Advanced Robotics

ENGG5402 Spring 2023

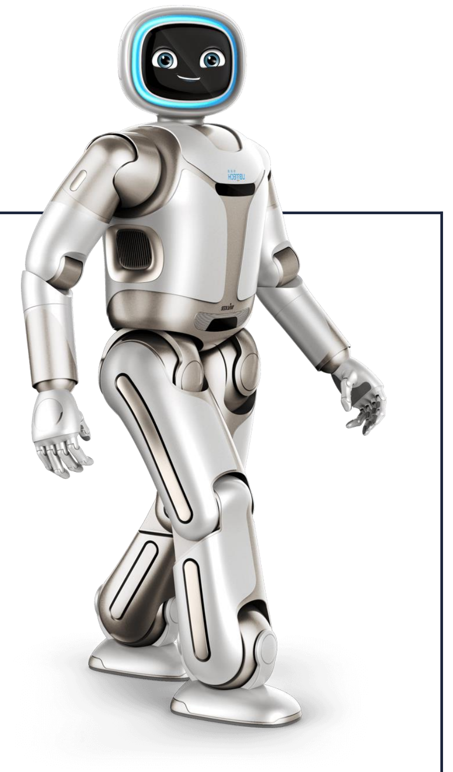
Fei Chen

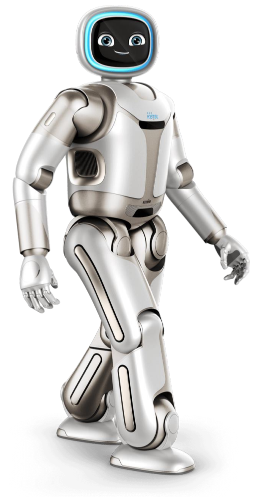
Topics:

- Interative Learning for Gravity Compensation

Readings:

- Siciliano: Sec. 8





Goal

- regulation of arbitrary equilibrium configurations in the presence of gravity
  - without explicit knowledge of robot dynamic coefficients (nor of the structure of the gravity term)
  - without the need of "high" position gain
  - without complex conditions on the control gains
- based on an iterative control scheme that uses
  1. PD control on joint position error + constant feedforward term
  2. iterative update of the feedforward term at successive steady- state conditions
- derive sufficient conditions for the global convergence of the iterative scheme with zero final error

Preliminaries

- robot dynamic model

$$M(q)\ddot{q} + c(q,\dot{q}) + g(q) = u$$

- available bound on the gradient of the gravity term

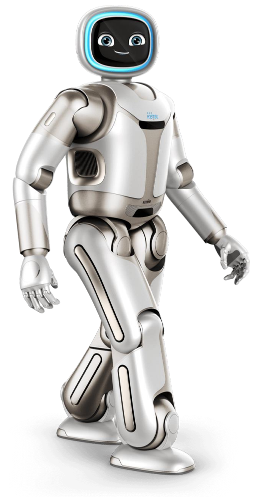
$$\left\|\frac{\partial g(q)}{\partial q}\right\| \leq \alpha$$

- regulation attempted with a joint-based PD law (without gravity cancellation nor compensation)

$$u = K_P(q_d - q) - K_D\dot{q} \quad K_P > 0, K_D > 0$$

- at steady state, there is a non-zero error left

$$q = \bar{q}, \dot{q} = 0 \quad g(\bar{q}) = K_P(q_d - \bar{q}) \quad \bar{e} = q_d - \bar{q} \neq 0$$



Iterative Control Scheme

- control law at the i-th iteration (for $i = 1, 2, \ldots$)

$$u = \gamma K_P(q_d - q) - K_D\dot{q} + u_{i-1} \quad \gamma > 0$$

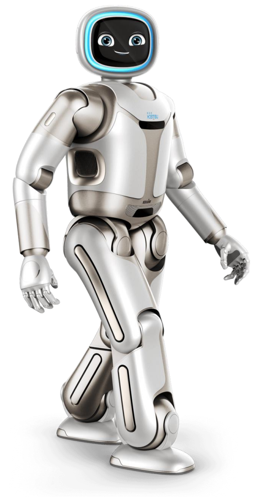
with a constant compensation term (feedforward)

  - $K_P > 0, K_D > 0$ are chosen diagonal for simplicity
  - q_0 is the initial robot configuration
  - $u_0 = 0$ is the 'easiest' initialization of the feedforward term
  - at the steady state of the i-th iteration ($q = q_i, \dot{q} = 0$), one has

$$g(q_i) = \gamma K_P(q_d - q_i) + u_{i-1}$$

- update law of the compensation term (for next iteration)

$$u_i = \gamma K_P(q_d - q_i) + u_{i-1} \quad [= g(q_i)]$$



Convergence Analysis

Theorem

$$(a)\lambda_{\min}(K_P) > \alpha$$
$$(b)\gamma \geq 2$$

guarantee that the sequence $\{q_0, q_1, q_2, \ldots\}$ converges to q_d (and $\dot{q} = 0$) from any initial value q_0 (and $\dot{q}_0$), i.e., globally

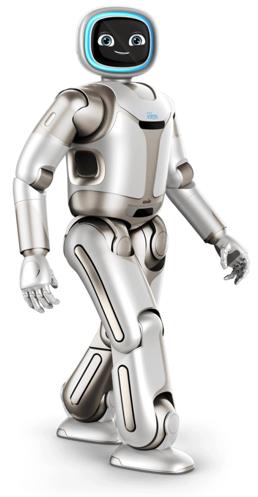
- condition (a) is sufficient for the global asymptotic stability of the desired equilibrium state when using

$$u = K_P(q_d - q) - K_D\dot{q} + g(q_d)$$

with a known gravity term and diagonal gain matrices

- the additional sufficient condition (b) guarantees the convergence of the iterative scheme, yielding

$$\lim_{i \to \infty} \|u_i\| = g(q_d)$$



Proof

- let $e_i = q_d - q_i$ be the error at the end of the i-th iteration; based on the update law, it is $u_i = g(q_i)$ and thus

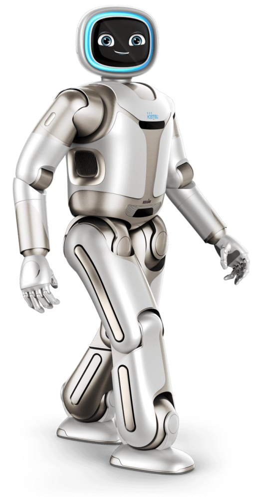
$$\|u_i - u_{i-1}\| = \|g(q_i) - g(q_{i-1})\| \leq \alpha\|q_i - q_{i-1}\|$$
$$\leq \alpha(\|e_i\| + \|e_{i-1}\|)$$

- on the other hand, from the update law it is

$$\|u_i - u_{i-1}\| = \gamma\|K_P e_i\|$$

- combining the two above relations under (a), we have

$$\gamma\alpha\|e_i\| < \gamma\lambda_{\min}(K_P)\|e_i\| < \gamma\|K_P e_i\| < \alpha(\|e_i\| + \|e_{i-1}\|)$$
$$\text{or } \|e_i\| < \frac{1}{\gamma}(\|e_i\| + \|e_{i-1}\|)$$



Proof

- condition (b) guarantees that the error sequence $\{e_0, e_1, e_2, \dots\}$

$$\|e_i\| < \frac{\frac{1}{\gamma}}{1 - \frac{1}{\gamma}} \|e_{i-1}\| = \frac{1}{\gamma - 1} \|e_{i-1}\|$$

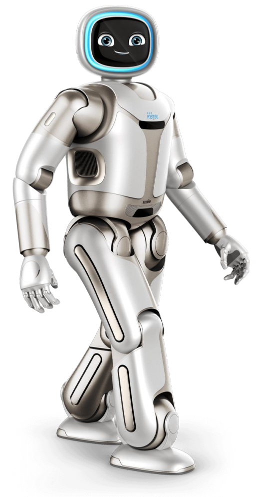
  is a contraction mapping, so that

$$\lim_{i \to \infty} \|e_i\| = 0$$

  with asymptotic convergence from any initial state

⇒ the robot progressively approaches the desired configuration through successive steady-state conditions

- K_P and K_D affect each transient phase
- coefficient γ drives the convergence rate of intermediate steady states to the final one



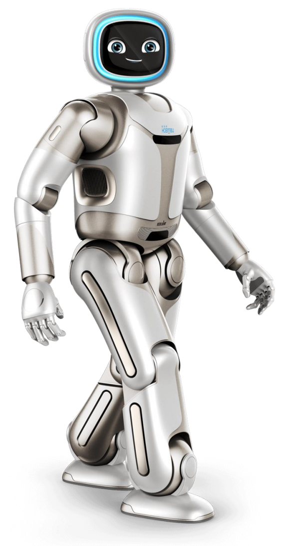
Basic Definitions

Remarks

- combining (a) and (b), the sufficient condition only requires the doubling of the proportional gain w.r.t. the known gravity case

$$\hat{K}_P = \gamma K_P \quad \Rightarrow \quad \boxed{\lambda_{\min}(\hat{K}_P) > 2\alpha}$$

- for a diagonal $\hat{K}_P$,this condition implies a (positive) lower bound on the single diagonal elements of the matrix
- again, it is only a sufficient condition
  - the scheme may converge even if this condition is violated ...
- the scheme can be interpreted as using an integral term
  - updated only in correspondence of a discrete sequence of time instants
  - with guaranteed global convergence (and implicit stability)



Example

Numerical results

- 3R robot with uniform links, moving in the vertical plane

$$l_1 = l_2 = l_3 = 0.5[m]$$
$$m_1 = 30, m_2 = 20, m_3 = 10[kg]$$

➡ $\alpha \cong 400$

- with saturations of the actuating torques

$$U_{1,max} = 800, U_{1,max} = 400, U_{1,max} = 200$$

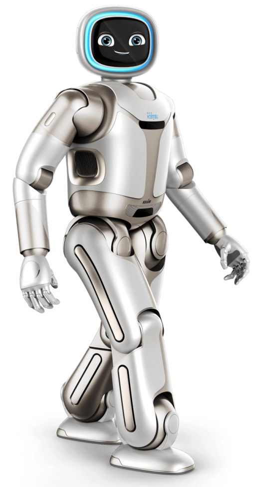
- three cases, from the downward position $q_0 = (0, 0, 0)$

I: $q_d = (\pi/2,0,0)$
II: $q_d = (3\pi/4,0,0)$

$$\begin{cases} \hat{K}_P = \text{diag}\{1000,600,280\} \\ K_D = \text{diag}\{200,100,20\} \end{cases}$$

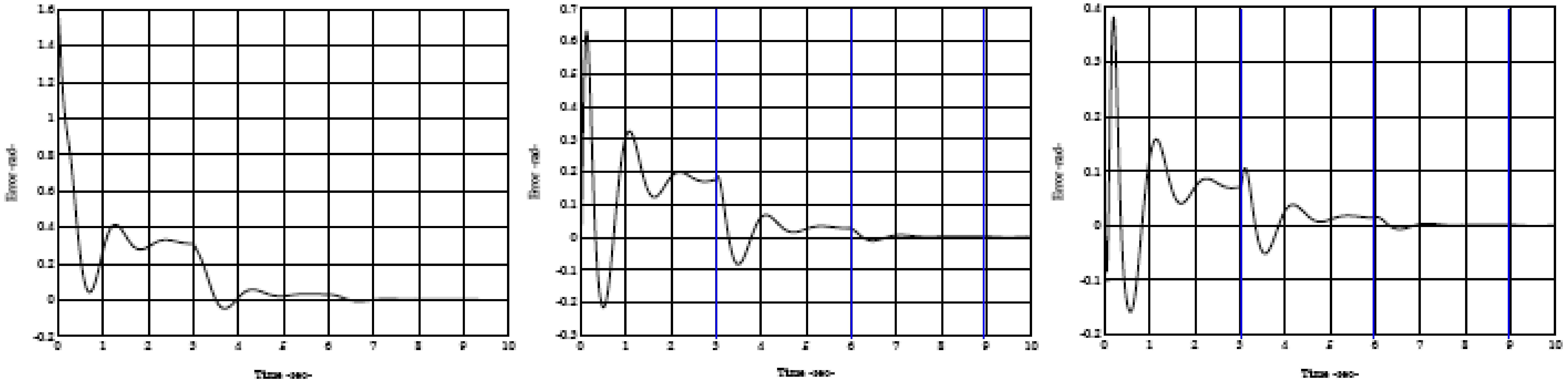
III: $q_d = (3\pi/4,0,0)$

$$\begin{cases} \hat{K}_P = \text{diag}\{500,500,500\} \\ K_D = \text{as before} \end{cases}$$

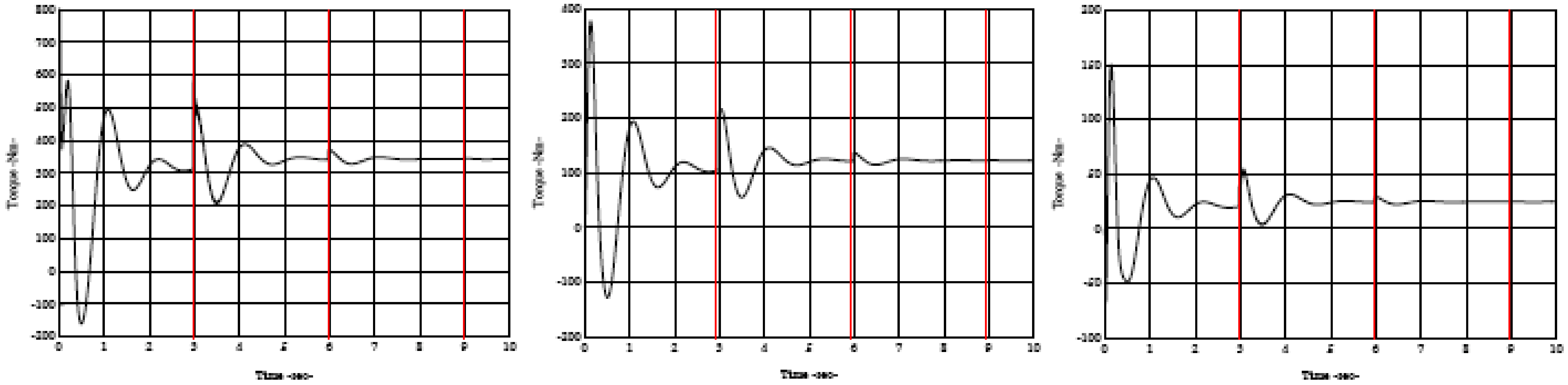


Example

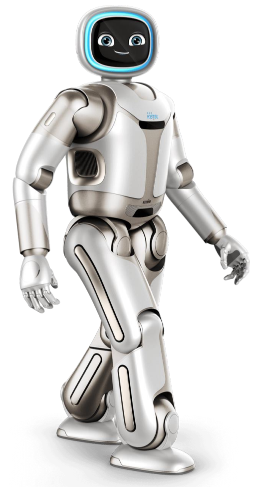
Case I: $q_d = (\frac{\pi}{2}, 0, 0)$



joint position errors (zero after 3 updates)

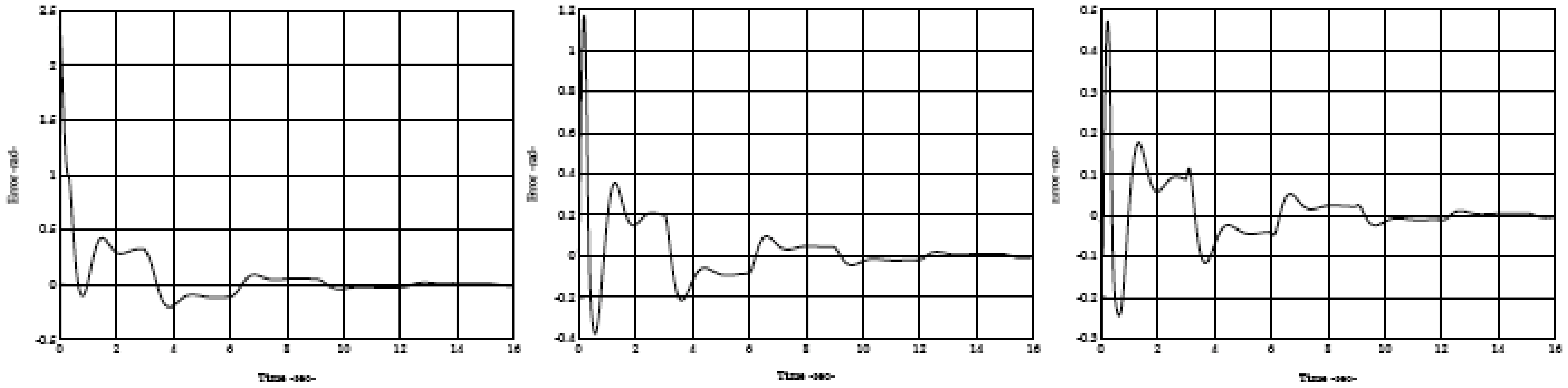


control torques

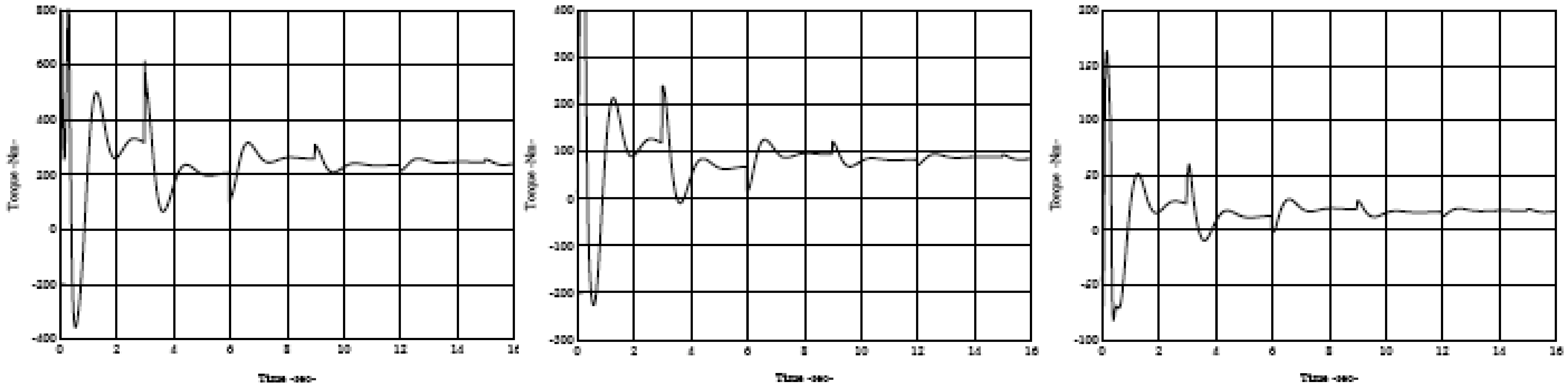


Basic Definitions

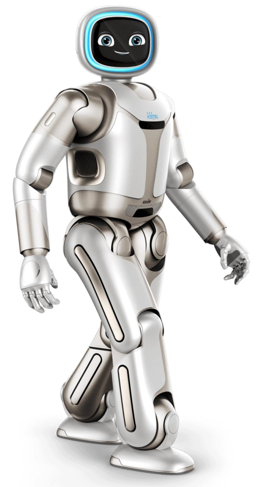
Case II: $q_d = (\frac{3\pi}{4}, 0, 0)$



joint position errors (zero after 5 updates)

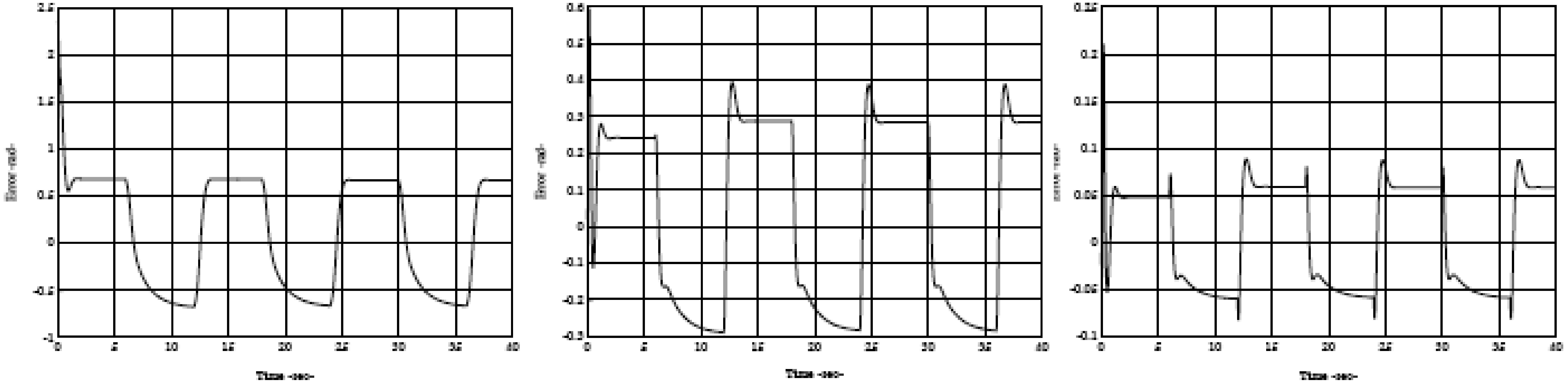


control torques

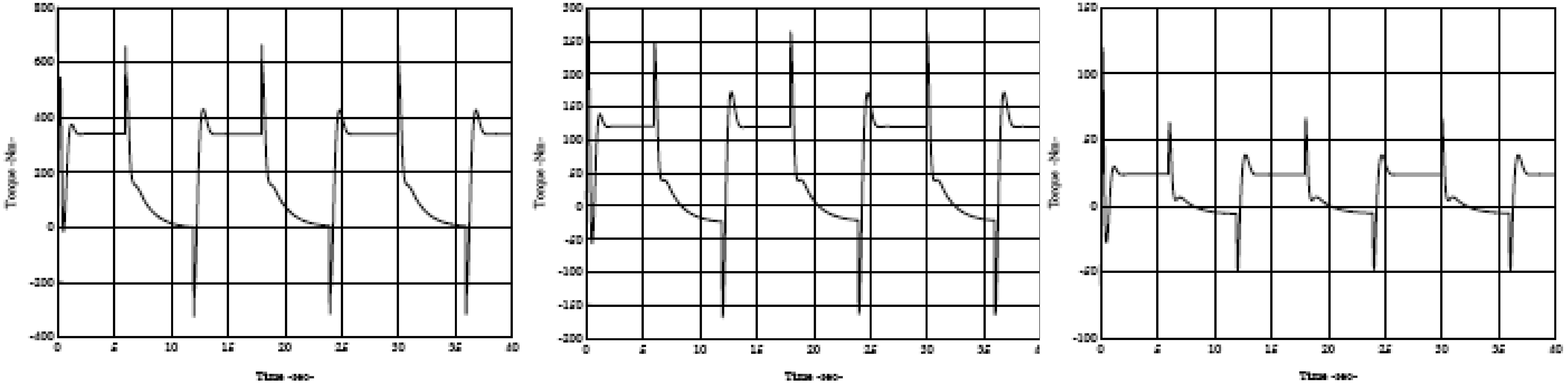


Basic Definitions

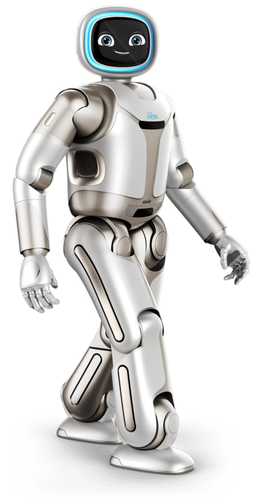
Case III: $q_d = (\frac{3\pi}{4}, 0, 0)$, reduced gains



joint position errors (limit cycles, no convergence!)

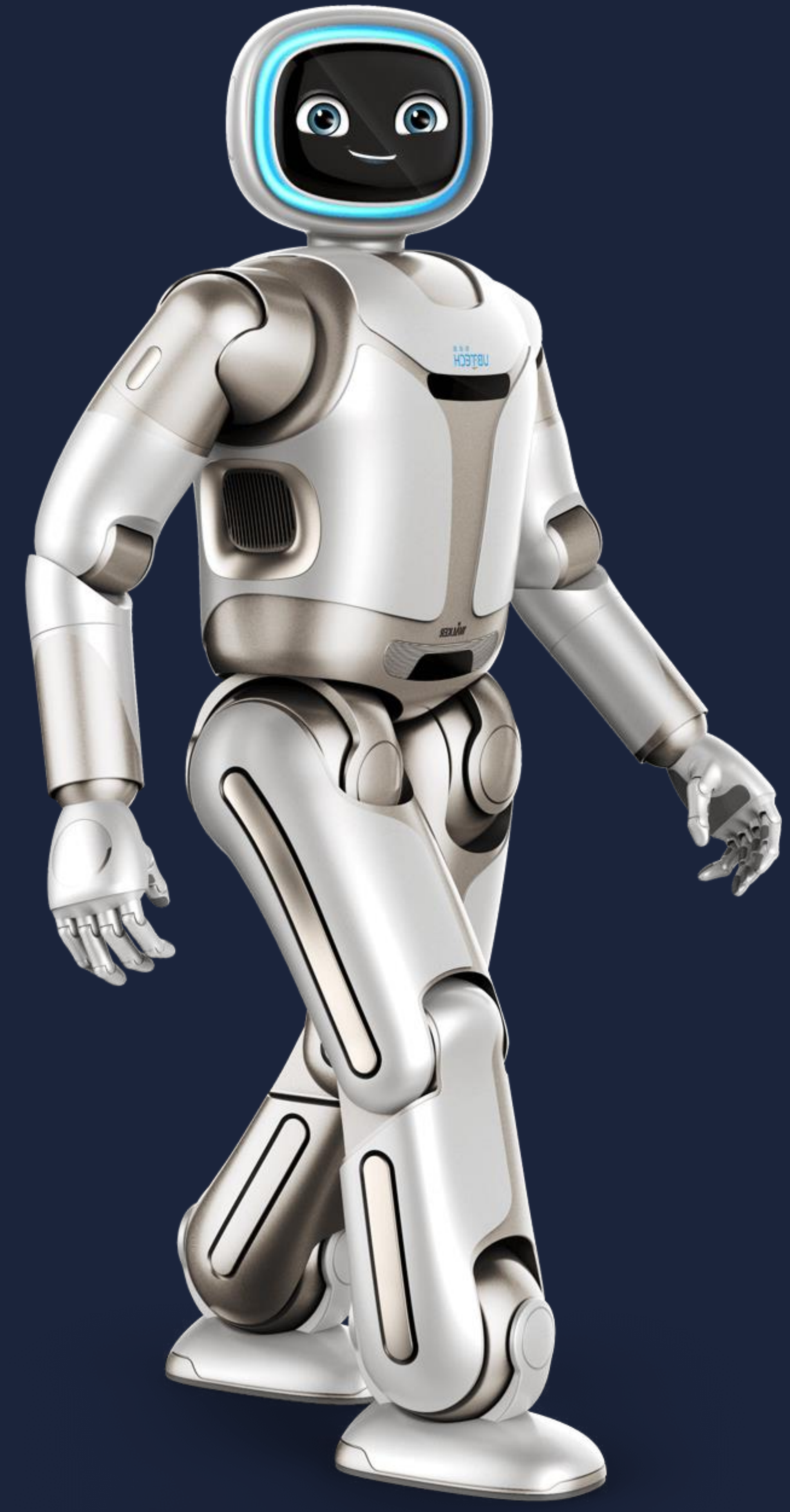


control torques



Final Comments

- only few iterations are needed for obtaining convergence, learning the correct gravity compensation at the desired q_d
- sufficiency of the condition on the P gain
  - even if violated, convergence can still be obtained (first two cases); otherwise, a limit motion cycle takes place between two equilibrium configurations that are both incorrect (as in the third case)
  - this shows how ‘distant’ is sufficiency from necessity
- analysis can be refined to get lower bounds on the K_{Pi} (diagonal case) that are smaller, but still sufficient for convergence
  - intuitively, lower values for K_{Pi} should be sufficient for distal joints
- in practice, update of the feedforward term occurs when the robot is close enough to a steady state (joint velocities and position variations are below suitable thresholds)



Q&A