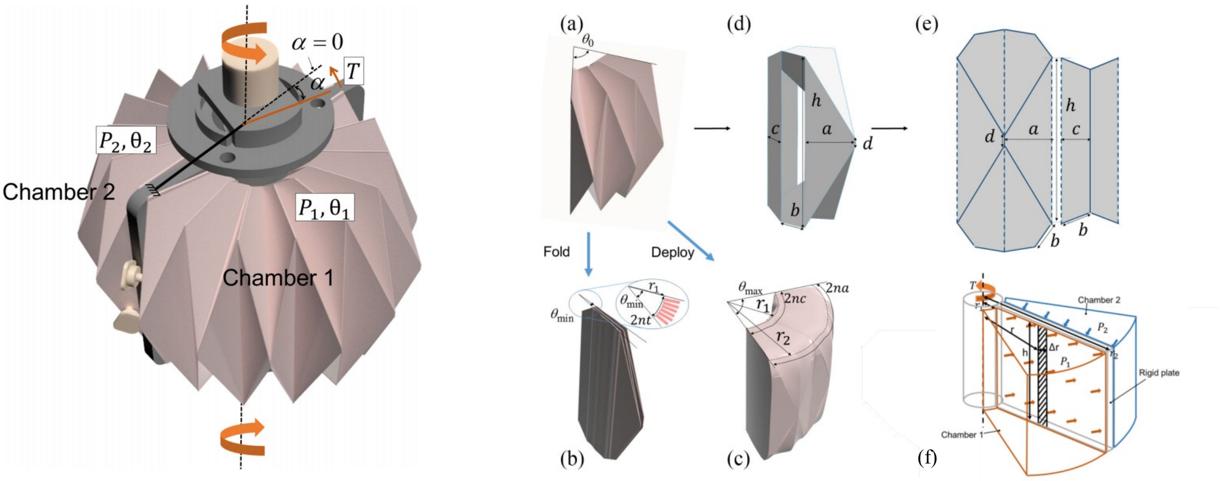


Fig. 1. The illustration of a classic rotary soft origami joint. (a) Joint neutral state. (b) Joint fully folding state. (c) Joint fully deployed state. (d) Geometrical parameters of one unit joint in neutral state. (e) Origami formation from the flat sheet.

Given a classic rotary soft origami joint as shown in Fig. 1, the rotary soft origami joint could achieve controllable bilateral rotation under dedicated pneumatic actuation. Please derive the model to illustrate the working principle of the soft origami rotary joint.

(1) The central angle of the origami chamber in the initial neutral state is θ_0, in a fully folded state under negative pneumatic actuation (vacuum) state is θ_{min}, in the maximum expanded state under positive pneumatic actuation is θ_{max}. r_1 and r_2 are the constants of the radius of the origami chamber as shown in Fig. 1. Please derive the calculation of (θ_{min}, θ_{max}, r_1, r_2) based on the geometrical design parameters (a, b, c, d, h, t, n) shown in Fig. 1.

(2) Using a cylindrical shape to simplify the inflated origami chamber, as shown in Fig 1(f), the T_1 and T_2 are the individual torque generated by chamber 1 and chamber 2 respectively. k is the inherent stiffness constant of the origami material. P_0 is the standard ambient air pressure. P_1 and P_2 are the actuation pressure for chamber1 and chamber2. h, r_1, r_2 are constants from the parameter design. Please derive the overall generated torque T of the origami joint.

(3) Please derive the joint stiffness K with the variables of $\partial P_1/\partial\alpha$ and $\partial P_2/\partial\alpha$.

(4) Assume the rotary origami joint is working in an adiabatic process, where $\frac{\partial P}{\partial V} = -\gamma\frac{P}{V}$ is provided, and γ is the specific heat ratio. Please further derive the expression of joint stiffness K based on the expansion of $\partial P_1/\partial\alpha$ and $\partial P_2/\partial\alpha$.

(5) Given M is the inertia matrix, Γ is the external load, F_f is the modeled friction, and $G = mgh\cos\alpha$ is the potential energy of mass m at a height of $h\cos\alpha$ in a gravitational field with constant g. Besides, the $\dot{m}$ is the mass flow rate, and $\dot{m}_{in}$ and $\dot{m}_{out}$ are the mass flow-in and mass flow-out rates through valves. The mass flow rate calculation is given as:

$$\dot{m} = \begin{cases} \dfrac{C_0 A C_1 P_u}{\sqrt{T_a}} & , \dfrac{P_d}{P_u} < 0.528 \\ \dfrac{C_0 A C_1 P_u}{\sqrt{T_a}} \left(\dfrac{P_d}{P_u}\right)^{\frac{1}{\gamma}} \sqrt{1 - \left(\dfrac{P_d}{P_u}\right)^{\frac{(\gamma-1)}{\gamma}}} & , \dfrac{P_d}{P_u} < 0.528 \end{cases}$$

where the discharge coefficient of the orifice C_0 is a dimensionless constant term in most engineering works, C_1 and C_2 are constants for a given fluid, and P_d and P_u are, respectively, the downstream and upstream pressures in the pneumatic circuit. Please derive Lagrange's equation of motion for the rotary joint and the dynamic air pressure for the joint chamber1 and chamber2.

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Part B: Summary of dominant variable stiffness approach for soft robots.
Please fill in the table based on the content we discussed in the variable stiffness topics of the course MAEG-5090.

Summary of variable stiffness approaches for soft robots				
Stiffness modulation approaches	Type of control	Principle	Materials/composing components	Robotic applications
1.				
2.				
3.				
4.				

End of Question 6 (Topics in Robotics) – MAEG 5090

(a) For any two quantum states, ρ_1, ρ_2, and any quantum channel, ξ, show that

$$F[\xi(\rho_1), \xi(\rho_2)] \geq F(\rho_1, \rho_2),$$

where $F(\rho_1, \rho_2)$ is the fidelity of two quantum states.

(b)Suppose we have a single qubit as the principal system which interacts with another single qubit as the environment through the transform

$$U = P_0 \otimes I + P_1 \otimes X$$

here X is the usual Pauli matrix (acting on the environment), and $P_0 \equiv |0\rangle\langle 0|, P_1 \equiv |1\rangle\langle 1|$ are projectors (acting on the principle system). Assume the initial state of the environment qubit is $|0\rangle$, write down the quantum channel acting on the principle system in terms of the Kraus operator representation.

End of Question 7 (Quantum Control & Quantum Information) – MAEG 5110

Question 8 (Nanomaterials and Nanotechnology: Fundamentals and Applications) – MAEG 5120

1.1 Electron microscopy is a powerful tool for sample inspection at nanoscale using high energy electrons as a radiation source. (i) Regarding the **interaction depth of the incident e-beam with a specimen**, compare the following three signals: (a) secondary electrons, (b) characteristic X-ray emission and (c) back-scattered electrons. (ii) Which signal has the highest sensitivity for sample inspection of its surface morphology, and which one is suitable for element analysis of a sample? Explain briefly why. (5 points)

1.2 List the pros and cons of using optical tweezer (OT) and magnetic tweezer (MT) for trapping and manipulation of micro-/nanoscale objects from the following three aspects (i) the range and precision of manipulation force (ii) flexibility and dexterity for manipulation (e.g. 2D/3D, possibility for batch process) (iii) the limitation of the manipulation objects (5 points)

End of Question 8 (Nanomaterials and Nanotechnology: Fundamentals and Applications) – MAEG 5120

Question 9 (Computational Mechanics) – MAEG 5130

A 2D weak form for heat conduction in a domain plotted in the figure below is:

Find smooth temperature field T with $T = 5$ on AB such that

$$3\int_{\Omega} (\nabla w)^T \cdot \nabla T d\Omega = -5\int_{\overrightarrow{BC}} w d\Gamma + \int_{\overrightarrow{CA}} w d\Gamma - 2\int_{\Omega} w d\Omega,$$

$\forall w$ that is smooth and vanishes on AB

Derive the strong form **using step-by-step calculation**.

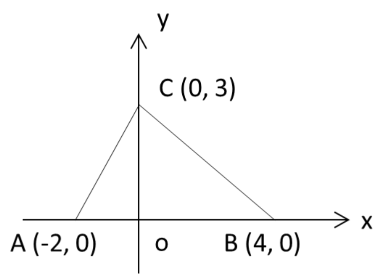
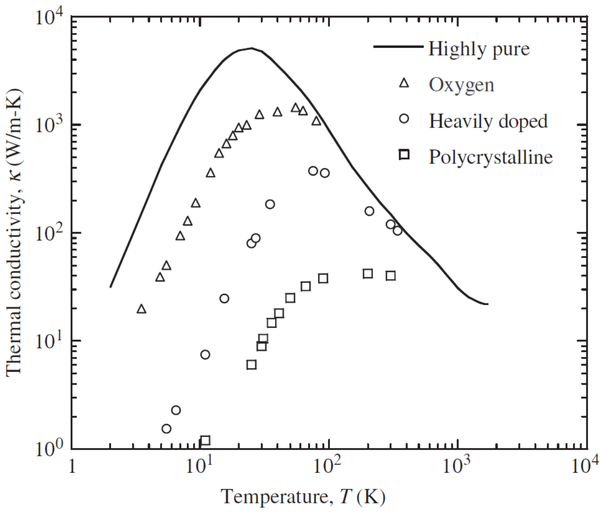


Figure. Domain for the problem

End of Question 9 (Computational Mechanics) – MAEG 5130

Answer each question concisely. (10 marks)

1. The following figure shows that different types of silicon could have very different thermal conductivity. Explain the possible reasons.



2. The following reaction pair (considering both forward and reverse reactions) occurs in methane (CH_4) combustion:

$$CH_4 + OH \underset{k_r}{\overset{k_f}{\leftrightarrow}} CH_3 + H_2O.$$

A student conducted experiments to successfully measure the rate coefficient of the forward reaction, which is given by the Arrhenius equation:

$$k_f\,(\text{cm}^3\ \text{mol}^{-1}\ \text{s}^{-1}) = 1.00\times10^6\ T^{2.182}\ \exp(-1231/T),$$

where T (K) is tempeature.

Determine the rate coefficient of the reverse reaction (k_r) at the temperature of 1500 K and pressure of 1 bar.

Gas constant: R_u = 8.314 J mol^{-1} K^{-1}. The thermodyanmic table is provided below.

Methane (CH_4) — $C_1H_4(g)$

Enthalpy Reference Temperature = T_r = 298.15 K; Standard State Pressure = p° = 0.1 MPa

	J·K⁻¹mol⁻¹			kJ·mol⁻¹			
T/K	C_p°	S°	$-[G^\circ-H^\circ(T_r)]/T$	$H^\circ-H^\circ(T_r)$	$\Delta_f H^\circ$	$\Delta_f G^\circ$	log K_f
0	0.	0.	INFINITE	−10.024	−66.911	−66.911	INFINITE
100	33.258	149.500	216.485	−6.698	−69.644	−64.353	33.615
200	33.473	172.577	189.418	−3.368	−72.027	−58.161	15.190
250	34.216	180.113	186.829	−1.679	−73.426	−54.536	11.395
298.15	35.639	186.251	186.251	0.	−74.873	−50.768	8.894
300	35.708	186.472	186.252	0.066	−74.929	−50.618	8.813
350	37.874	192.131	186.694	1.903	−76.461	−46.445	6.932
400	40.500	197.356	187.704	3.861	−77.969	−42.054	5.492
450	43.374	202.291	189.053	5.957	−79.422	−37.476	4.350
500	46.342	207.014	190.614	8.200	−80.802	−32.741	3.420
600	52.227	215.987	194.103	13.130	−83.308	−22.887	1.993
700	57.794	224.461	197.840	18.635	−85.452	−12.643	0.943
800	62.932	232.518	201.675	24.675	−87.238	−2.115	0.138
900	67.601	240.205	205.532	31.205	−88.692	8.616	−0.500
1000	71.795	247.549	209.370	38.179	−89.849	19.492	−1.018
1100	75.529	254.570	213.162	45.549	−90.750	30.472	−1.447
1200	78.833	261.287	216.895	53.270	−91.437	41.524	−1.807
1300	81.744	267.714	220.558	61.302	−91.945	52.626	−2.115
1400	84.305	273.868	224.148	69.608	−92.308	63.761	−2.379
1500	86.556	279.763	227.660	78.153	−92.553	74.918	−2.609

Methyl (CH_3) — $C_1H_3(g)$

Enthalpy Reference Temperature = T_r = 298.15 K; Standard State Pressure = p° = 0.1 MPa

	J·K⁻¹mol⁻¹			kJ·mol⁻¹			
T/K	C_p°	S°	$-[G^\circ-H^\circ(T_r)]/T$	$H^\circ-H^\circ(T_r)$	$\Delta_f H^\circ$	$\Delta_f G^\circ$	log K_f
0	0.	0.	INFINITE	−10.407	149.031	149.031	INFINITE
100	33.395	155.665	226.465	−7.080	147.800	147.438	−77.014
200	35.638	179.380	197.609	−3.646	146.868	147.432	−38.505
250	37.162	187.495	194.799	−1.826	146.297	147.637	−30.847
298.15	38.693	194.170	194.170	0.	145.687	147.950	−25.920
300	38.753	194.410	194.171	0.072	145.663	147.964	−25.763
350	40.392	200.506	194.649	2.050	144.997	148.400	−22.147
400	42.041	206.008	195.730	4.111	144.320	148.932	−19.449
450	43.668	211.054	197.156	6.254	143.644	149.549	−17.359
500	45.252	215.737	198.783	8.477	142.976	150.241	−15.696
600	48.288	224.258	202.333	13.155	141.682	151.817	−13.217
700	51.174	231.921	206.021	18.130	140.477	153.602	−11.462
800	53.926	238.935	209.703	23.386	139.383	155.554	−10.157
900	56.527	245.438	213.316	28.910	138.411	157.635	−9.149
1000	58.954	251.521	216.836	34.686	137.557	159.817	−8.348
1100	61.187	257.247	220.252	40.694	136.814	162.080	−7.697
1200	63.217	262.659	223.562	46.916	136.167	164.406	−7.156
1300	65.048	267.793	226.769	53.331	135.602	166.783	−6.701
1400	66.690	272.675	229.875	59.919	135.105	169.201	−6.313
1500	68.156	277.327	232.885	66.663	134.662	171.652	−5.977

Hydroxyl (OH) — $H_1O_1(g)$

Enthalpy Reference Temperature = T_r = 298.15 K; Standard State Pressure = p° = 0.1 MPa

	J·K⁻¹mol⁻¹			kJ·mol⁻¹			
T/K	C_p°	S°	$-[G^\circ-H^\circ(T_r)]/T$	$H^\circ-H^\circ(T_r)$	$\Delta_f H^\circ$	$\Delta_f G^\circ$	log K_f
0	0.	0.	INFINITE	−9.172	38.390	38.390	INFINITE
100	32.627	149.590	210.980	−6.139	38.471	37.214	−19.438
200	30.777	171.592	186.471	−2.976	38.832	35.803	−9.351
250	30.283	178.402	184.204	−1.450	38.930	35.033	−7.320
298.15	29.986	183.708	183.708	0.	38.987	34.277	−6.005
300	29.977	183.894	183.709	0.055	38.988	34.248	−5.963
350	29.780	188.499	184.073	1.549	39.019	33.455	−4.993
400	29.650	192.466	184.880	3.035	39.029	32.660	−4.265
450	29.567	195.954	185.921	4.515	39.020	31.864	−3.699
500	29.521	199.066	187.082	5.992	38.995	31.070	−3.246
600	29.527	204.447	189.542	8.943	38.902	29.493	−2.568
700	29.663	209.007	192.005	11.902	38.764	27.935	−2.085
800	29.917	212.983	194.384	14.880	38.598	26.399	−1.724
900	30.264	216.526	196.651	17.888	38.416	24.884	−1.444
1000	30.676	219.736	198.801	20.935	38.230	23.391	−1.222
1100	31.124	222.680	200.840	24.024	38.046	21.916	−1.041
1200	31.586	225.408	202.775	27.160	37.867	20.458	−0.891
1300	32.046	227.955	204.615	30.342	37.697	19.014	−0.764
1400	32.492	230.346	206.368	33.569	37.535	17.583	−0.656
1500	32.917	232.602	208.043	36.839	37.381	16.163	−0.563

Cont'd/……

Water (H_2O) **$H_2O_1(g)$**

Enthalpy Reference Temperature = T_r = 298.15 K   Standard State Pressure = $p°$ = 0.1 MPa

T/K	$C_p°$ (J·K⁻¹mol⁻¹)	$S°$ (J·K⁻¹mol⁻¹)	$-[G°-H°(T_r)]/T$ (J·K⁻¹mol⁻¹)	$H°-H°(T_r)$ (kJ·mol⁻¹)	$\Delta_f H°$ (kJ·mol⁻¹)	$\Delta_f G°$ (kJ·mol⁻¹)	log K_f
0	0.	0.	INFINITE	−9.904	−238.921	−238.921	INFINITE
100	33.299	152.388	218.534	−6.615	−240.083	−236.584	123.579
200	33.349	175.485	191.896	−3.282	−240.900	−232.766	60.792
298.15	33.590	188.834	188.834	0.	−241.826	−228.582	40.047
300	33.596	189.042	188.835	0.062	−241.844	−228.500	39.785
400	34.262	198.788	190.159	3.452	−242.846	−223.901	29.238
500	35.226	206.534	192.685	6.925	−243.826	−219.051	22.884
600	36.325	213.052	195.550	10.501	−244.758	−214.007	18.631
700	37.495	218.739	198.465	14.192	−245.632	−208.812	15.582
800	38.721	223.825	201.322	18.002	−246.443	−203.496	13.287
900	39.987	228.459	204.084	21.938	−247.185	−198.083	11.496
1000	41.268	232.738	206.738	26.000	−247.857	−192.590	10.060
1100	42.536	236.731	209.285	30.191	−248.460	−187.033	8.881
1200	43.768	240.485	211.730	34.506	−248.997	−181.425	7.897
1300	44.945	244.035	214.080	38.942	−249.473	−175.774	7.063
1400	46.054	247.407	216.341	43.493	−249.894	−170.089	6.346
1500	47.090	250.620	218.520	48.151	−250.265	−164.376	5.724

3. To assess the velocity scale of the smallest eddy, the Kolmogorov velocity, V, can be used. It is related to the kinematic viscosity, υ, and the turbulence dissipation rate, ε. Derive the relationship between Kolmogorov velocity and viscosity and dissipation rate using dimensional analysis.

4. To derive the momentum equation for Reynolds-Averaged Navier-Stokes (RANS), the following term needs to be derived:

$$\overline{u_j \frac{\partial u_i}{\partial x_j}}$$

Use the following two terms to express the above term. Please write down the detailed derivation procedure to receive the marks.

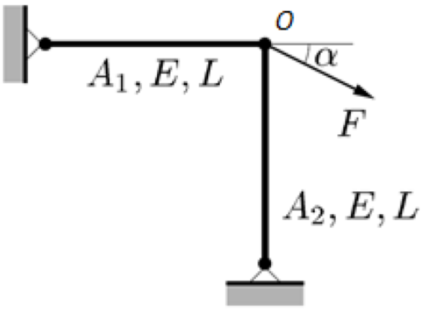
$$\frac{\partial^2 (\overline{u_i' u_j'})}{\partial x_j} \quad , \quad \overline{u_j} \frac{\partial \overline{u_i}}{\partial x_j}$$

End of Question 10 (Advanced Heat Transfer and Fluid Mechanics) – MAEG 5150

Question 11 (Design for Additive Manufacturing) – MAEG 5160

Category: This question relates to the size optimization of the truss structures.

Main question Q: Consider a two-bar truss structure consisting of bars of length L and Young's modulus E, placed at right angle perpendicular to each other according to figure below. The force $F > 0$ is applied at an angle $\alpha = 45°$. The problem is to find the circular cross-sectional areas A_1 and A_2 such that the weight of the truss is minimized under constraints on stresses and Euler buckling. ρ is the density of the truss material.



Sub-questions (the order of the sub-questions also serves as the hint/approach to solving the problem, i.e. the answer of sub-Q1 could be the input for sub-Q2) and marking scheme (total 10 marks):

Sub-Q1 (3 points): The stress σ in the truss constraints are $|\sigma_i| \le \sigma_0, \quad i = 1, 2,$ for a given stress bound $\sigma 0 > 0$. Please draw the free-body diagram for point O (1 point) and express the stress constraints to be imposed in the optimization problem in terms of circular cross-sectional areas A1 and A2. (2 points).

Sub-Q2 (3 points): Concerning instability, which (of Bar 1 and Bar 2) bar may have buckling? (1 point). If we want to obtain a safety factor of 4 against Euler buckling, what is the buckling load for a hinged-hinged column? (1 point) Based on this, what is the constraint on the bar cross-section? (1 point)

Sub-Q3 (2 points): Given the answers above, what is the optimization problem expressed in equations?

Sub-Q4 (2 points): Given the case/values of $\sigma_0 = \frac{E}{100}, \quad \sqrt{\frac{F}{\sigma_0}} = \frac{L}{4}.$, what will be the optimal cross-section area values A_1 and A_2 in terms of L?

End of Question 11 (Design for Additive Manufacturing) – MAEG 5160