Advanced Robotics

ENGG5402 Spring 2023

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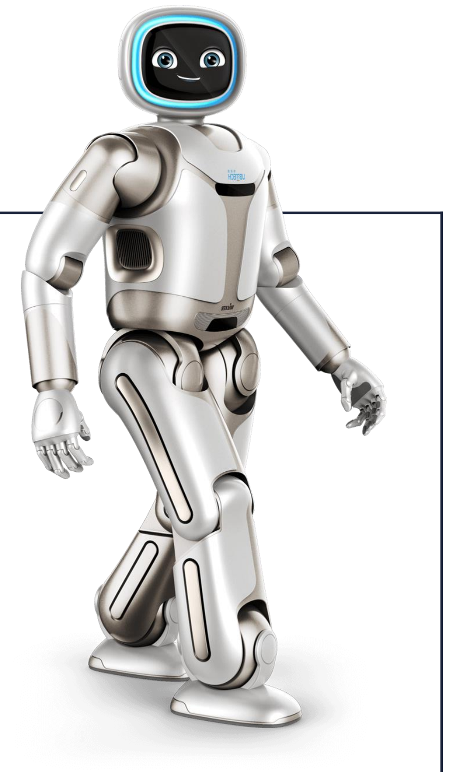
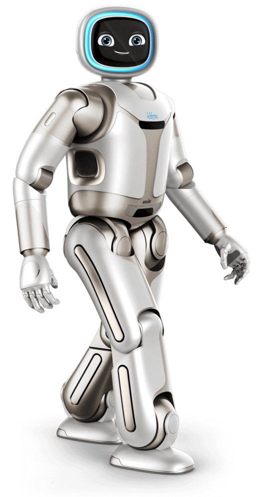


Topics:

- Adaptive Trajectory Control

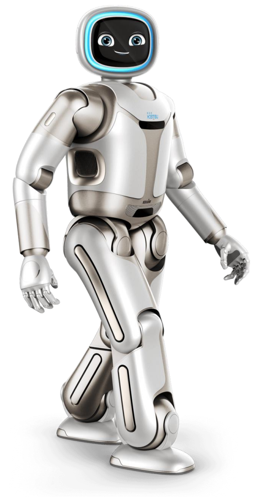
Readings:

- Siciliano: Sec. 8.5



Motivation and Approach

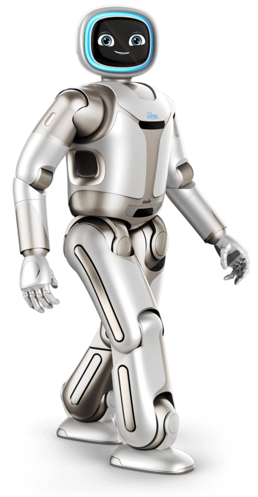
- need of adaptation in robot motion control laws
  - large uncertainty on the robot dynamic parameters
  - poor knowledge of the inertial payload
- characteristics of direct adaptive control
  - direct aim is to bring to zero the state trajectory error, i.e., position and velocity errors
  - no need to estimate on line the true values of the dynamic coefficients of the robot (as opposed to indirect adaptive control)
- main tool and methodology
  - linear parametrization of robot dynamics
  - nonlinear control law of the dynamic type (the controller has its own ‘states’)



Summary of Robot Parameters

- parameters assumed to be known
  - kinematic description based, e.g., on Denavit-Hartenberg parameters ($\{\alpha_i, d_i, a_i, i = 1, \dots, \mathrm{N}\}$ in case of all revolute joints), including link lengths (kinematic calibration)
- uncertain parameters that can be identified off line $\Rightarrow p \ll 10 \times N$
  - masses m_i, positions r_{ci} of CoMs, and inertia matrices I_i of each link,
  - appearing in combinations (dynamic coefficients)
- parameters that are (slowly) varying during operation
  - viscous F_{Vi}, dry F_{Di}, and stiction F_{Si} friction at each joint $\Rightarrow 1 - 3 \times N$
- unknown and abruptly changing parameters
  - mass, CoM, inertia matrix of the payload w.r.t. the tool center point

when a payload is firmly attached to the robot E-E, only the 10 parameters of the last link are modified, influencing however most part of the robot dynamics

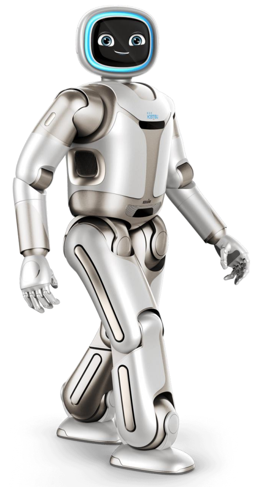


Goal of Adaptive Control

- given a twice-differentiable desired joint trajectory $q_d(t)$
  - with known desired velocity $\dot{q}_d(t)$ and acceleration $\ddot{q}_d(t)$
  - possibly obtained by kinematic inversion + joint interpolation
- execute this trajectory under large dynamic uncertainties
  - with a trajectory tracking error vanishing asymptotically

$$e = q_d - q \rightarrow 0 \quad \dot{e} = \dot{q}_d - \dot{q} \rightarrow 0$$

  - guaranteeing global stability, no matter how far are the initial estimates of the unknown/uncertain parameters from their true values and how large is the initial trajectory error
- identification is not of particular concern: in general, the estimates of dynamic coefficients will not to converge to the true ones!
- if this convergence is a specific extra requirement, then one should use (more complex) indirect adaptive schemes



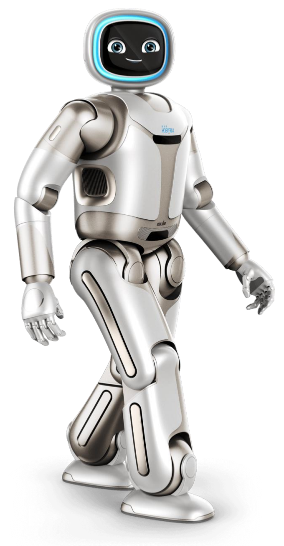
Linear Parameterization

$$M(q)\ddot{q} + S(q,\dot{q})\dot{q} + g(q) + F_V\dot{q} = u$$

- there exists always a (p-dimensional) vector a of dynamic coefficients, so that the robot model takes the linear form

$$Y(q,\dot{q},\ddot{q})a = u$$

- vector a contains only unknown or uncertain coefficients
- each component of a is in general a combination of the robot physical parameters (not necessarily all of them)
- the model regression matrix Y depends linearly on $\ddot{q}$, quadratically on $\dot{q}$ (for the terms related to kinetic energy), and nonlinearly (trigonometrically) on q



Controllers

Trajectory controllers (based on model estimates)

- inverse dynamics feedforward (FFW) + PD (linear) control

$$u = \underbrace{\widehat{M}(q_d)\ddot{q}_d + \hat{S}(q_d, \dot{q}_d)\dot{q}_d + \hat{g}(q_d) + \hat{F}_V\dot{q}_d}_{\hat{u}_d} + K_P e + K_D \dot{e}$$

- (nonlinear) control based on feedback linearization (FBL)

$$u = \widehat{M}(q)(\ddot{q}_d + K_P e + K_D \dot{e}) + \hat{S}(q, \dot{q})\dot{q} + \hat{g}(q) + \hat{F}_V\dot{q}$$

$$\boxed{\widehat{M}, \hat{S}, \hat{g}, \hat{F}_V \quad \Leftrightarrow \quad \text{estimate } \hat{a}}$$

- approximate estimates of dynamic coefficients may lead to instability with FBL due to temporary 'non-positive' PD gains (e.g., $\widehat{M}(q)K_P < 0$!)
- not easy to turn these laws in adaptive schemes: inertia inversion/use of acceleration (FBL); bounds on PD gains (FFW)

Controllers

A control law easily made 'adaptive'

- nonlinear trajectory tracking control (without cancellations) having global asymptotic stabilization properties

$$u = \widehat{M}(q)\ddot{q}_d + \hat{S}(q,\dot{q})\dot{q}_d + \hat{g}(q) + \hat{F}_V\dot{q}_d + K_P e + K_D\dot{e}$$

- a natural adaptive version would require ...

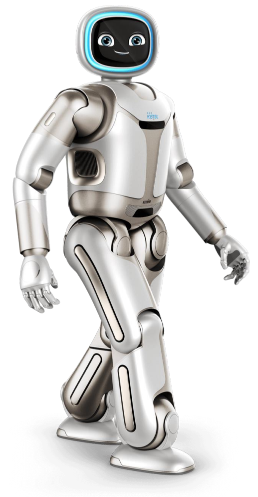
$\dot{\hat{a}} =$ designing a suitable update law (in continuous time)

  - without extra assumptions, it can be shown only that joint velocities become eventually "clamped" to those of the desired trajectory (zero velocity error), but a permanent residual position error is left

- idea: on-line modification with a reference velocity

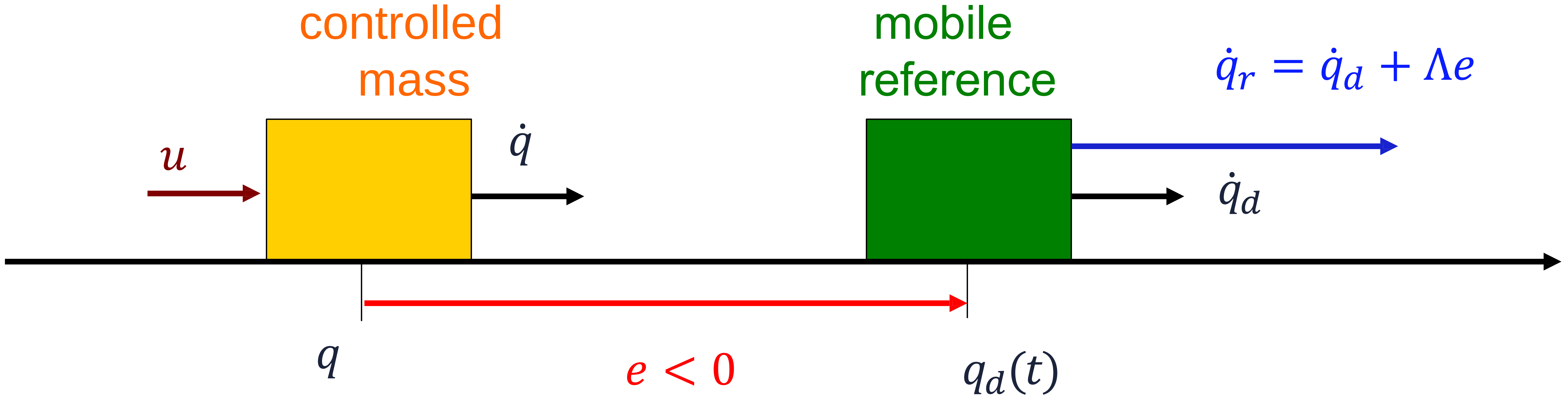
$$\dot{q}_d \quad \rightarrow \quad \boxed{\dot{q}_r = \dot{q}_d + \Lambda(q_d - q)} \qquad \Lambda > 0$$

typically $\Lambda = K_D^{-1}K_P$ (all matrices will be chosen diagonal)



Intuitive Interpretation of $\dot{q}_r$

- elementary case
  - a mass ‘lagging behind’ its mobile reference ($e > 0$) on a linear rail

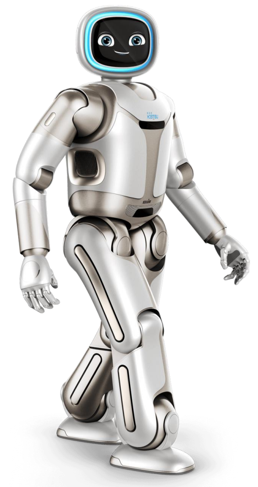


➡ ‘enhanced’ velocity error $s = \dot{q}_r - \dot{q} > \dot{q}_d - \dot{q} = \dot{e}$

$$u = K_D s = K_D(\dot{q}_r - \dot{q}) = K_D(\dot{q}_d + \Lambda e - \dot{q}) = K_D \dot{e} + \underbrace{K_D \Lambda}_{K_P} e$$

- a mass ‘leading in front’ of its mobile reference ($e < 0$)

➡ in a symmetric way, a ‘reduced’ velocity error will appear ($s < \dot{e}$)



Adaptive Control Law Design

- substituting $\dot{q}_r = \dot{q}_d + \Lambda e, \ddot{q}_r = \ddot{q}_d + \Lambda \dot{e}$ in the previous nonlinear controller for trajectory tracking

$$u = \widehat{M}(q)\ddot{q}_r + \hat{S}(q,\dot{q})\dot{q}_r + \hat{g}(q) + \hat{F}_V\dot{q}_r + K_P e + K_D \dot{e}$$
$$= Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\hat{a} + K_P e + K_D \dot{e}$$

dynamic parameterization of the control law using current estimates (note here the 4 arguments in $Y(\cdot)$!)

PD stabilization (diagonal matrices, > 0)

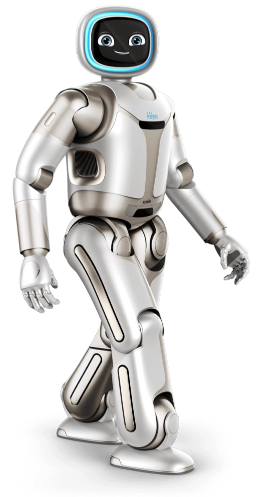
- update law for the estimates of the dynamic coefficients ($\hat{a}$ becomes the p-dimensional state of the dynamic controller)

$$\dot{\hat{a}} = \Gamma Y^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)(\dot{q}_r - \dot{q})$$

'modified' velocity error s

$\boldsymbol{\Gamma} > 0$ (diagonal)

estimation gains (variation rate of estimates)



Proof

Asymptotic stability of trajectory error

Theorem

The introduced adaptive controller makes the tracking error along the desired trajectory globally asymptotically stable

$$e = q_d - q \to 0, \dot{e} = \dot{q}_d - \dot{q} \to 0$$

Proof

- a Lyapunov candidate for the closed-loop system (robot + dynamic controller) is given by

$$V = \frac{1}{2} s^T M(q) s + \frac{1}{2} e^T R e + \frac{1}{2} \tilde{a}^T \Gamma^{-1} \tilde{a} \geq 0$$

$s = \dot{q}_r - \dot{q} (= \dot{e} + \Lambda e)$

modified velocity error

$R > 0$

constant matrix (to be specified later)

$\tilde{a} = a - \hat{a}$

error in parametric estimation

$$V = 0 \quad \Leftrightarrow \quad \hat{a} = a, \quad q = q_d, \quad s = 0 \quad (\Rightarrow \dot{q} = \dot{q}_d)$$

Proof

- the time derivative of V is

$$\dot{V} = \frac{1}{2} s^T \dot{M}(q) s + s^T M(q) \dot{s} + e^T R \dot{e} - \tilde{a}^T \Gamma^{-1} \dot{\hat{a}}$$

since $\dot{\tilde{a}} = -\dot{\hat{a}}$ $(\dot{a} = 0)$

- the closed-loop dynamics is given by

$$M(q)\ddot{q} + S(q,\dot{q})\dot{q} + g(q) + F_V \dot{q}$$
$$= \widehat{M}(q)\ddot{q}_r + \hat{S}(q,\dot{q})\dot{q}_r + \hat{g}(q) + \hat{F}_V \dot{q}_r + K_P e + K_D \dot{e}$$

subtracting the two sides from $M(q)\ddot{q}_r + S(q,\dot{q})\dot{q}_r + g(q) + F_V \dot{q}_r$ leads to

$$M(q)\dot{s} + (S(q,\dot{q}) + F_V)s = \widetilde{M}(q)\ddot{q}_r + \tilde{S}(q,\dot{q})\dot{q}_r + \tilde{g}(q) + \tilde{F}_V \dot{q}_r - K_P e - K_D \dot{e}$$

with $\widetilde{M} = M - \widehat{M}, \tilde{S} = S - \hat{S}, \quad \tilde{g} = g - \hat{g}, \quad \tilde{F}_V = F_V - \hat{F}_V$

Proof

- from the property of linearity in the dynamic coefficients, it follows

$$M(q)\dot{s} + (S(q,\dot{q}) + F_V)s = Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\tilde{a} - K_P e - K_D \dot{e}$$

- substituting in $\dot{V}$, together with $\dot{\hat{a}} = \Gamma Y^T s$, and using the skew- symmetry of matrix $\dot{M} - 2S$ we obtain

$$\dot{V} = \frac{1}{2} s^T [\dot{M}(q) - 2S(q,\dot{q})]s - s^T F_V s + s^T Y \tilde{a} - s^T (K_P e + K_D \dot{e})$$
$$+ e^T R \dot{e} - \tilde{a}^T Y^T s = -s^T F_V s - s^T (K_P e + K_D \dot{e}) + e^T R \dot{e}$$

- replacing $s = \dot{e} + \Lambda e$ and being $F_V = F_V^T$ (diagonal)

$$\dot{V} = -e^T (\Lambda^T F_V \Lambda + \Lambda^T K_P) e - e^T \left(2\Lambda^T F_V + \Lambda^T K_D + K_P - R \right) \dot{e} - \dot{e}^T (F_V + K_D) \dot{e}$$

quadratic form in $e, \dot{e}$!

Proof

- defining now (all matrices are diagonal!)

$$\Lambda = K_D^{-1} K_P > 0 \qquad R = 2K_P(I + K_D^{-1} F_V) > 0$$

leads to

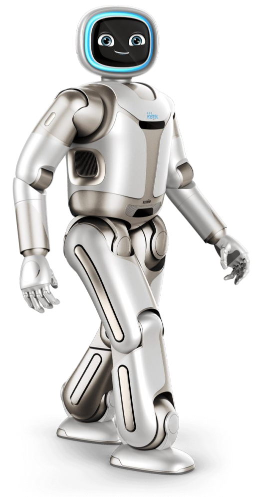
$$\dot{V} = -e^T \Lambda^T (F_V + K_D) \Lambda e - \dot{e}^T (F_V + K_D) \dot{e}$$
$$= -e^T K_P K_D^{-1} (F_V + K_D) K_D^{-1} K_P e - \dot{e}^T (F_V + K_D) \dot{e} \leq 0$$

and thus

$$\dot{V} = 0 \Leftrightarrow e = \dot{e} = 0$$

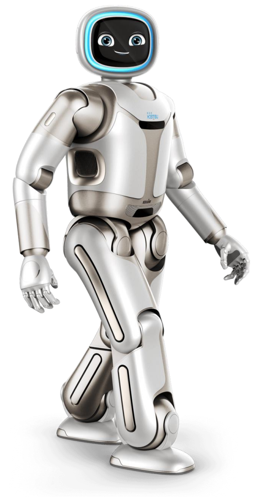
the thesis follows from Barbalat lemma + LaSalle theorem

the set of states of convergence has zero trajectory error and a constant value for $\hat{a}$, not necessarily the true one ($\tilde{a} \neq 0$)



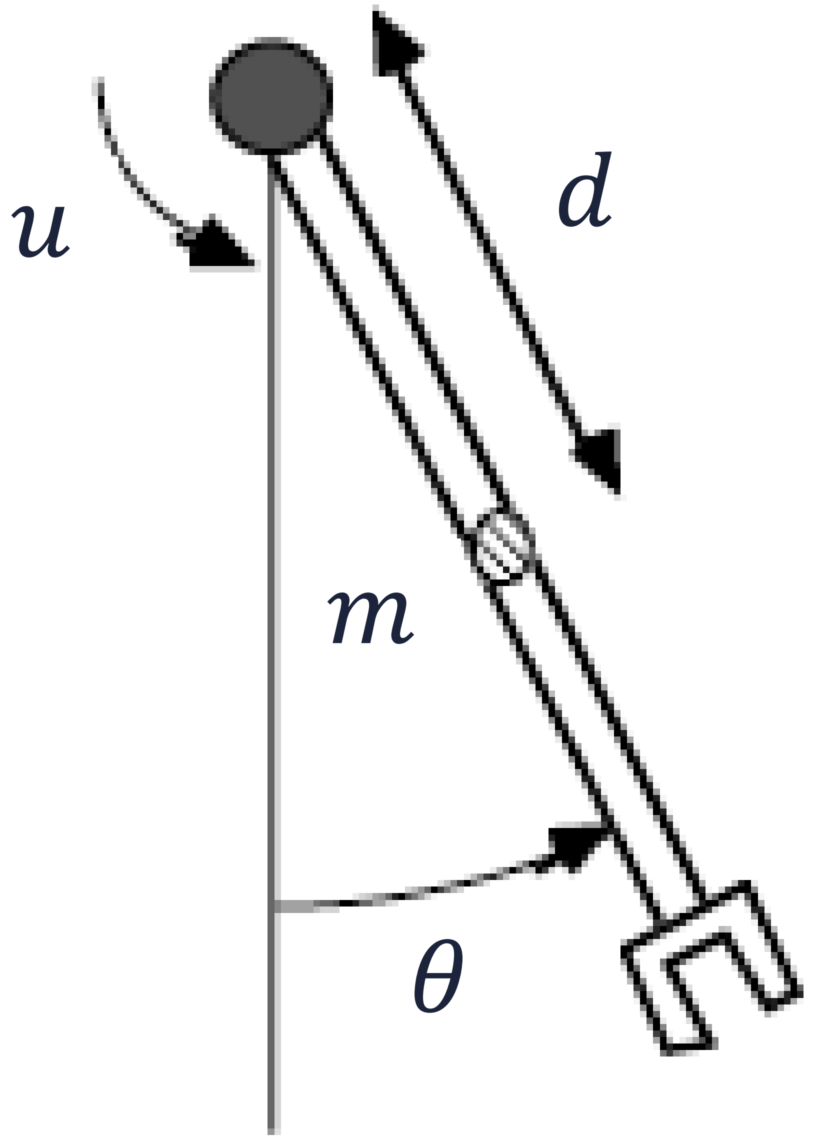
Comments

- if the desired trajectory $q_d(t)$ is persistently exciting, then also the estimates of the dynamic coefficients converge to their true values
- condition of persistent excitation
  - for linear systems: # of frequency components in the desired trajectory should be at least twice as large as # of unknown coefficients
  - for nonlinear systems: the condition can be checked only a posteriori (a certain motion integral should be permanently lower bounded)
- in case of known absence of (viscous) friction ($F_V \equiv 0$), the same proof applies (a bit easier in the final part)
- the adaptive controller does not require the inverse of the inertia matrix (true or estimated), nor the actual robot acceleration (only the desired acceleration), nor further lower bounds on $K_P > 0, K_D > 0$
- adaptation can be also used only for a subset of dynamic coefficients, the remaining being known ($Ya = Y_{adapt}\hat{a}_{adapt} + Y_{known}a_{known}$)
- the non-adaptive version (using accurate estimates) is a static tracking controller based on the passivity property of robot dynamics



Case Study

Case study: Single-link under gravity



model $I\ddot{\theta} + mgd\sin\theta + f_V\dot{\theta} = u$ (with friction)

linear parameterization

$$Y(\theta,\dot{\theta},\ddot{\theta})a = [\ddot{\theta} \quad \sin\theta \quad \dot{\theta}]\begin{bmatrix} I \\ mgd \\ f_V \end{bmatrix} = u$$

adaptive controller

$$u = \hat{I}\ddot{\theta}_r + \widehat{mgd}\sin\theta + \hat{f}_V\dot{\theta} + k_P e + k_D\dot{e}$$

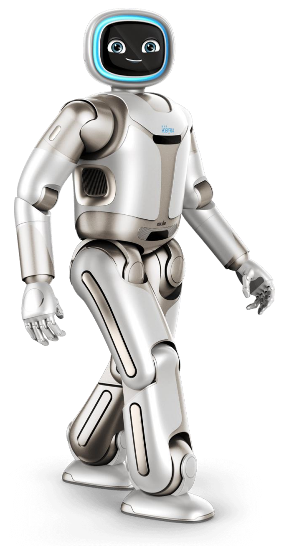
$$\dot{\hat{a}} = \begin{pmatrix} \dot{\hat{I}} \\ \dot{\widehat{mgd}} \\ \dot{\hat{f}}_V \end{pmatrix} = \begin{pmatrix} \gamma_1\ddot{\theta}_r \\ \gamma_2\sin\theta \\ \gamma_3\dot{\theta}_r \end{pmatrix}(\dot{\theta}_r - \dot{\theta})$$

$$e = \theta_d - \theta$$

$$\dot{\theta}_r = \dot{\theta}_d + \frac{k_P}{k_D}e$$

$\Lambda > 0$

$$\gamma_i > 0, i = 1,2,3$$



Case Study

Simulation data

- real dynamic coefficients

$$I = 7.5, \quad mgd = 6, \quad f_V = 1$$

- initial estimates

$$\hat{I} = 5, \quad \widehat{mgd} = 5, \quad \widehat{f_V} = 2$$

- control parameters

$$k_P = 25, \quad k_D = 10, \quad \gamma_i = 5, \quad i = 1,2,3$$

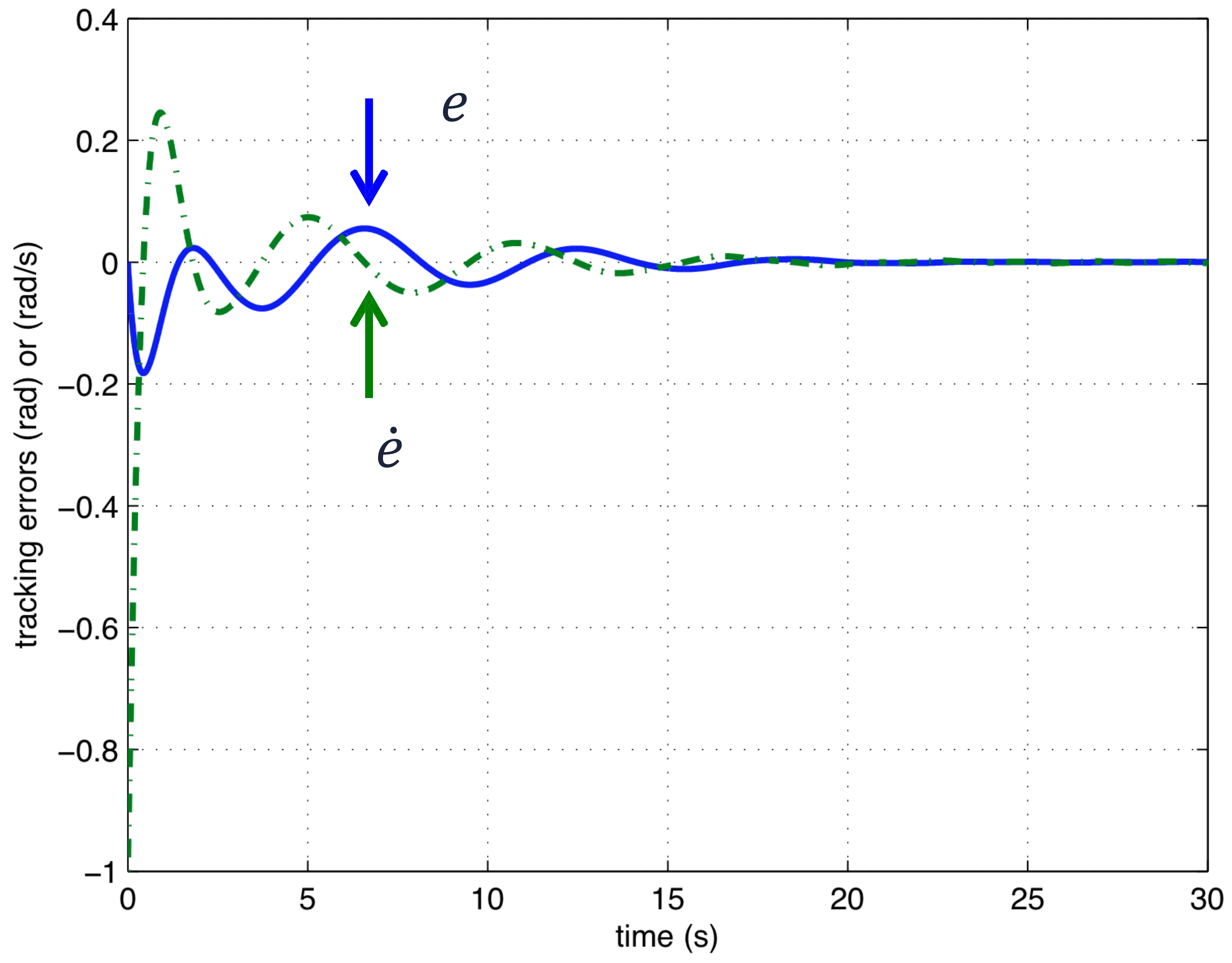
- test trajectories (starting with $\theta(0) = 0, \dot{\theta} = 0$)
  - first

$$\theta_d(t) = -\sin t$$

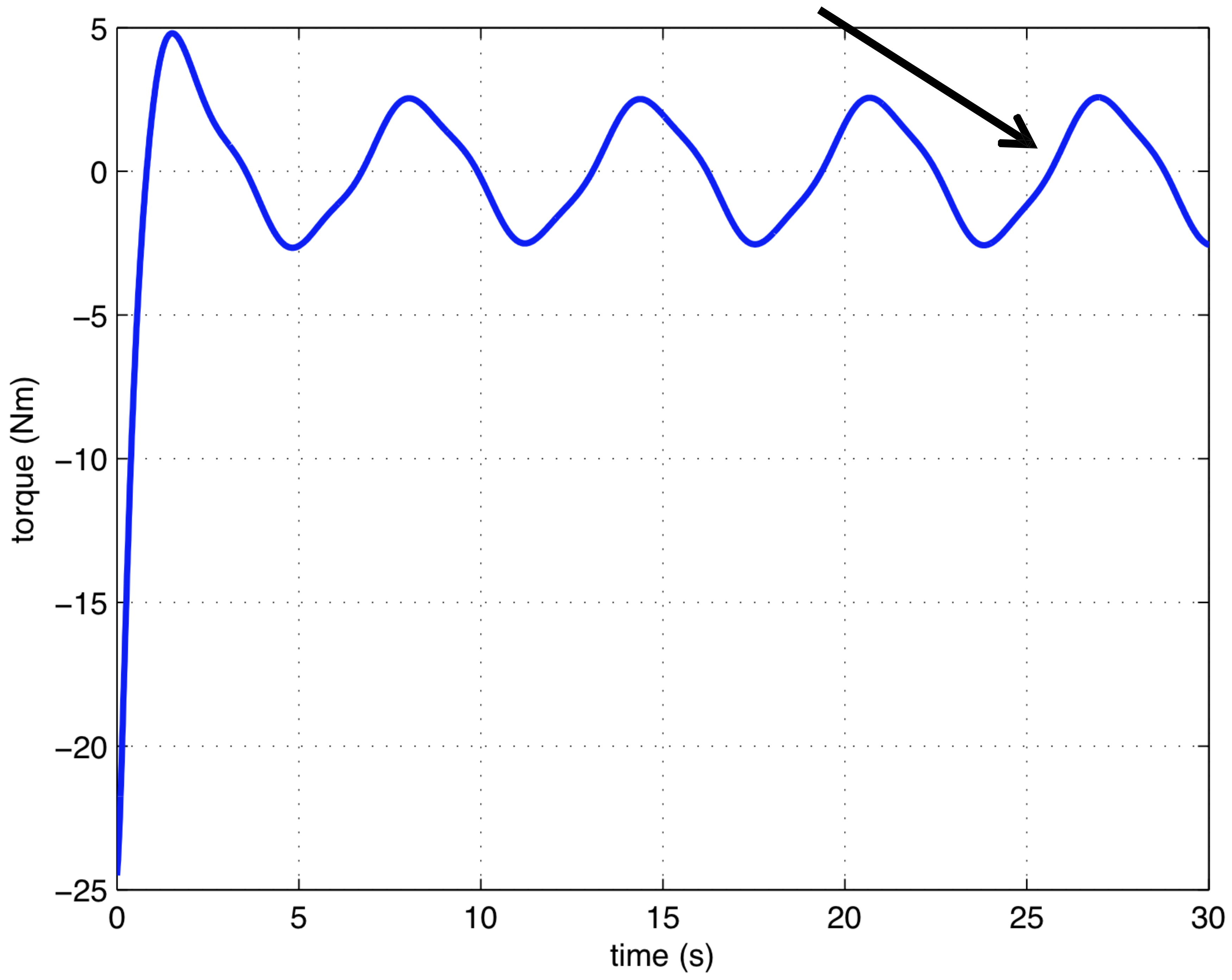
Case Study

Results

note the nonlinear system dynamics (no sinusoidal regime at steady state!)



position and velocity errors

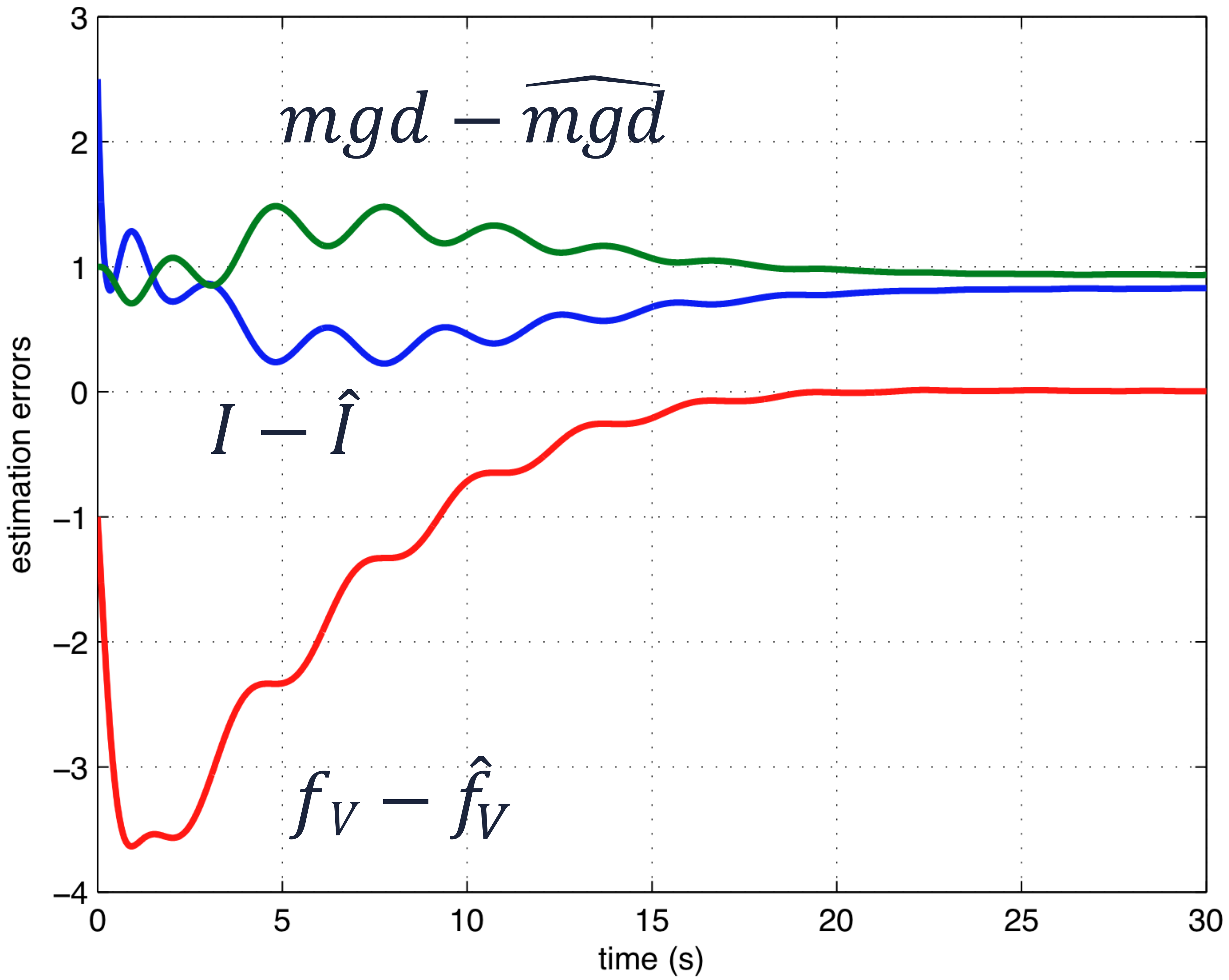


control torque

$$\theta_d(t) = -\sin t$$

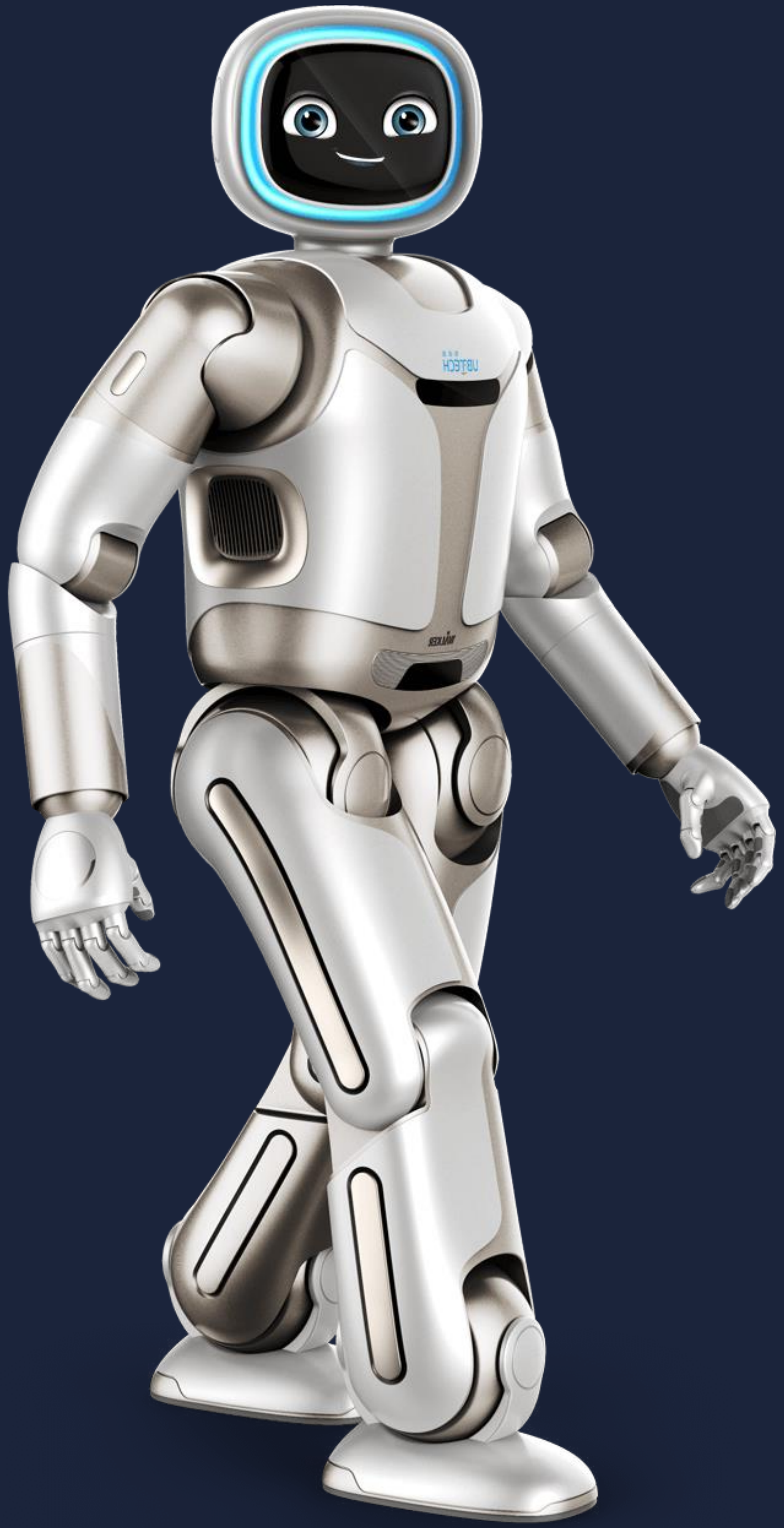
Case Study

Estimates of dynamic coefficients



only the estimate of the viscous friction coefficient converges to the true value

errors $\tilde{a} = a - \hat{a}$



Q&A