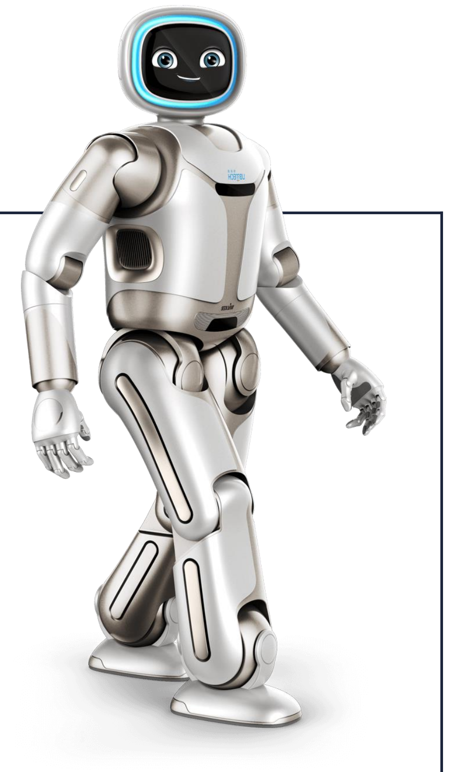
Advanced Robotics

ENGG5402 Spring 2023

Fei Chen



Topics:

- Control in the Cartesian Space

Readings:

- Siciliano: Sec. 8.6

Regulation Controller

Regulation of robot Cartesian pose

- "PD +" type control for regulation problems
  - proportional to the Cartesian pose error, with a derivative term (on velocity) + cancellation/compensation of gravity in joint space
- robot
  - dynamics $M(q)\ddot{q} + S(q,\dot{q})\dot{q} + g(q) = u$
  - kinematics $p = f(q) \rightarrow \quad \dot{p} = J(q)\dot{q}$

dimension of spaces

joint $= n$

Cartesian $= m$

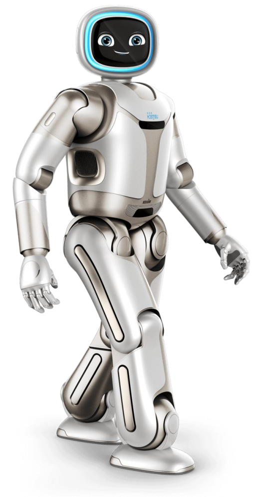
- goal: asymptotic stabilization of the end-effector pose

$$p = p_d, \dot{q} = 0 \quad \rightarrow \quad \dot{p}_d = 0$$

Note:

if $n = m$, then $\dot{q} = 0 \Leftrightarrow \dot{p} = 0$ up to singularities

if $n > m$, then the goal is not uniquely associated to a complete robot state: $n - m$ joint coordinates are missing…



Regulation Controller

A Cartesian regulation law

(*) $u = J^T(q)K_P(p_d - p) - K_D\dot{q} + g(q)$ $\quad$ $K_P, K_D > 0$ (symmetric)

Theorem

under the control law (*), the robot state will converge asymptotically to the set $A = \{\dot{q} = 0, q: K_P(p_d - f(q)) \in N(J^T(q))\}$

$$\supseteq \{\dot{q} = 0, q: f(q) = p_d\}$$

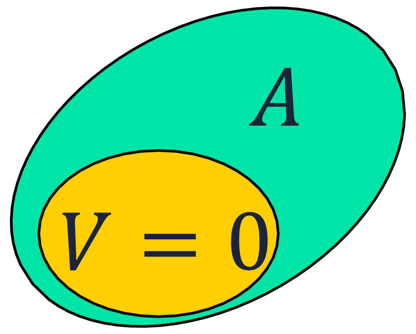
Proof

define $e_P = p_d - p$ (Cartesian error) and the associated Lyapunov-like candidate function

$$V = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \frac{1}{2}e_P^T K_P e_P$$

with

$$V = 0 \Leftrightarrow (q, \dot{q}) \in \{\dot{q} = 0, q: f(q) = p_d\} \subseteq A$$



Proof

Proof (cont)

differentiating
$$V = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \frac{1}{2} e_p^T K_P e_P \geq 0$$

$$\begin{aligned}
\dot{V} &= \dot{q}^T \left(M\ddot{q} + \frac{1}{2}\dot{M}\dot{q} \right) - e_p^T K_P \dot{p} \\
&= \dot{q}^T \left(u - S\dot{q} - g + \frac{1}{2}\dot{M}\dot{q} \right) - e_p^T K_P \dot{p} \\
&= \dot{q}^T (J^T K_P e_P - K_D \dot{q} + g - g) - e_p^T K_P J \dot{q} \\
&= -\dot{q}^T K_D \dot{q} \leq 0
\end{aligned}$$

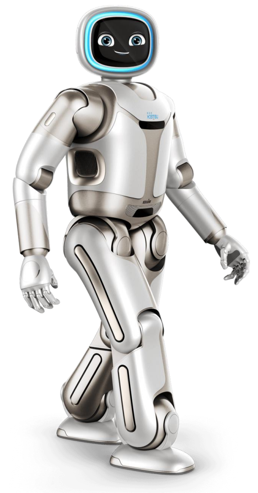
with $\dot{V} = 0 \Leftrightarrow \dot{q} = 0$

in this situation, the closed-loop equations become

$$M(q)\ddot{q} + g(q) = J^T(q) K_P e_P + g(q) \quad \Rightarrow \quad \ddot{q} = M^{-1}(q) J^T(q) K_P e_P$$

$$\Rightarrow \quad \boxed{\ddot{q} = 0 \Leftrightarrow K_P e_P \in N(J^T(q))}$$

by applying LaSalle theorem, the thesis follows



Corollary

for a given initial state $(q(0), \dot{q}(0))$, if the robot does not encounter any singularity of $J^T(q)$ (configurations where $\rho(J^T) < m \leq n$) during its motion, then there is asymptotic stabilization to one single state $(m = n)$ or to a set of states $(m < n)$ such that

$$e_P = 0, \dot{q} = 0$$

Note: singular configurations q of $J^T(q)$ coincide with those of $J(q)$

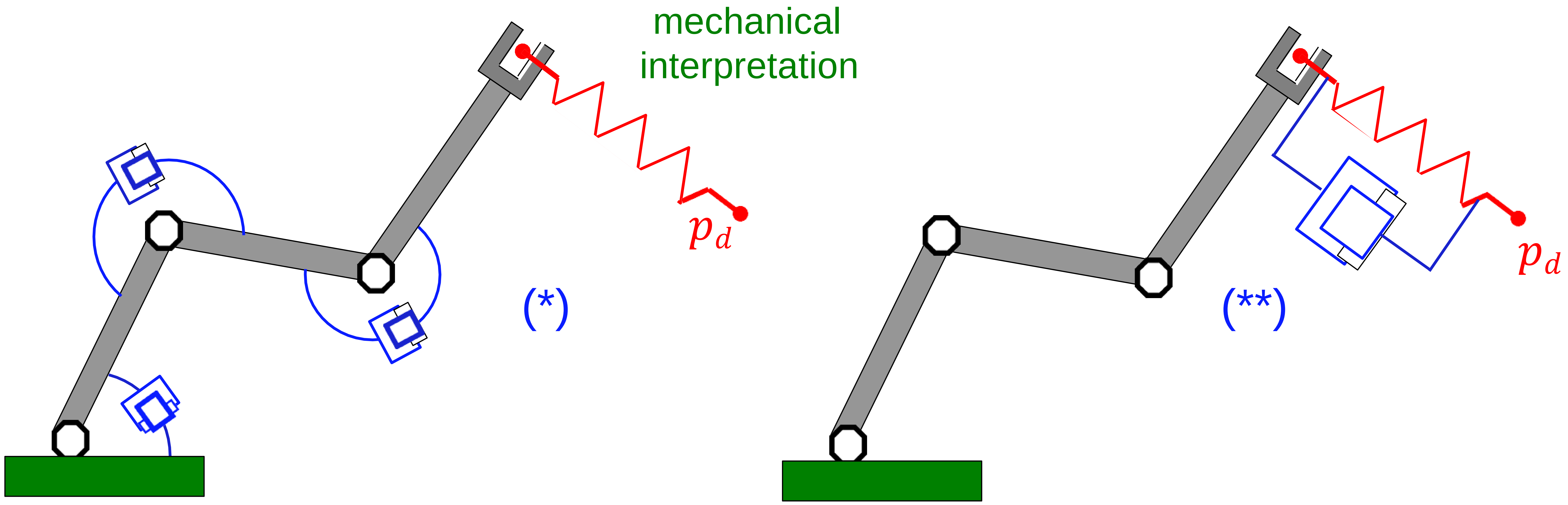
Variant

A possible variant for regulation

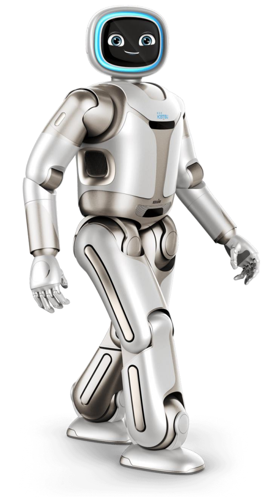
"all Cartesian" PD control + gravity cancellation in joint space

(**) $$u = J^T(q)[K_P(p_d - p) - K_D\dot{p}] + g(q)$$

$K_P, K_D > 0$ (symmetric)



J^T transforms the "virtual" elastic, for (*), or visco-elastic, for (**), force/torque acting on the end-effector into control torques at the joints



FBL in Cartesian Space

Feedback linearization in Cartesian space

robot $M(q)\ddot{q} + c(q,\dot{q}) + g(q) = u$

output $y = p, \quad p = f(q)$ — Cartesian position/orientation

assume: $M = N$

algorithm

differentiate the output(s) as many times as needed up to the appearance of (at least one of) the input torque(s), then verify if it is possible to solve for the input = "inversion"

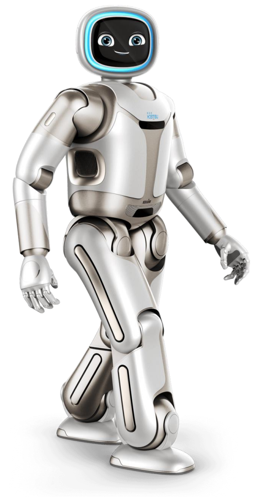
uniform "relative degree" $\rho = 2$ for all outputs →

$$y = f(q)$$
$$\dot{y} = J(q)\dot{q}$$
$$\ddot{y} = J(q)\ddot{q} + \dot{J}(q)\dot{q}$$
$$= J(q)M^{-1}(q)u - J(q)M^{-1}(q)[c(q,\dot{q}) + g(q)]$$
$$+ \dot{J}(q)\dot{q}$$

from the dynamic model

Theorem

for a non-redundant robot, it is possible to exactly linearize and decouple the dynamic behavior at the Cartesian level if and only if $\det J(q) \neq 0$



FBL in Cartesian Space

Feedback linearization in Cartesian space
(in the right coordinates!)

control law $u = M(q)J^{-1}(q)a + c(q,\dot{q}) + g(q) - M(q)J^{-1}(q)\dot{J}(q)\dot{q}$

$$= \beta(q)a + \alpha(q,\dot{q})$$

$$\ddot{y} = \ddot{p} = J(q)M^{-1}(q)u - J(q)M^{-1}(q)[c(q,\dot{q}) + g(q)] + \dot{J}(q)\dot{q} = a$$

$p, \dot{p}$ are the so-called "**linearizing**" coordinates

closed-loop equations (in the joint space)

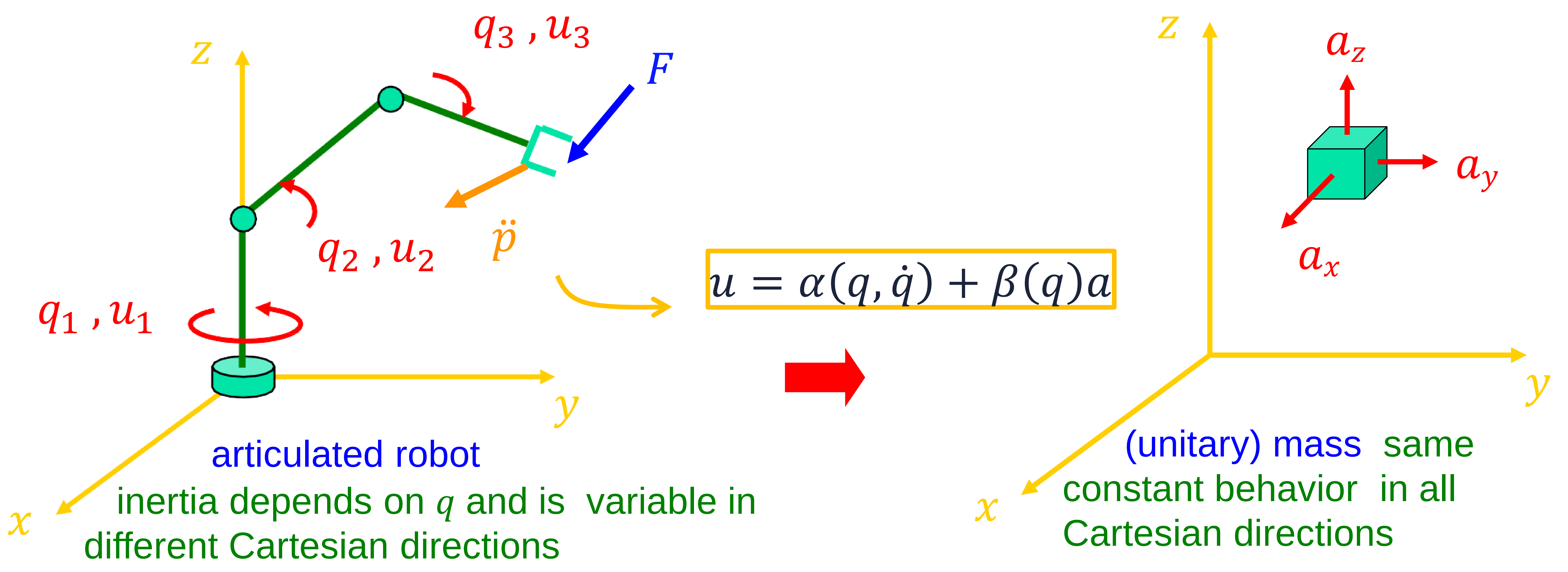
$M^{-1} *$
$$M\ddot{q} + c + g = MJ^{-1}a + c + g - MJ^{-1}\dot{J}\dot{q}$$

$$\ddot{q} = J^{-1}(q)a - J^{-1}(q)(\dot{q})\dot{q}$$

purely kinematic equations

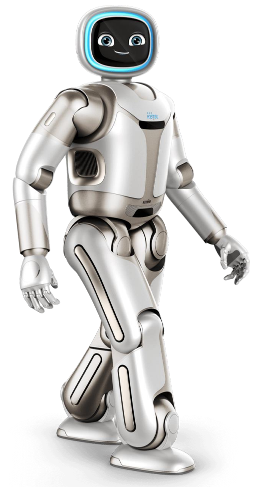
(but still **nonlinear** and **coupled**!!)

Physical Interpretation



when a control force F is applied at the end-effector

- the uncontrolled robot will accelerate with $\ddot{p}$ in a different direction
- the (unitary) mass accelerates in the same direction of applied force F



Alternative Derivation

Alternative derivation (in purely Cartesian terms)

the previous exact linearizing and decoupling law can be rewritten in Cartesian terms using a control force/torque F

$$u = M(q)J^{-1}(q)a + c(q,\dot{q}) + g(q) - M(q)J^{-1}(q)\dot{J}(q)\dot{q}$$

joint torque u is moved to the Cartesian space as $F = J^{-T}(q)u$ (for $m = n$)

$$\begin{aligned} F &= [J^{-T}MJ^{-1}]a \\ &\quad +[J^{-T}c - J^{-T}MJ^{-1}\dot{J}\dot{q}] \\ &\quad +[J^{-T}g] \\ &= M_p a + c_p + g_p \end{aligned}$$

Cartesian inertia [= M_pp]

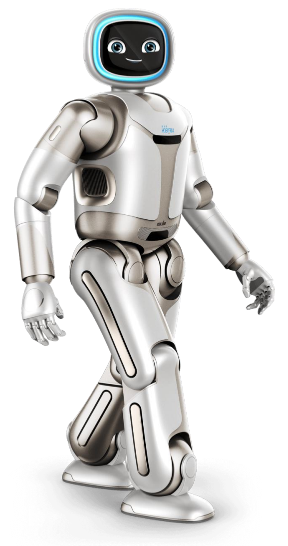
Cartesian Coriolis/centrifugal

terms Cartesian gravity

this is the feedback linearization law applied to the **Cartesian dynamic model** of the robot

$$M_p(p)\ddot{p} + c_p(p,\dot{p}) + g_p(p) = F$$

$$\ddot{p} = a$$



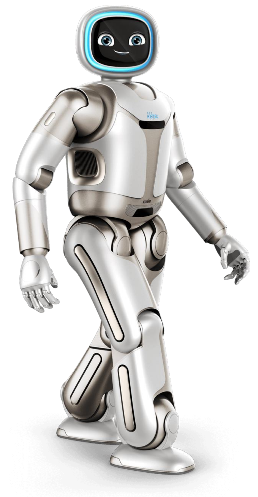
Comments

- the design of a Cartesian trajectory tracking control is completed by stabilizing the tracking error in the m independent chains of double integrators, i.e., by setting

$$a_i = \ddot{p}_{di} + K_{Di}(\dot{p}_{di} - \dot{p}_i) + K_{Pi}(p_{di} - p_i)$$

scalars
$K_{Pi} > 0, K_{Di} > 0$
$i = 1, \dots, m$

- in redundant ($m < n$) robots: replacing J^{-1} with $J^{\#}$ in the control law, we obtain input-output decoupling and linearization, but not exact linearization of the whole state dynamics
  - there is an remaining internal dynamics of dimension $n - m$ left
- the Cartesian pose/velocity can either be directly measured by external sensors (cameras) or computed through the direct and differential kinematics of the robot arm

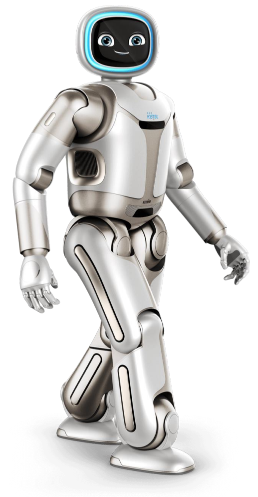


Comments

- the transient behavior of the Cartesian error along a desired trajectory is exponentially stable (with arbitrary eigenvalues assigned by choosing the diagonal gains of K_P, K_D)
- when applied to the case $p_d =$ constant (regulation task), the control law becomes

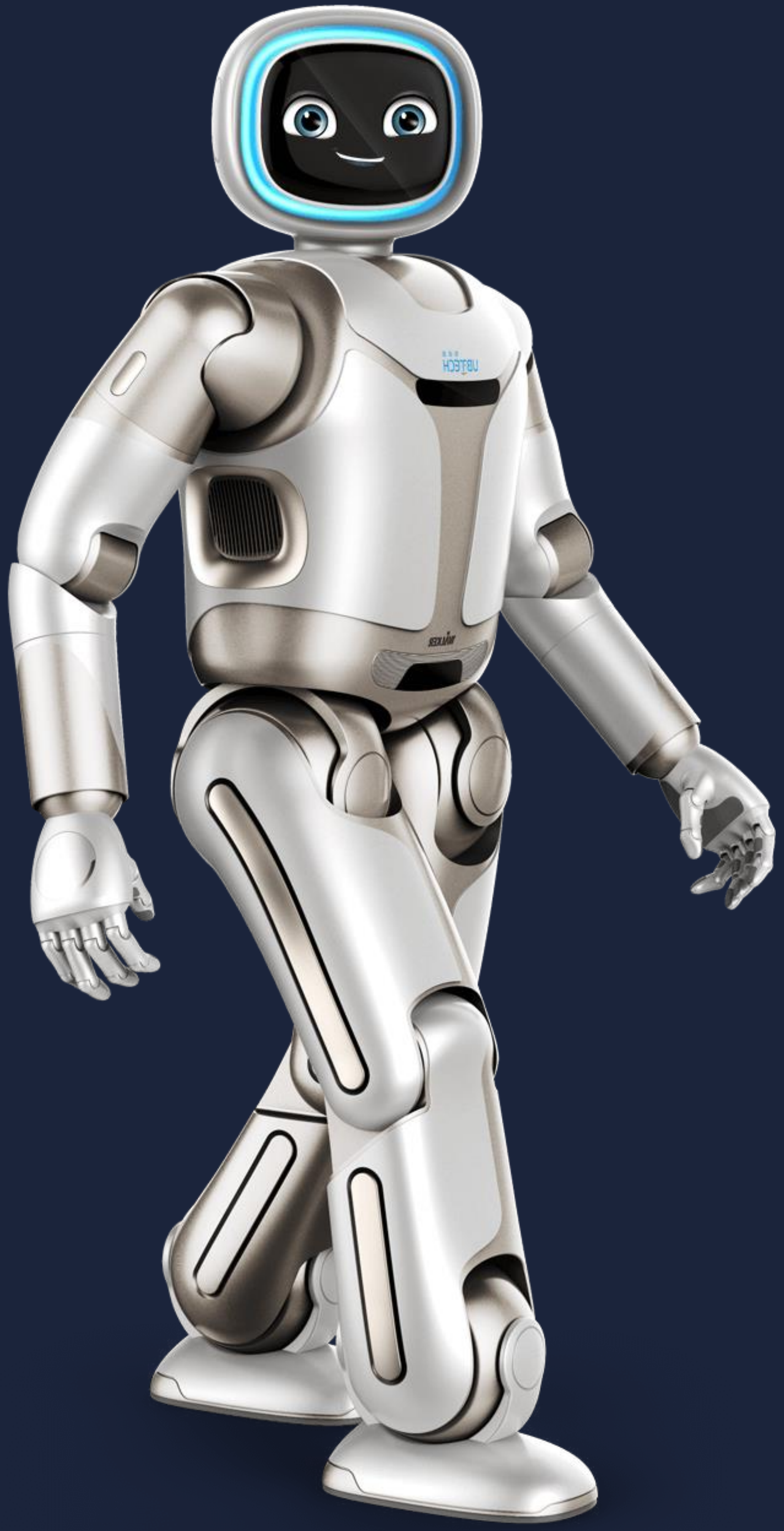
$$u = M(q)J^{-1}(q)[K_P e_P - K_D J(q)\dot{q}] + c(q,\dot{q}) + g(q) - M(q)J^{-1}(q)\dot{J}(q)\dot{q}$$

which is computationally more expensive than a control law designed directly for regulation, such as the previous laws (*) or (**), but keeps the additional property of obtaining an exponentially stable transient error



Conclusion

- most of the control laws presented in the joint space (i.e., driven by a joint error) can be translated with relative ease to the Cartesian space, e.g.
  - regulation with constant gravity compensation
  - adaptive regulation
  - robust control for trajectory tracking
  - adaptive control for trajectory tracking
- the main issues are related to
  - presence of kinematic singularities, both for the Jacobian transpose or the Jacobian inverse control laws, needs some suitable singularity robustness modifications
  - presence of kinematic redundancy ($m < n$) needs the use of an additional stabilizing null-space torque for the extra $n - m$ generalized coordinates (locally, $n - m$ joint variables)



Q&A