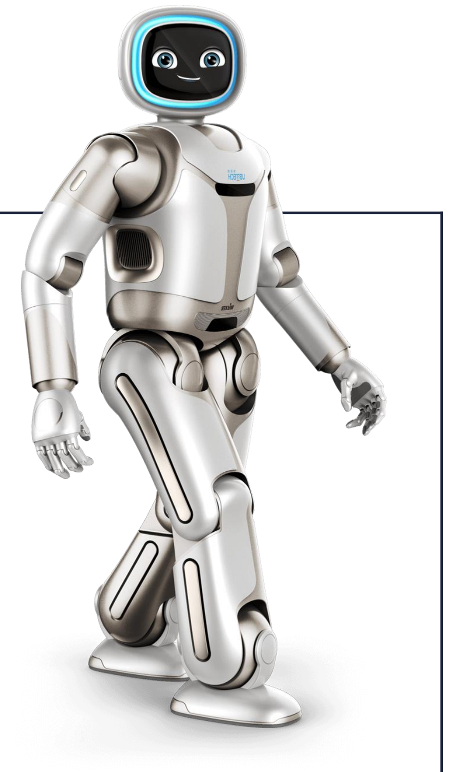
Advanced Robotics

ENGG5402 Spring 2023

Fei Chen



Topics:

- Direct kinematics

Readings:

- Siciliano: Sec. 2.8

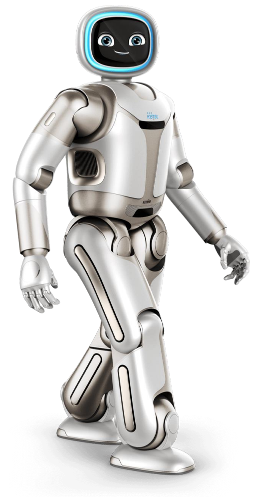
Kinematics of Robot Manipulators

- Study of…

  geometric and timing aspects of robot motion, without reference to the causes producing it

- Robot seen as…

  an (open) kinematic chain of rigid bodies interconnected by (revolute or prismatic) joints



Motivations

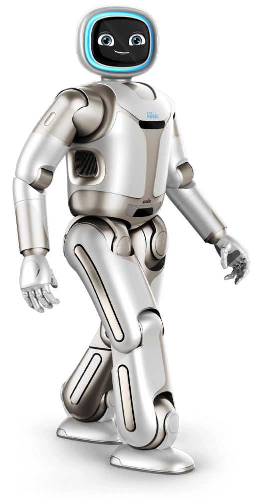
- Functional aspects
  - Definition of robot workspace
  - Calibration (*not covered in ENGG5402*)

- Operational aspects

task execution (actuation by motors)	⇄	task definition and performance
Joint-Actuator Space		Task Space

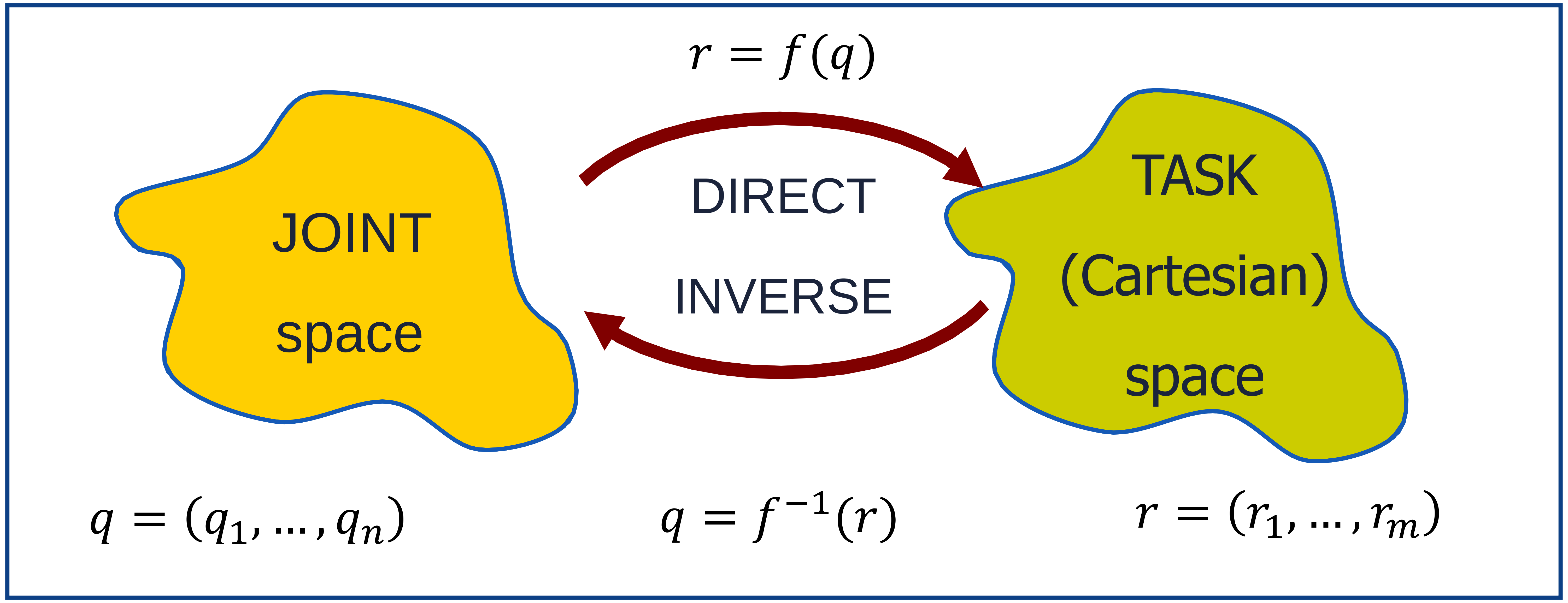
two different “spaces” related by kinematic (and dynamic) maps

- trajectory planning
- programming
- motion control

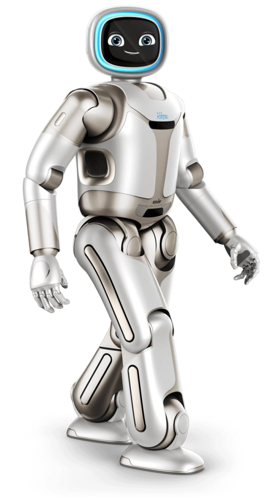


Kinematics

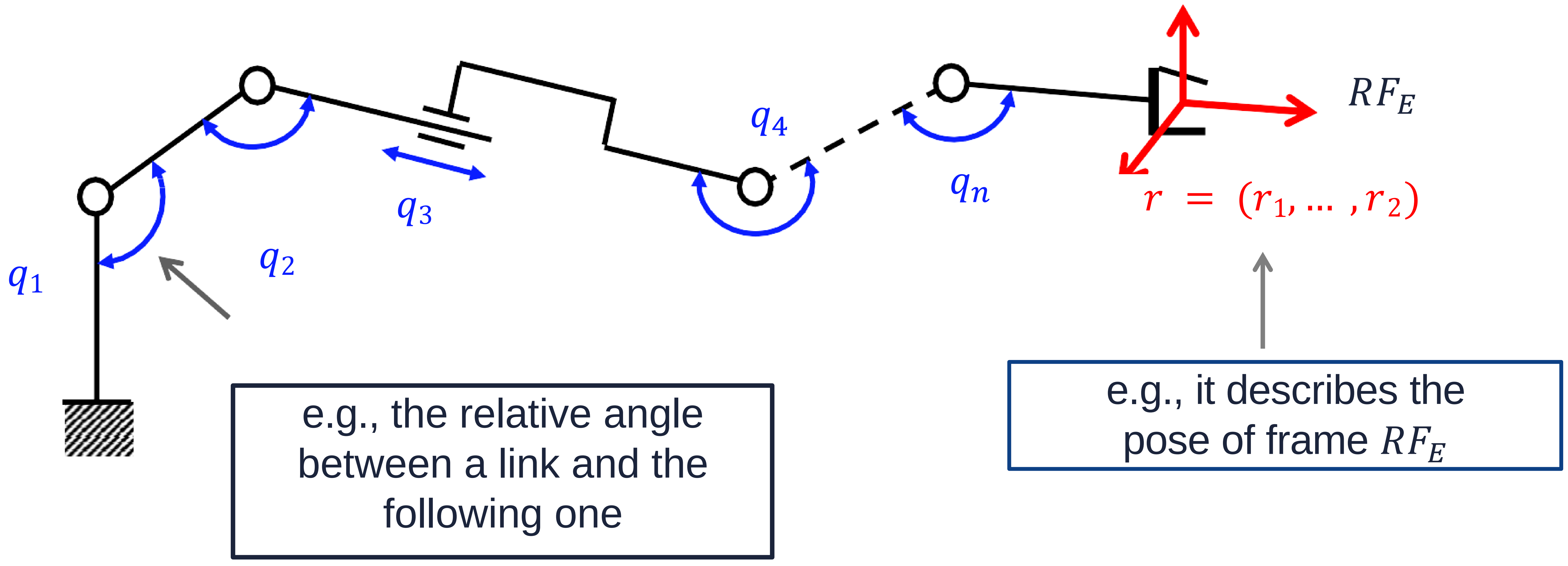
Kinematics (formulation and parameterizations)



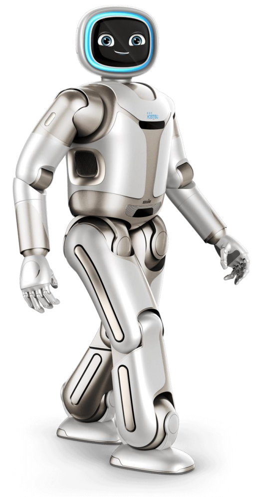
- choice of parameterization q
  - unambiguous and minimal characterization of robot configuration
  - $n =$ # degrees of freedom $(DOF) =$ # robot joints (rotational or translational)
- choice of parameterization r
  - compact description of position and/or orientation (pose) variables of interest to the required task
  - usually, $m \leq n$ and $m \leq 6$ (but none of these is strictly necessary)



Open Kinematic Chains

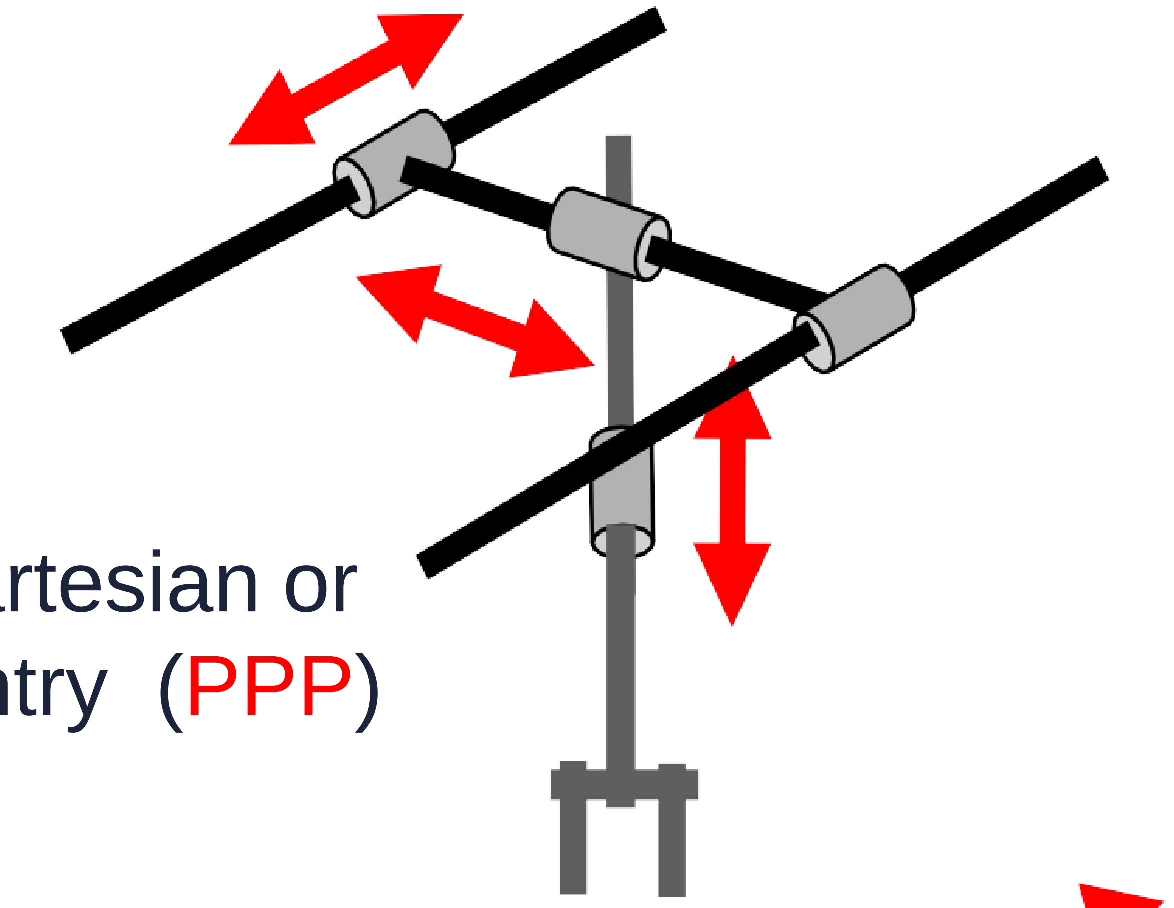


- $m = 2$
  - pointing in space
  - positioning in the plane
- $m = 3$
  - orientation in space
  - positioning and orientation in the plane
- $m = 5$
  - positioning and pointing in space (like for spot welding)
- $m = 6$
  - positioning and orientation in space
  - positioning of two points in space (e.g., end-effector and elbow)



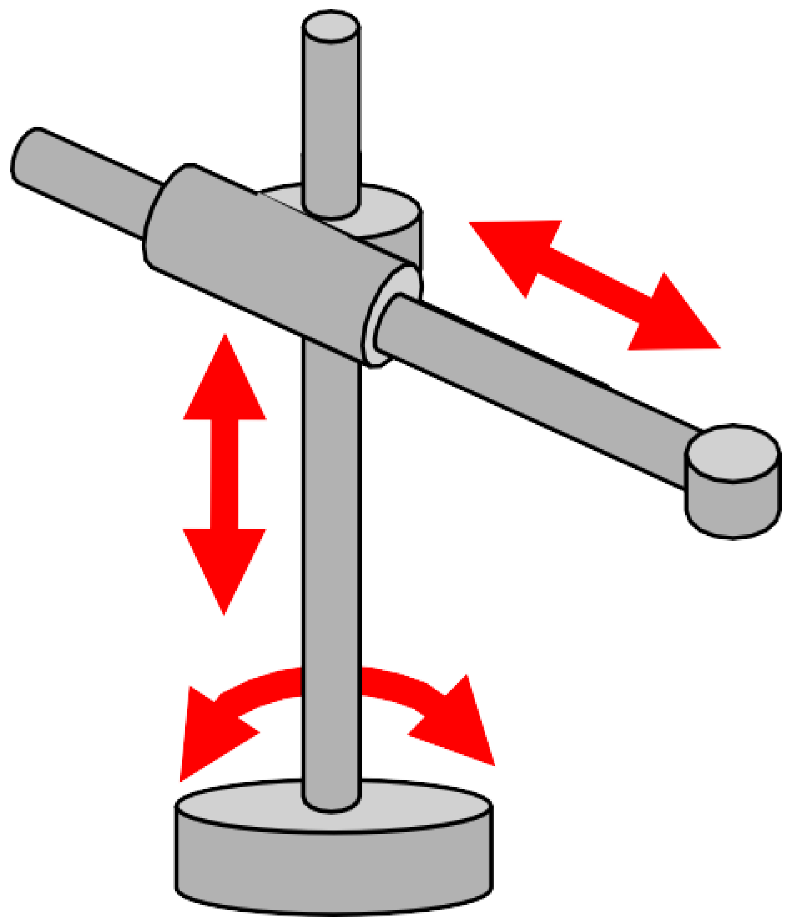
Classification

Classification by kinematic type (first 3 dofs only)

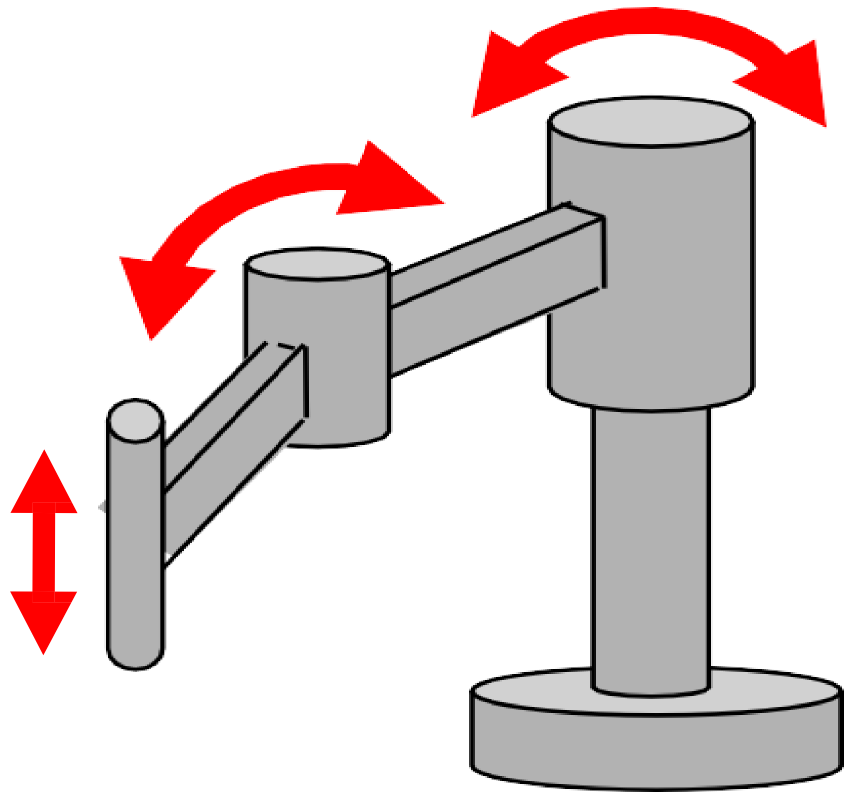


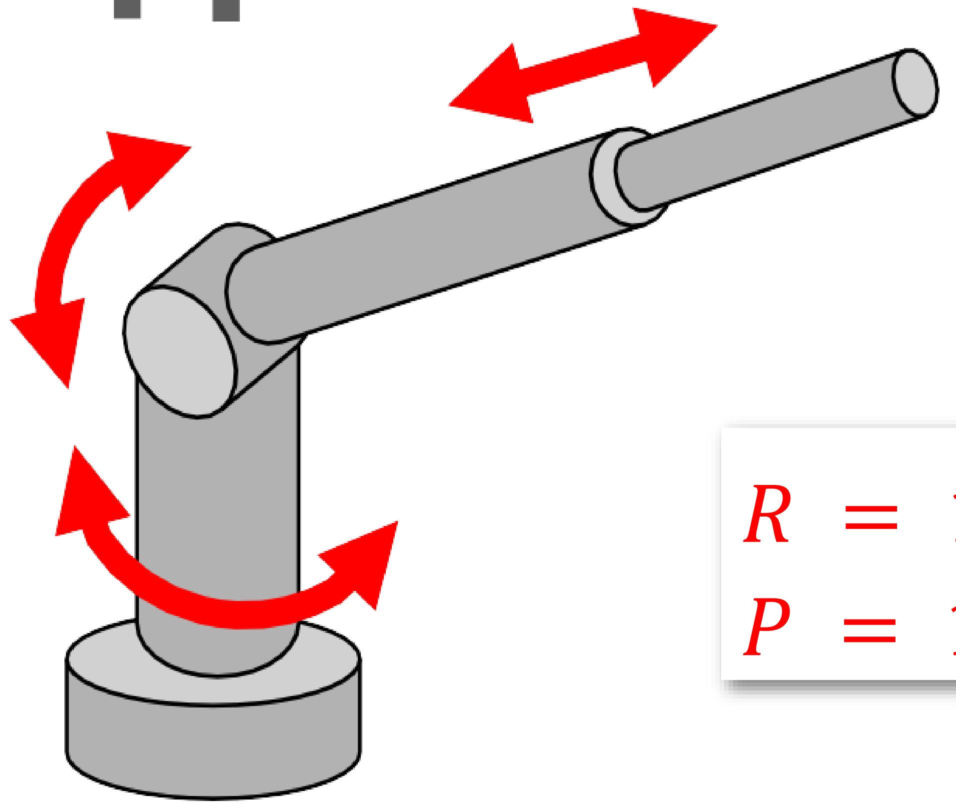
Cartesian or gantry (PPP)

cylindric (RPP)



SCARA (RRP)



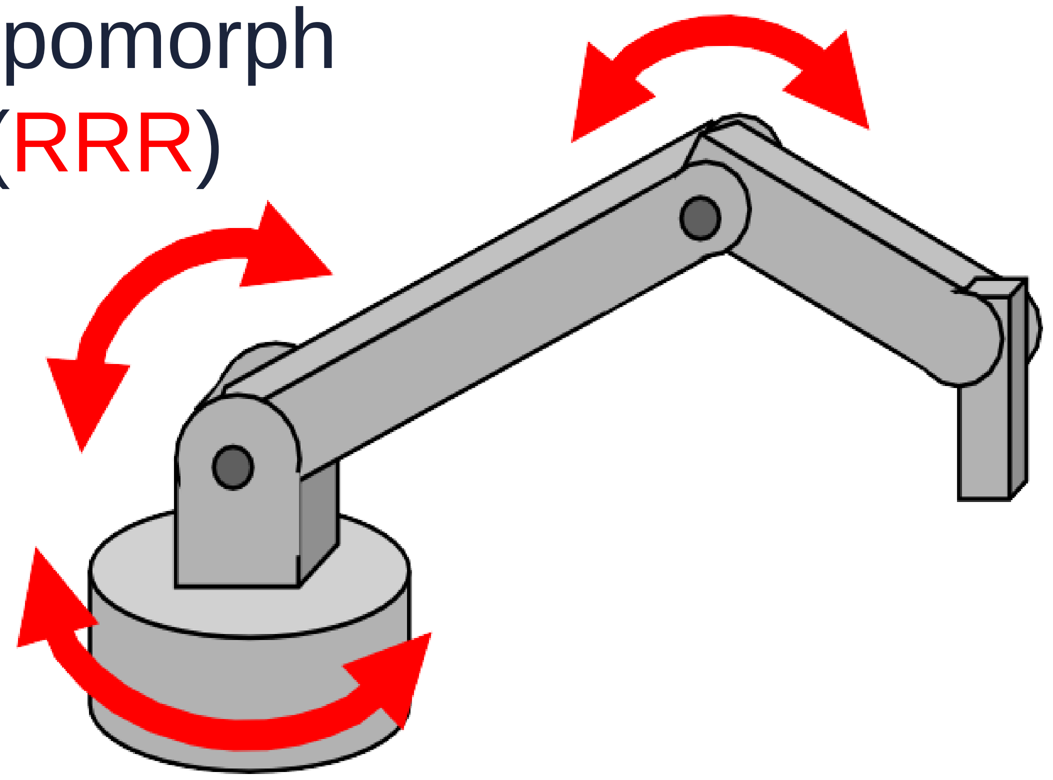


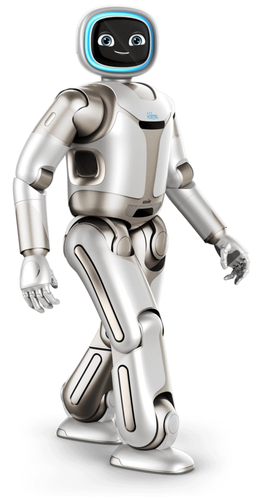
polar or spherical (RRP)

$R = 1 - dof$ rotational (revolute) joint

$P = 1 - dof$ translational (prismatic) joint

articulated or anthropomorphic (RRR)





Direct Kinematic Map

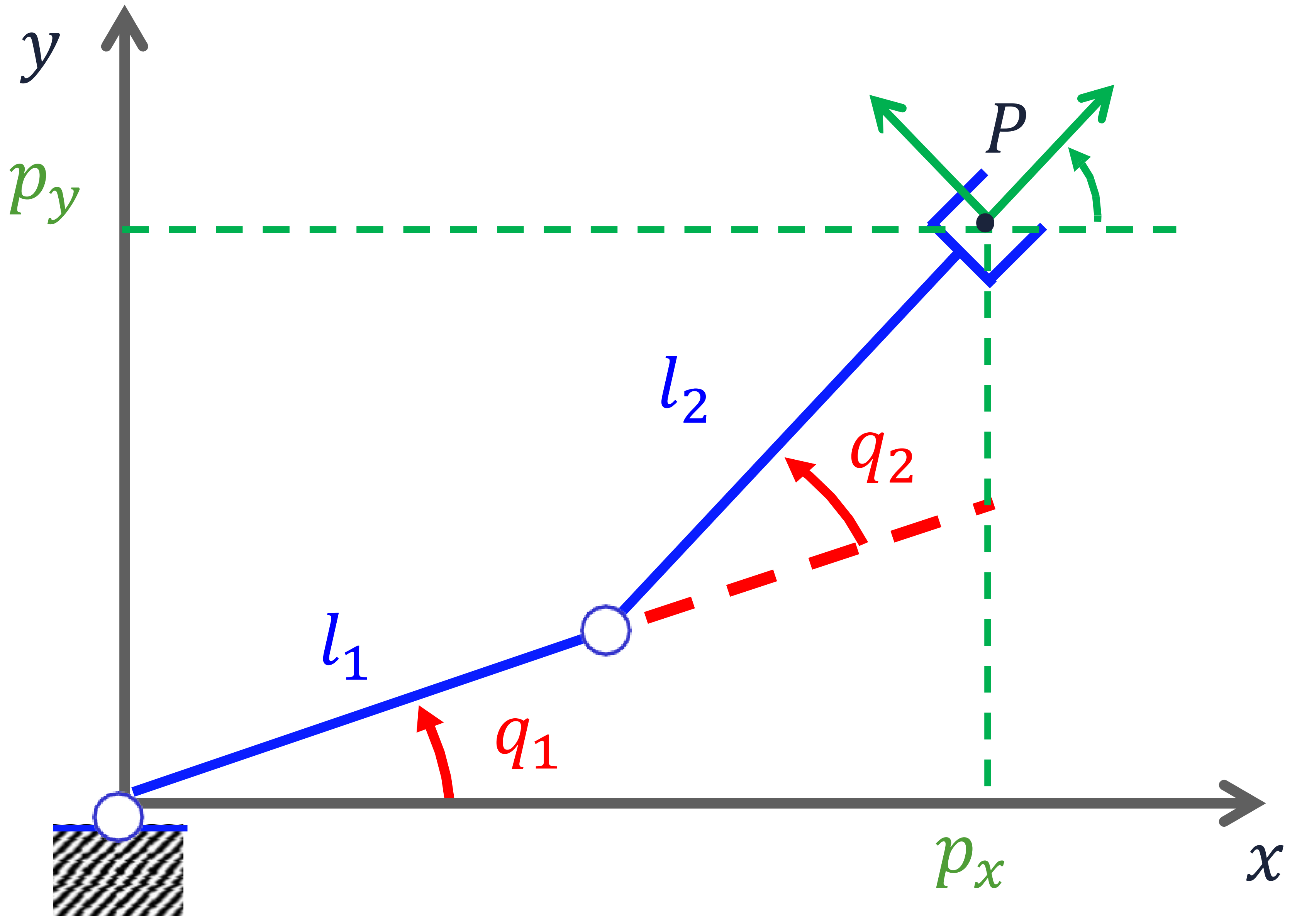
- The structure of the direct kinematics function depends on the chosen r

$$r = f_r(q)$$

- Methods for computing $f_{\mathrm{r}}(q)$
  - geometric/by inspection
  - systematic: assigning frames attached to the robot links and using homogeneous transformation matrices

Direct Kinematic Map

Direct kinematics of 2R planar robot (just using inspection…)

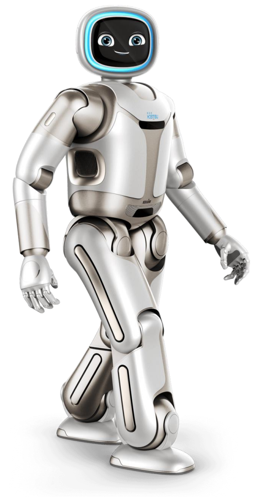


$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \quad n = 2$$

$$r = \begin{bmatrix} p_x \\ p_y \\ \phi \end{bmatrix} \quad m = 3$$

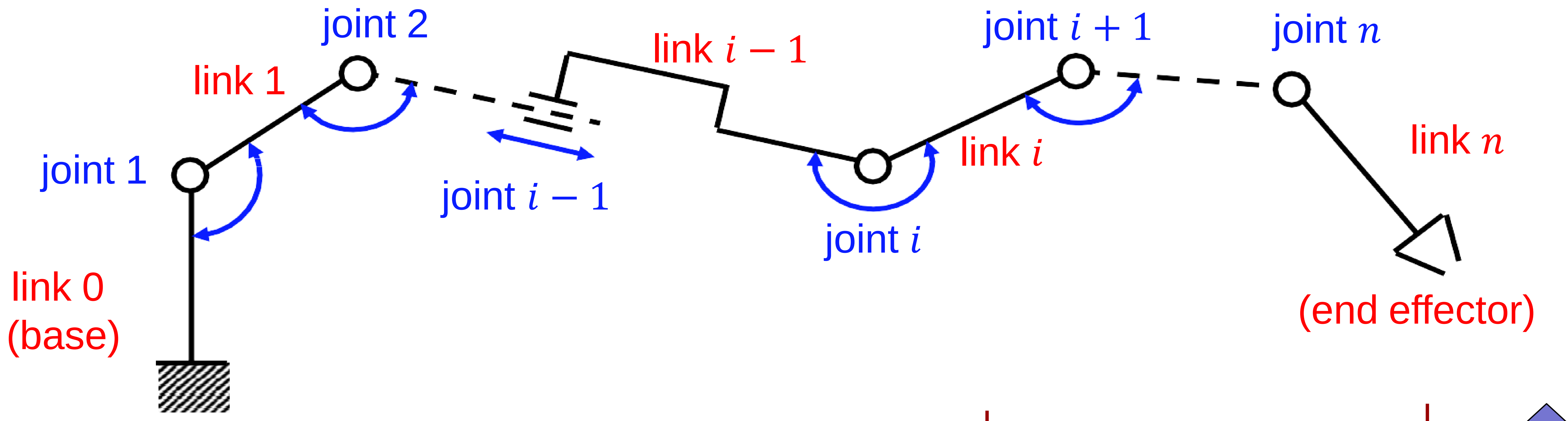
$$p_x = l_1 \cos q_1 + l_2 \cos(q_1 + q_2)$$
$$p_y = l_1 \sin q_1 + l_2 \sin(q_1 + q_2)$$
$$\phi = q_1 + q_2$$

for more general cases, we need a ‘method’!



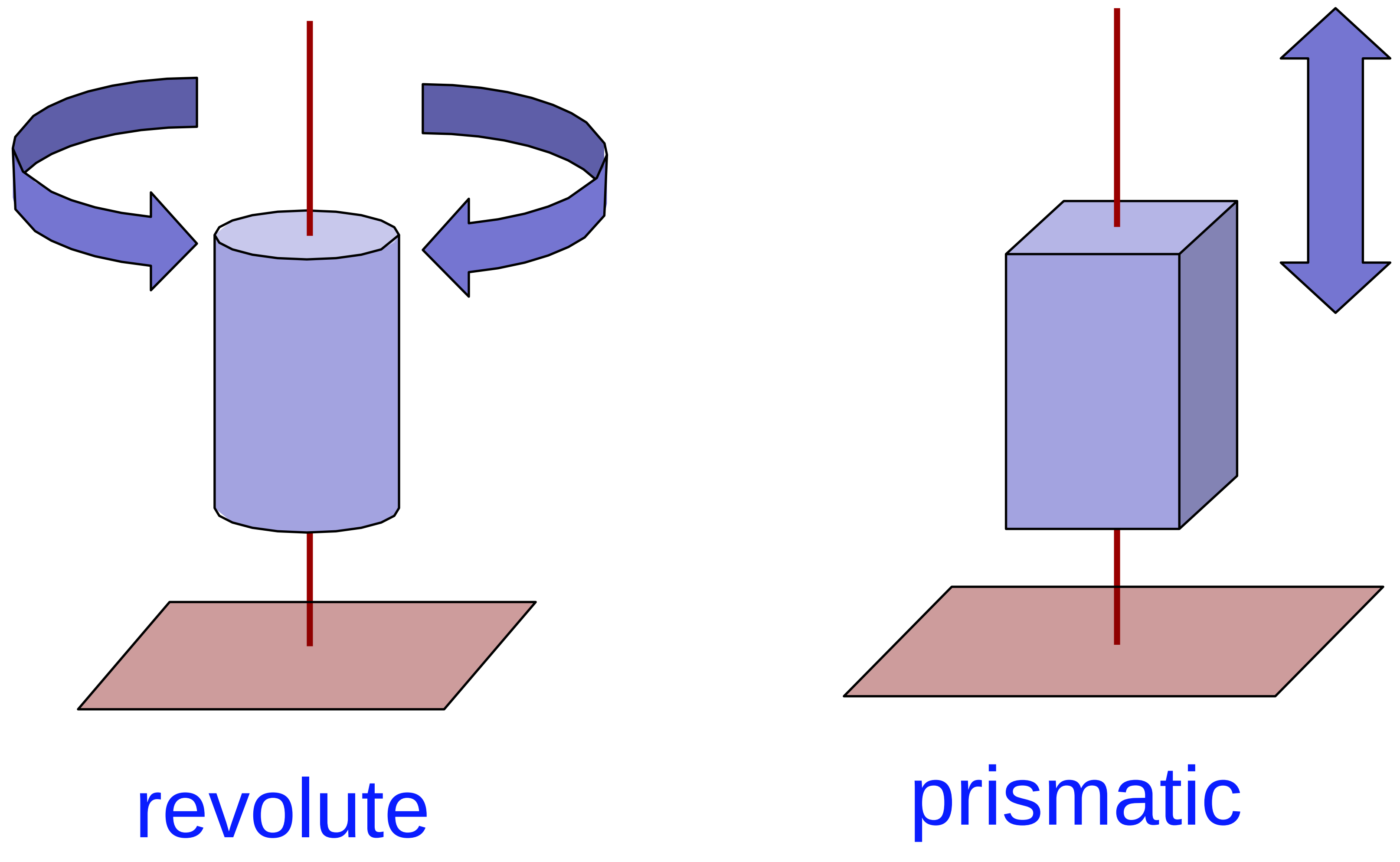
Direct Kinematic Map

Numbering links and joints



icon representation of joint types

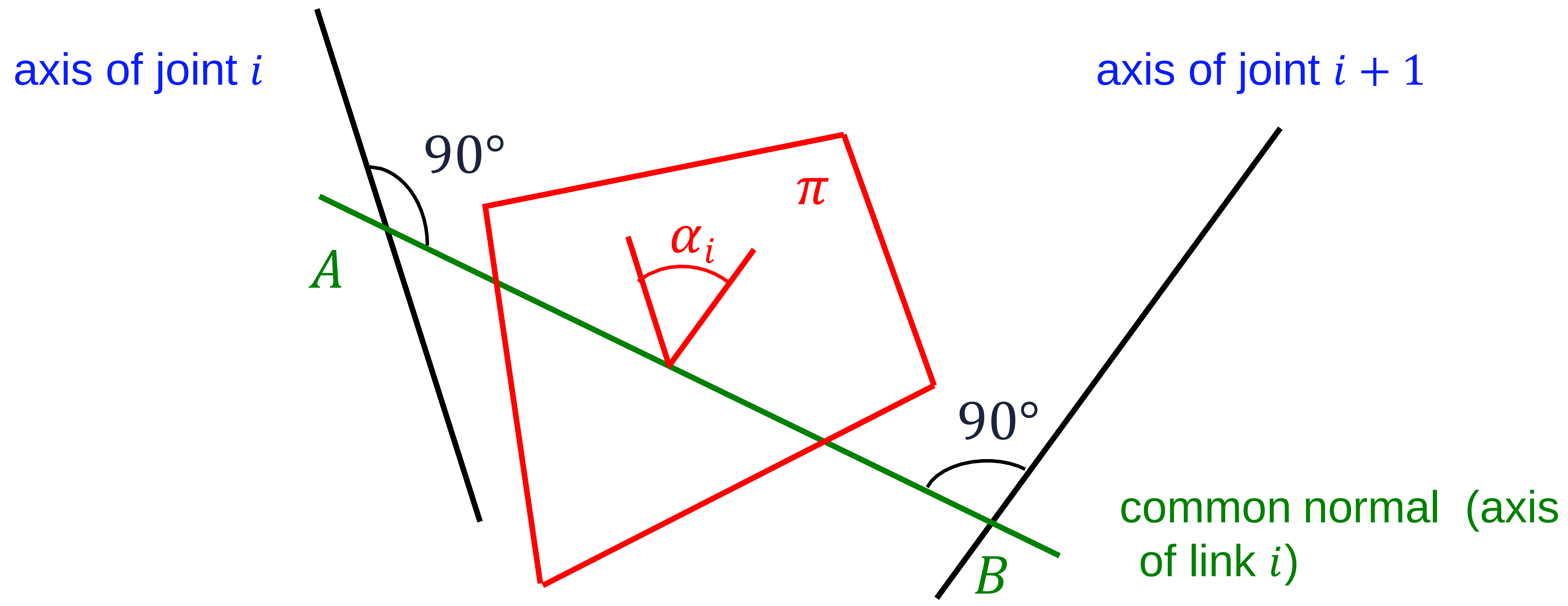
for the manipulator skeleton



J. Denavit and R.S. Hartenberg, "A kinematic notation for lower-pair mechanisms based on matrices," *Trans. ASME J. Applied Mechanics*, **23**: 215–221, 1955

Denavit-Hartenberg (DH) Layout

Spatial relation between joint axes



a_i = **displacement** AB between joint axes (always well defined)

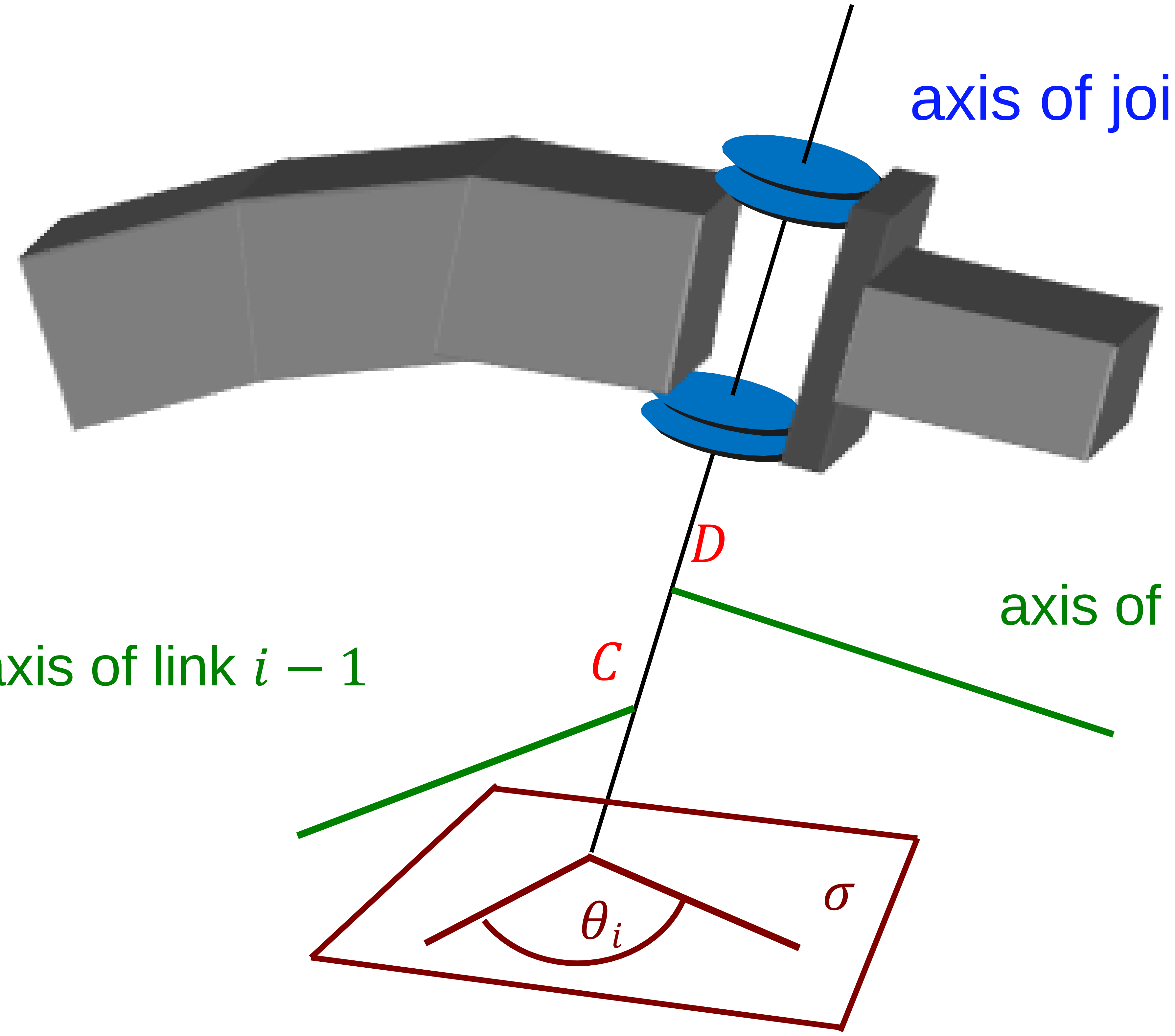
α_i = **twist angle** between joint axes

— projected on a plane π orthogonal to the link axis

with sign (pos/neg)!

Denavit-Hartenberg (DH) Layout

Spatial relation between link axes

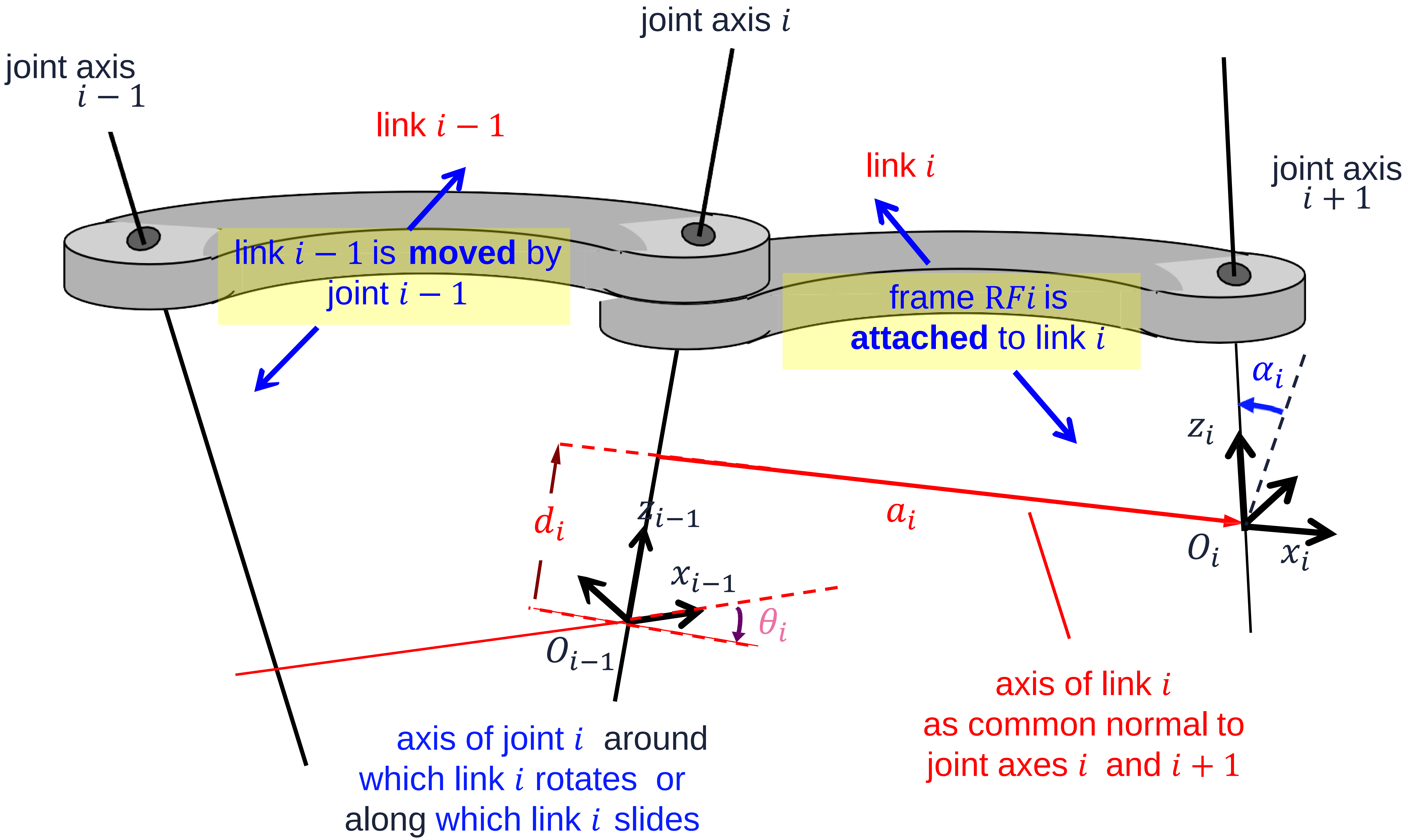


d_{i} = **displacement** CD (a variable if joint i is prismatic)
θ_i = **angle between link axes** (a variable if joint i is revolute)
— projected on a plane σ orthogonal to the joint axis

with sign (pos/neg)!

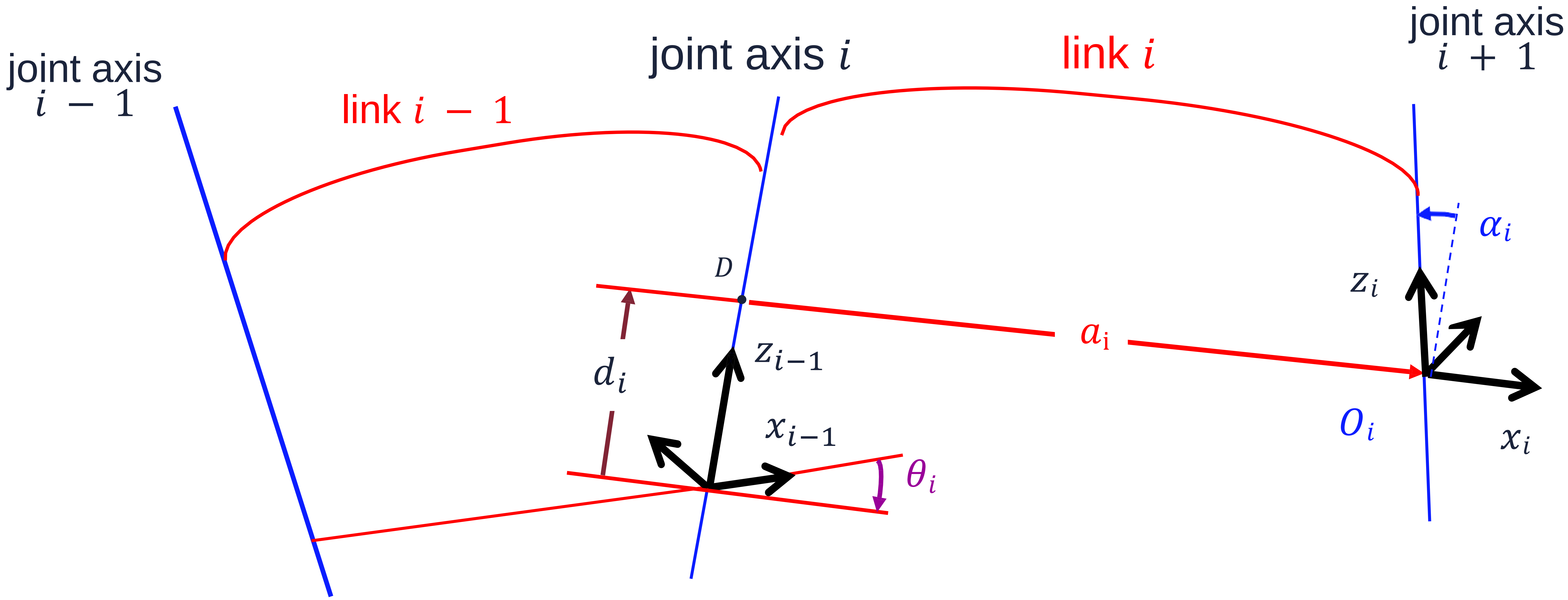
Denavit-Hartenberg (DH) Layout

Denavit-Hartenberg (DH) frame



Denavit-Hartenberg (DH) Layout

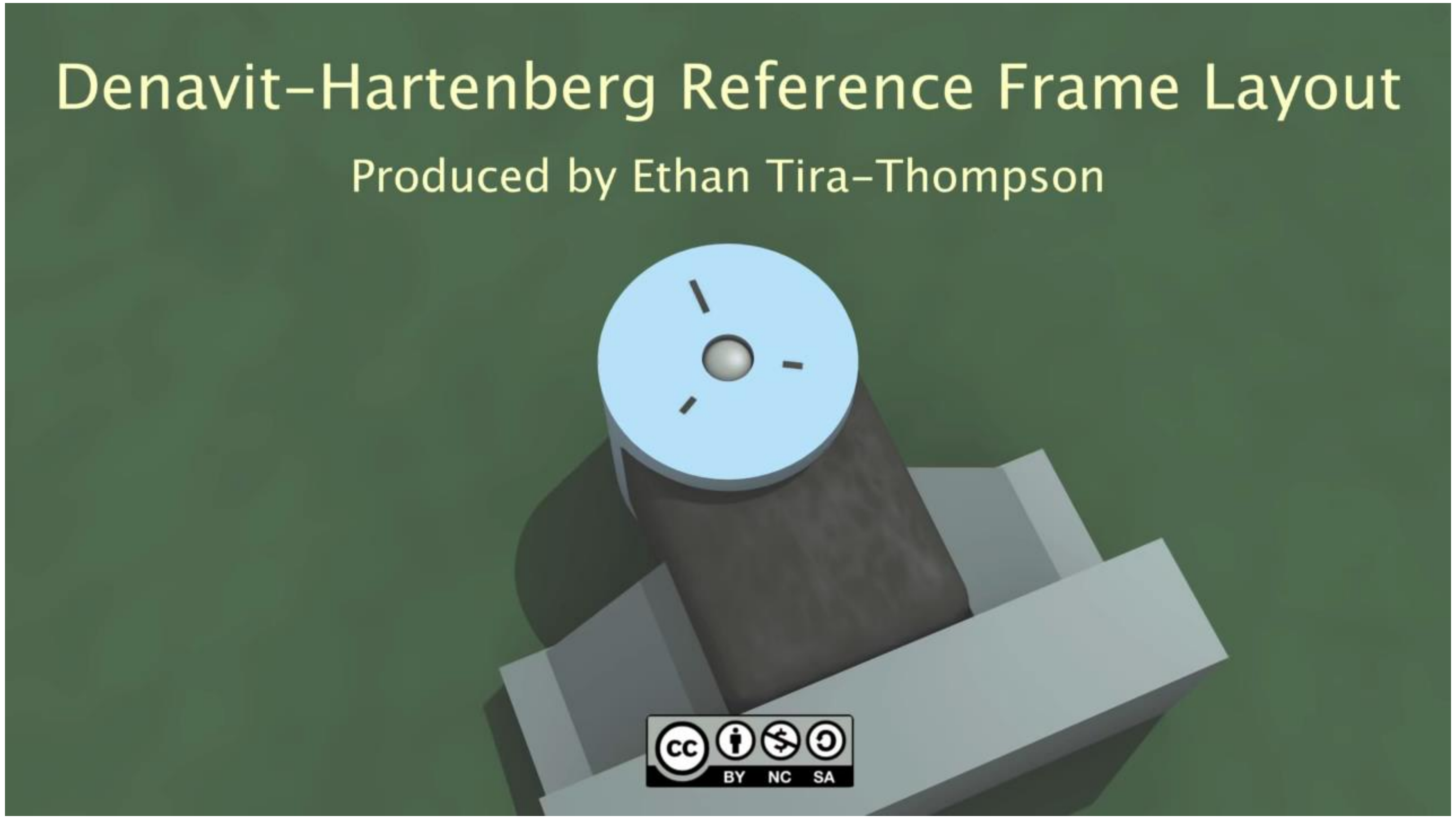
Definition of DH parameters



- unit vector z_i along axis of joint $i+1$
- unit vector x_i along the common normal to joint i and $i+1$ axes ($i \rightarrow i+1$)
- a_i = distance DO_i, + if oriented as x_i, always constant (= 'length' of link i)
- d_i = distance $O_{i-1}D$, + if oriented as z_{i-1}, variable if joint i is PRISMATIC
- α_i = twist angle from z_{i-1} to z_i around x_i, + if CCW, always constant
- θ_i = angle from x_{i-1} to x_i around z_{i-1}, + if CCW, variable if joint i is REVOLUTE

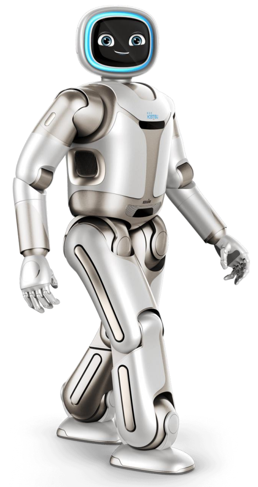
Denavit-Hartenberg (DH) Layout

DH layout made simple (a popular 3-minute illustration)



https://www.youtube.com/watch?v=rA9tm0gTln8

- **note**: the author of this video uses r in place of a, and does not add subscripts!



Ambiguities in Defining DH Frames

- frame 0 : origin and x_0 axis are arbitrary
- frame n : z_n axis is not specified
  - however, x_n must intersect and be chosen orthogonal to z_{n-1}
- positive direction of z_{i-1} (up/down on axis of joint i) is arbitrary
  - choose one, and try to 'avoid flipping over' to the next one
- positive direction of x_i (back/forth on axis of link i) is arbitrary
  - if successive joint axes are incident, we often take $x_i = z_{i-1} \times z_i$
  - when natural, follow the direction 'from base to tip'
- if z_i and z_{i-1} are parallel (common normal not uniquely defined)
  - O_i chosen arbitrarily along z_i, still trying to 'zero out' parameters
- if z_{i-1} and z_i are coincident, normal x_i axis can be chosen at will
  - this case occurs only if the two joints are of different kind (P/R or R/P)
  - again, try using 'simple values' (e.g., 0 or $\pm\pi/2$) for constant angles

Homogeneous Transformation

Homogeneous transformation (between successive DH frames)
(from i-1 to frame i)

- roto-translation (screw motion) around and along z_{i-1}

$$
{}^{i-1}A_{i'}(q_i) = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

the product of these two matrices commutes!

rotational joint $\Rightarrow q_i = \theta_i$

prismatic joint $\Rightarrow q_i = d_i$

- roto-translation (screw motion) around and along x_i

$$
{}^{i-1}A_i = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

← always a constant matrix

Homogeneous Transformation

Denavit-Hartenberg matrix

J. Denavit and R.S. Hartenberg, "A kinematic notation for lower-pair mechanisms based on matrices," *Trans. ASME J. Applied Mechanics*, **23**: 215–221, 1955

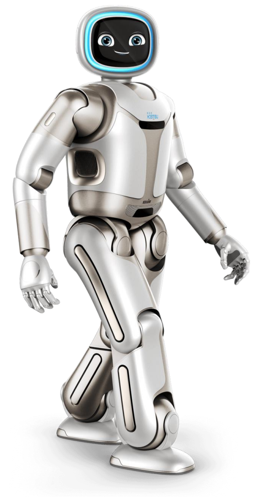
$$
{}^{i-1}A_i(q_i) = {}^{i-1}A_{i'}(q_i)\,{}^{i'}A_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i\sin\theta_i & \sin\alpha_i\sin\theta_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\alpha_i\cos\theta_i & -\sin\alpha_i\cos\theta_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

compact notation: $c = \cos, s = \sin$

super-compact notation (if feasible): $c_i = \cos q_i\,, s_i = \sin q_i$

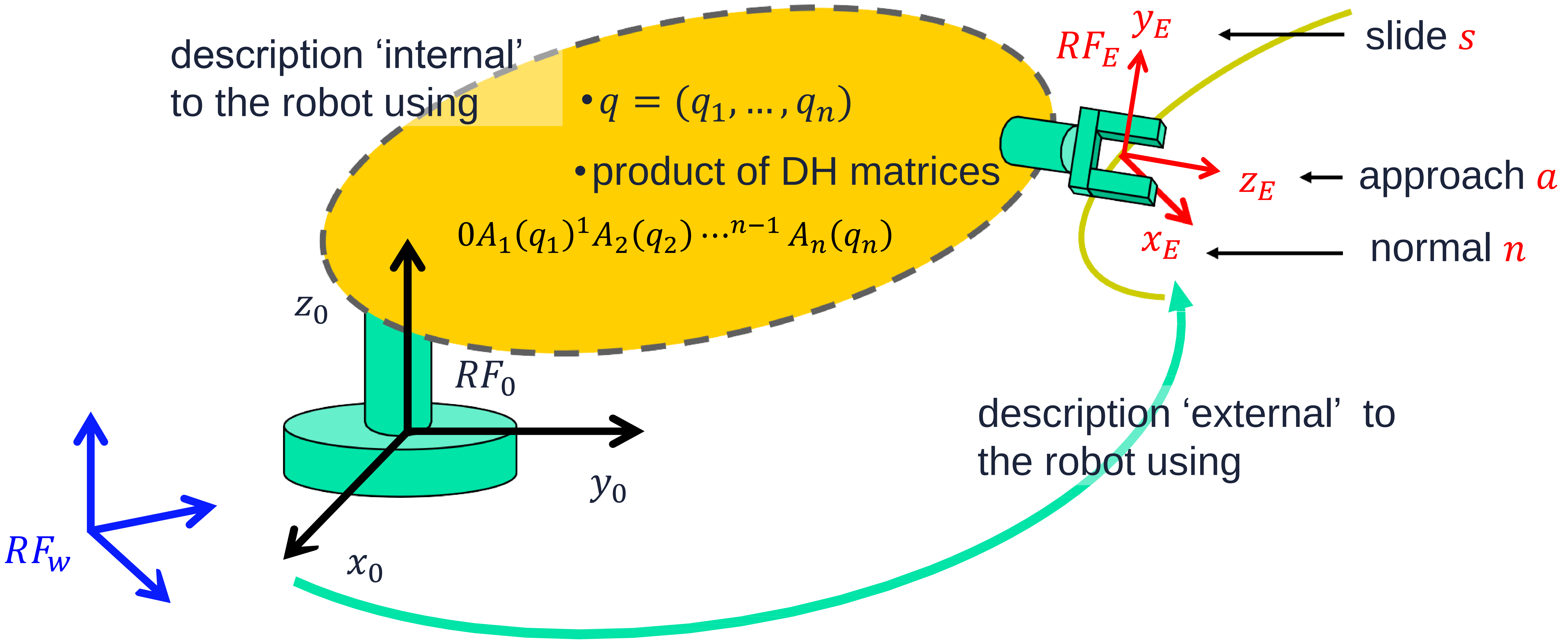
$c_{ij} = \cos(q_i + q_j), s_{ij} = \sin(q_i + q_j)$

…



Direct Kinematics

Direct kinematics of robot manipulators



$$
{}^wT_E = {}^wT_0\,{}^0A_1(q_1)\,{}^1A_2(q_2)\cdots{}^{n-1}A_n(q_n)\,{}^nT_E
$$

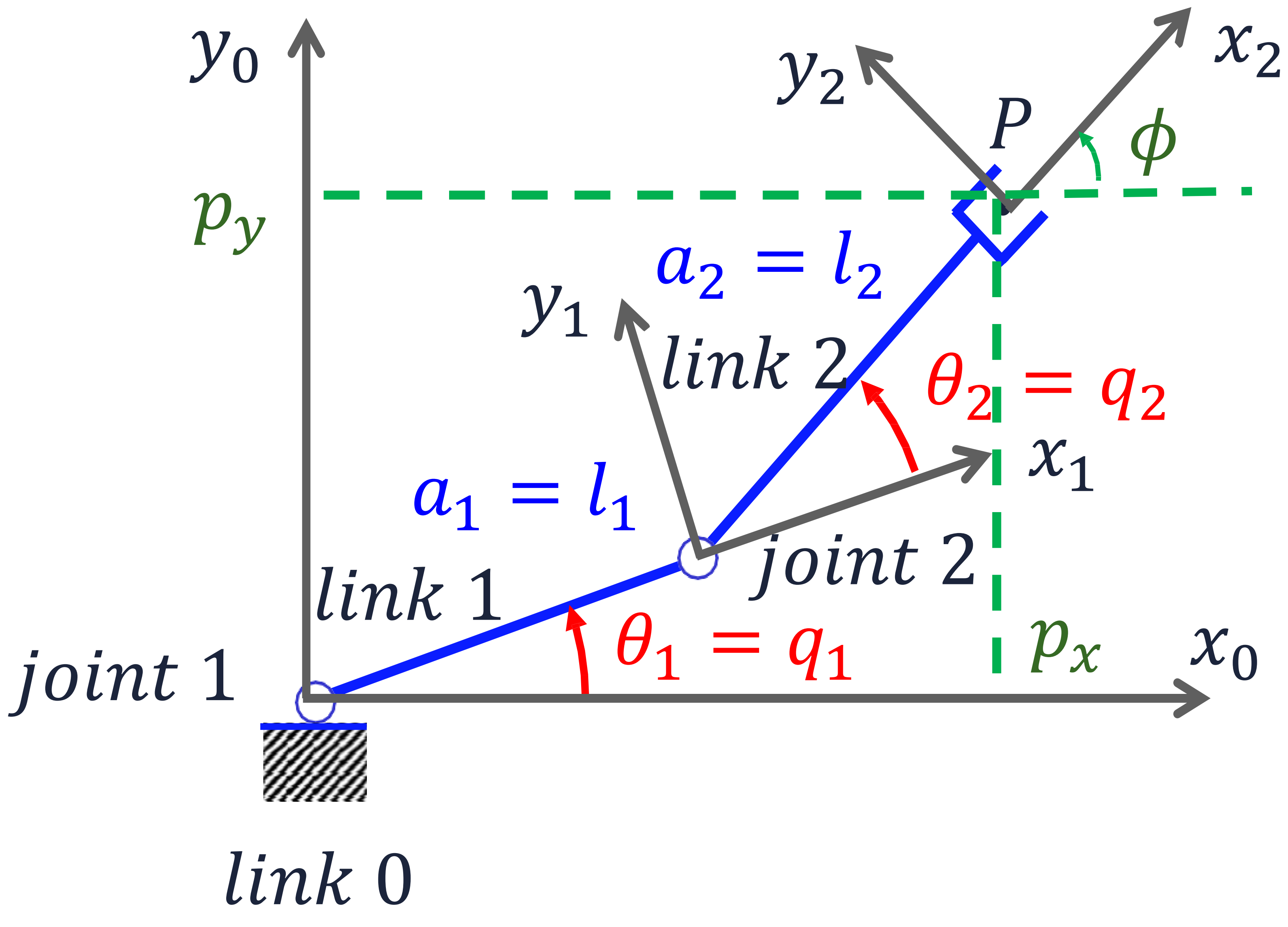
$$
r = f_r(q)
$$

alternative representations of the **direct kinematics**

- $${}^wT_E = \begin{bmatrix} {}^wR_E & {}^wp_{wE} \\ 0^T & 1 \end{bmatrix}$$
- $r = (r_1, \dots, r_m)$

Direct Kinematics

Direct kinematics of 2R planar robot (using DH frame assignment…)



z_0, z_1, z_2 outgoing from plane

i	α_i	a_i	d_i	θ_i
1	0	l_i	0	q_i
2	0	l_2	0	q_2

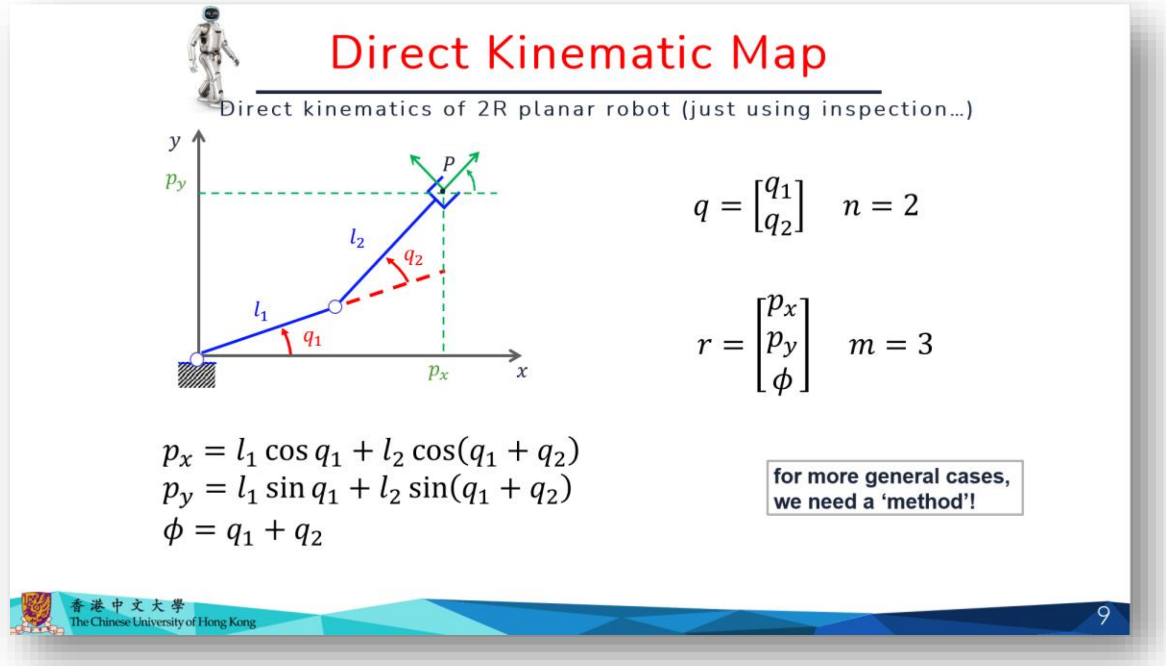
$$^0A_1(\theta_1) = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & l_1 c\theta_1 \\ s\theta_1 & c\theta_1 & 0 & l_1 s\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

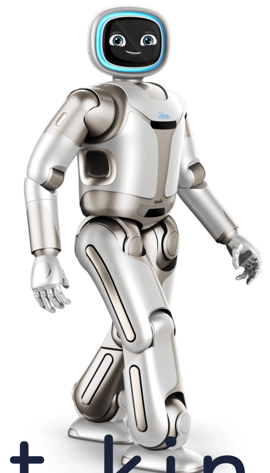
$$^1A_2(\theta_2) = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & l_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & l_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2(q) = \begin{bmatrix} c_{12} & -s_{12} & 0 & l_1 c_1 + l_2 c_{12} \\ s_{12} & c_{12} & 0 & l_1 s_1 + l_2 s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} p \\ 1 \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \\ 0 \\ 1 \end{bmatrix} = {}^0A_2(q) \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 S_1 + l_2 S_{12} \\ 0 \\ 1 \end{bmatrix}$$

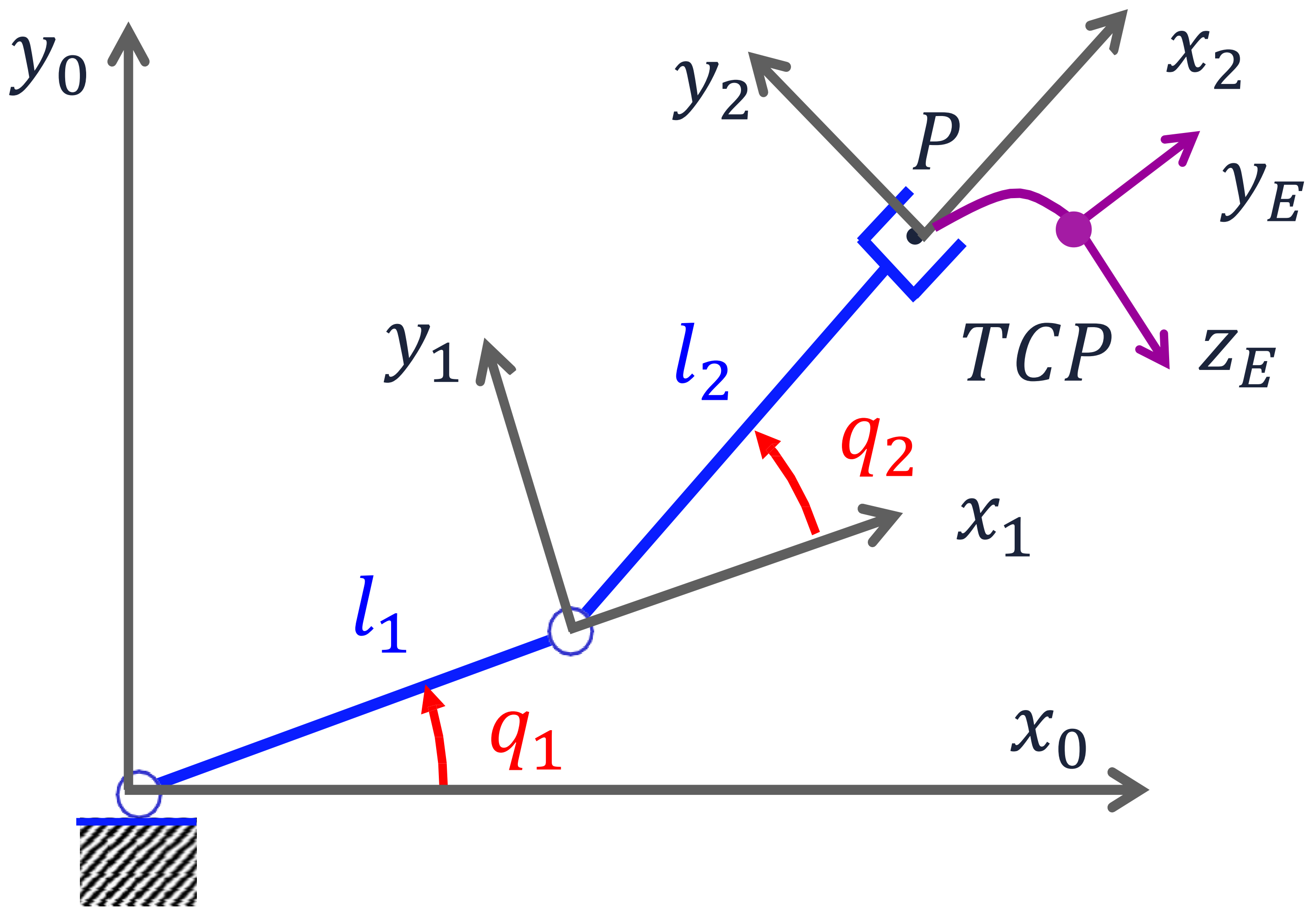
$\phi = q_1 + q_2$ (extracted from $^0R_2(q)$)





Direct Kinematics (2R PR)

Direct kinematics of 2R planar robot (TCP location on the robot end effector)



i	α_i	a_i	d_i	θ_i
1	0	l_i	0	q_i
2	0	l_2	0	q_2

$$
{}^0A_2(q) = \begin{bmatrix} c_{12} & -s_{12} & 0 & l_1c_1 + l_2c_{12} \\ s_{12} & c_{12} & 0 & l_1s_1 + l_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

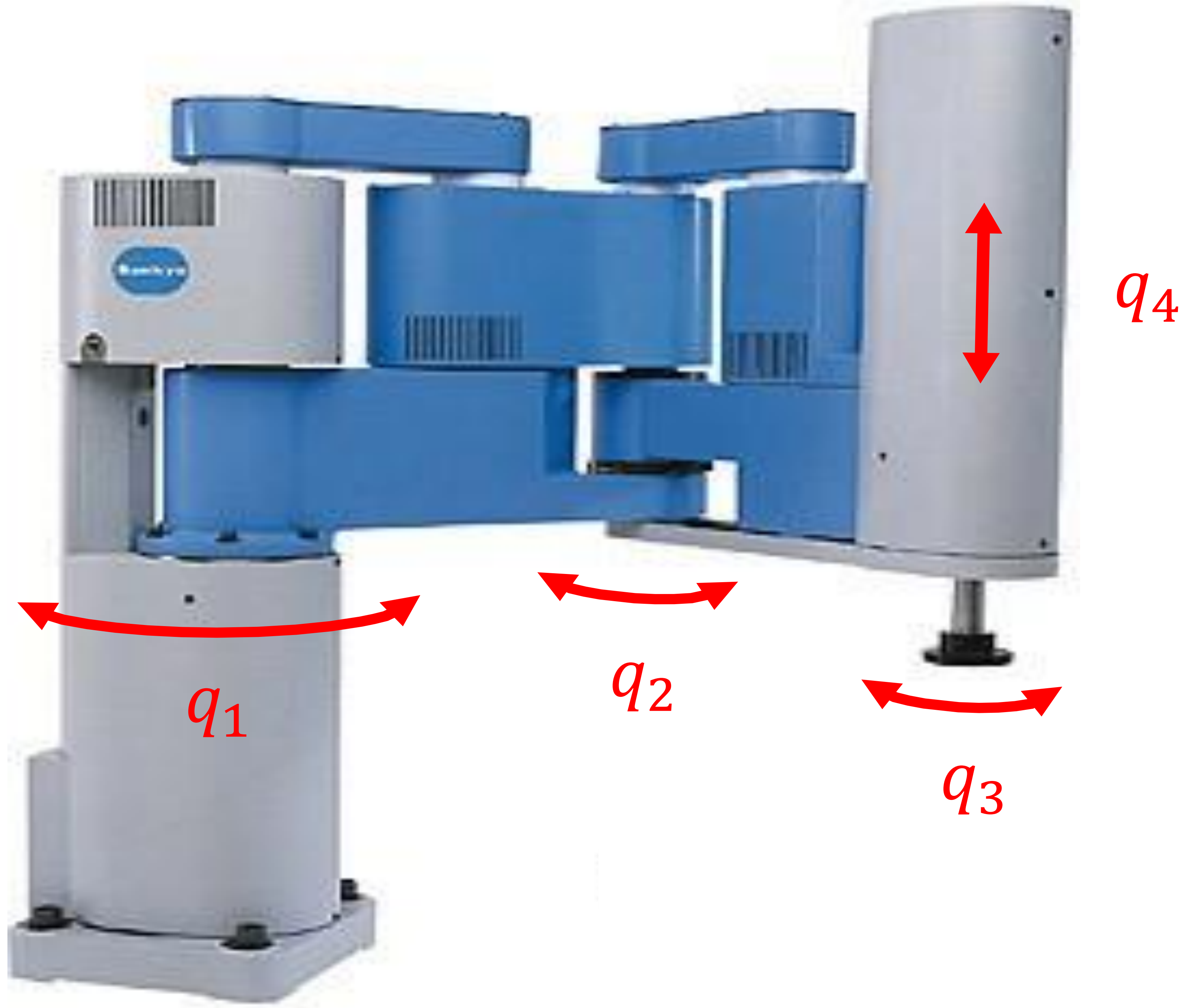
Tool Center Point TCP and associated end-effector frame RF_E

$$
{}^2T_E = \begin{bmatrix} 0 & 1 & 0 & {}^2TCP_x \\ 0 & 0 & -1 & {}^2TCP_y \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^0TCP(q) \\ 1 \end{bmatrix} = \begin{bmatrix} {}^0TCP_x(q) \\ {}^0TCP_y(q) \\ 0 \\ 1 \end{bmatrix} = {}^0A_2(q) \begin{bmatrix} {}^2TCP_x \\ {}^2TCP_y \\ 0 \\ 1 \end{bmatrix} = {}^0A_2(q) \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = {}^0A_2(q)\,{}^2T_E
$$

Direct Kinematics (SCARA)

DH assignment for a SCARA robot

Sankyo SCARA 8438



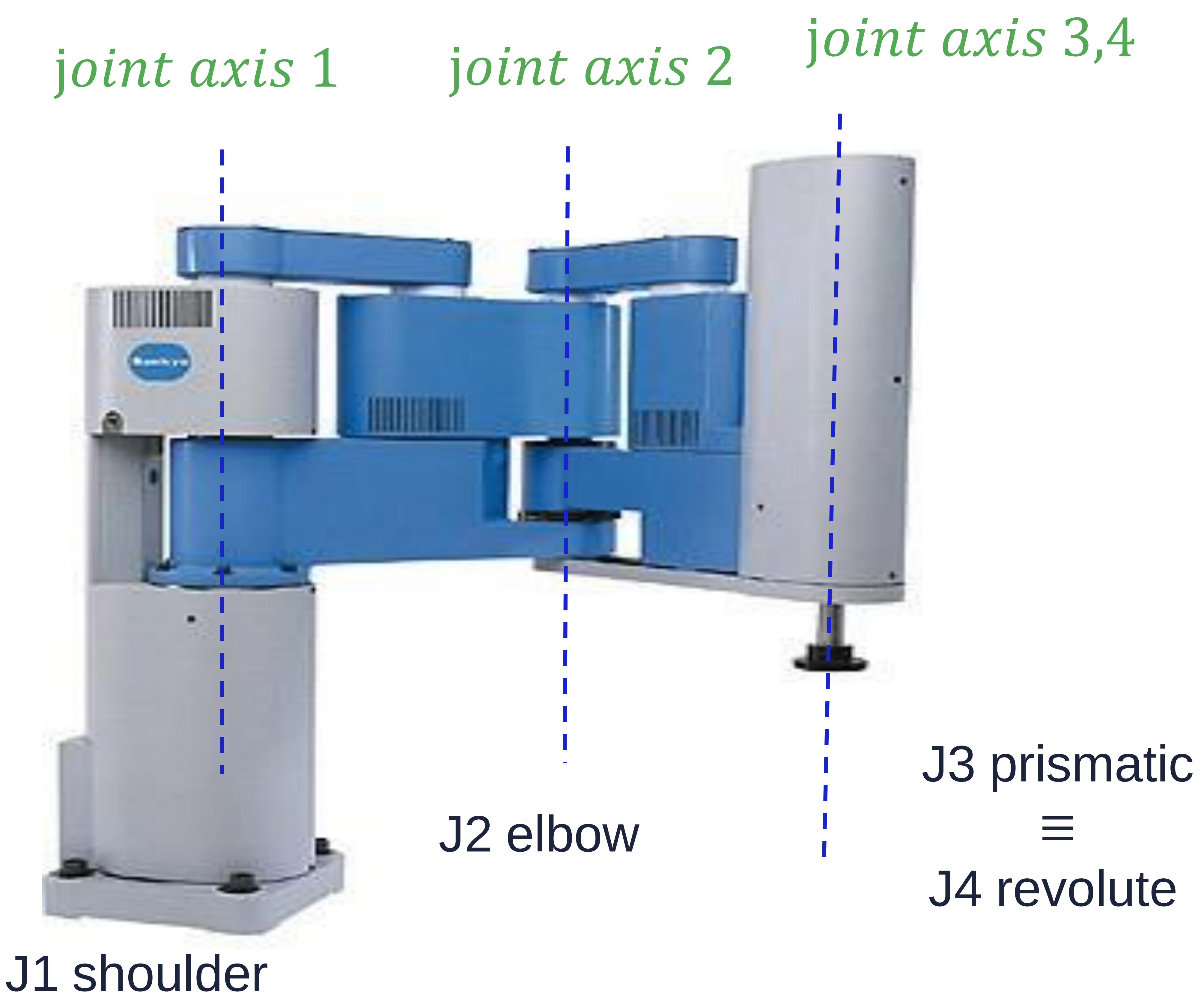
Sankyo SCARA SR 8447

Direct Kinematics (SCARA)

Step 1: joint axes

all parallel
(or coincident)

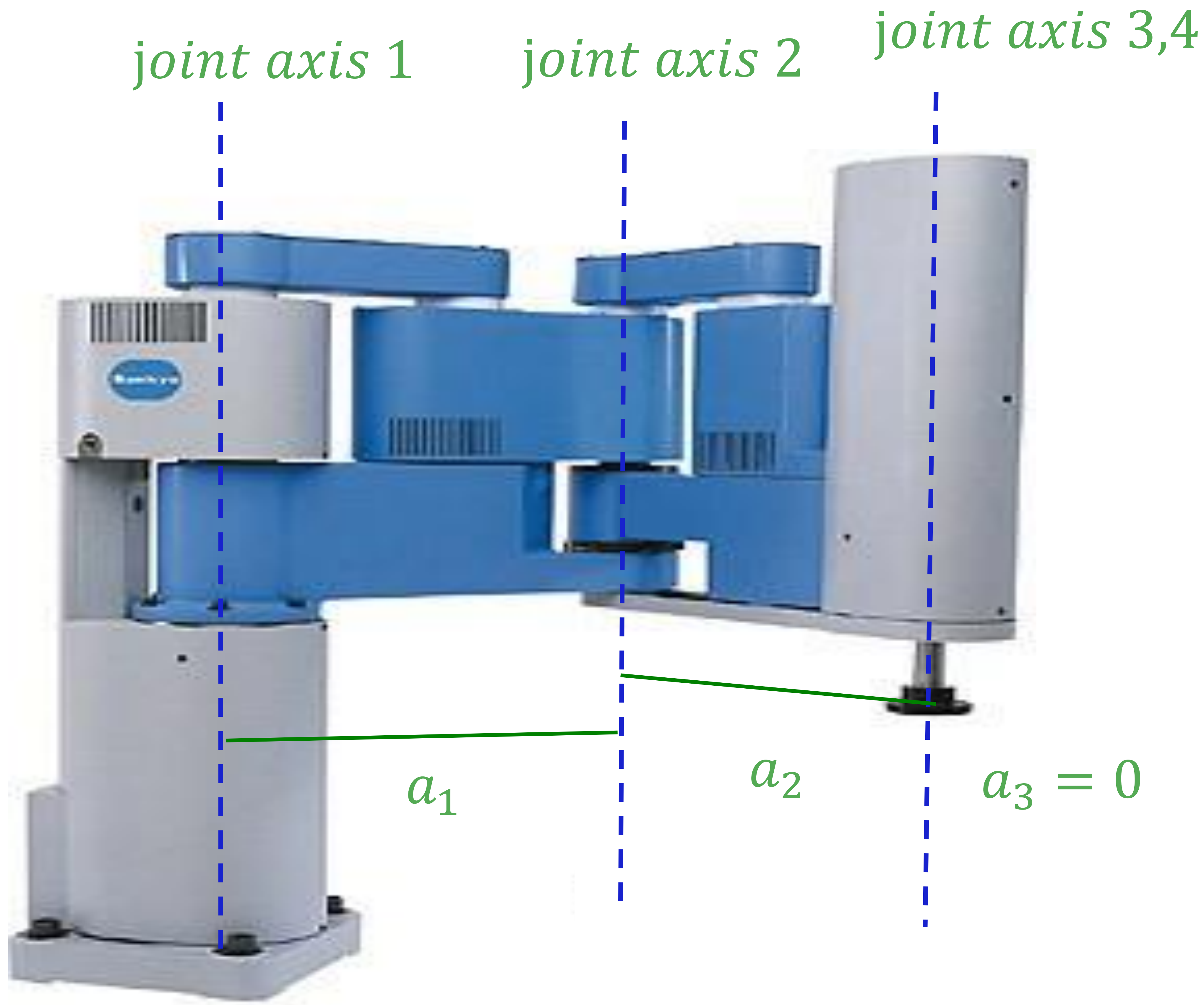
twist angles
$\alpha_i = 0$ or π



Direct Kinematics (SCARA)

Step 2: link axes

the vertical 'heights' of
the link axes
are arbitrary
(for the time being)

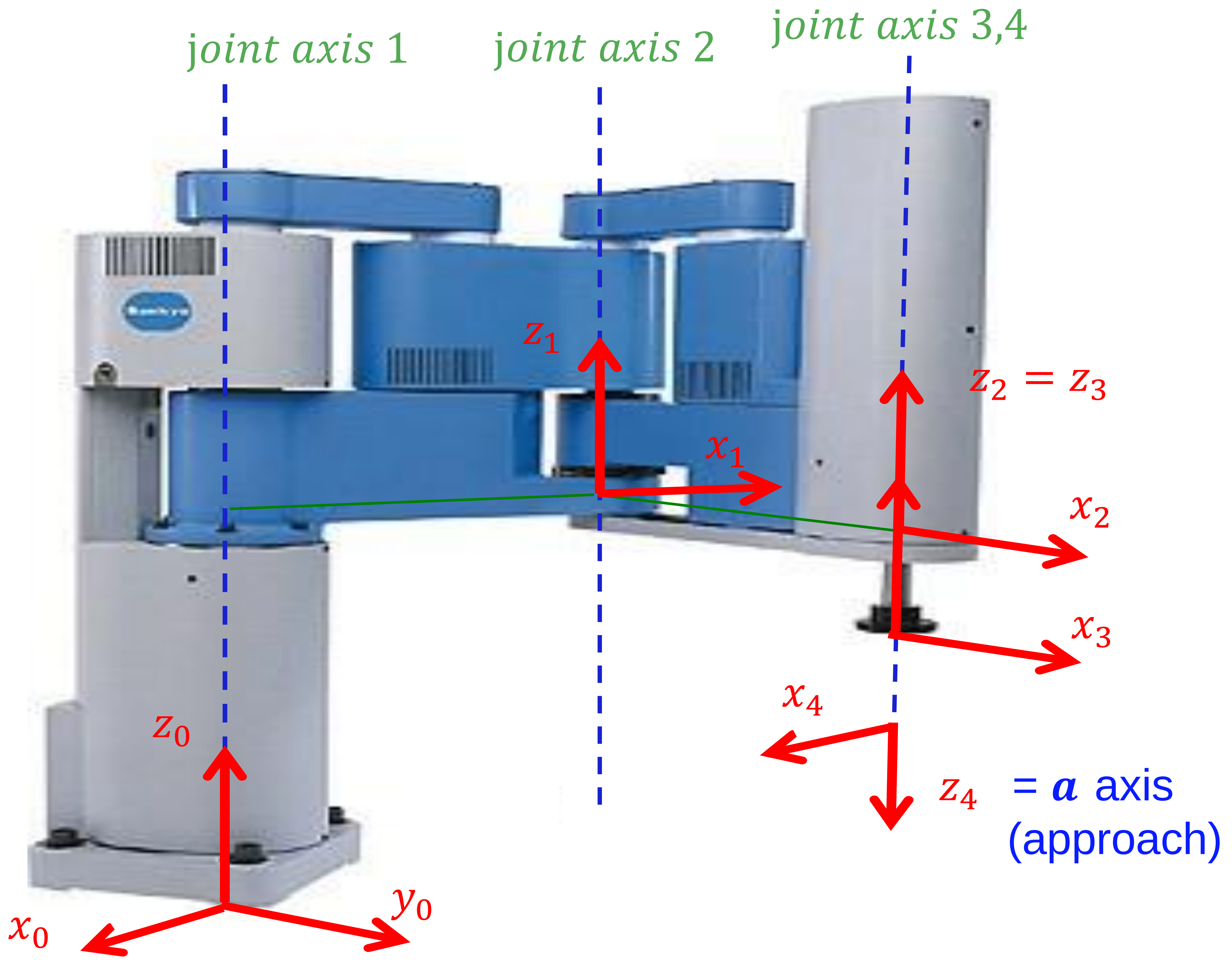


Direct Kinematics (SCARA)

Step 3: frames

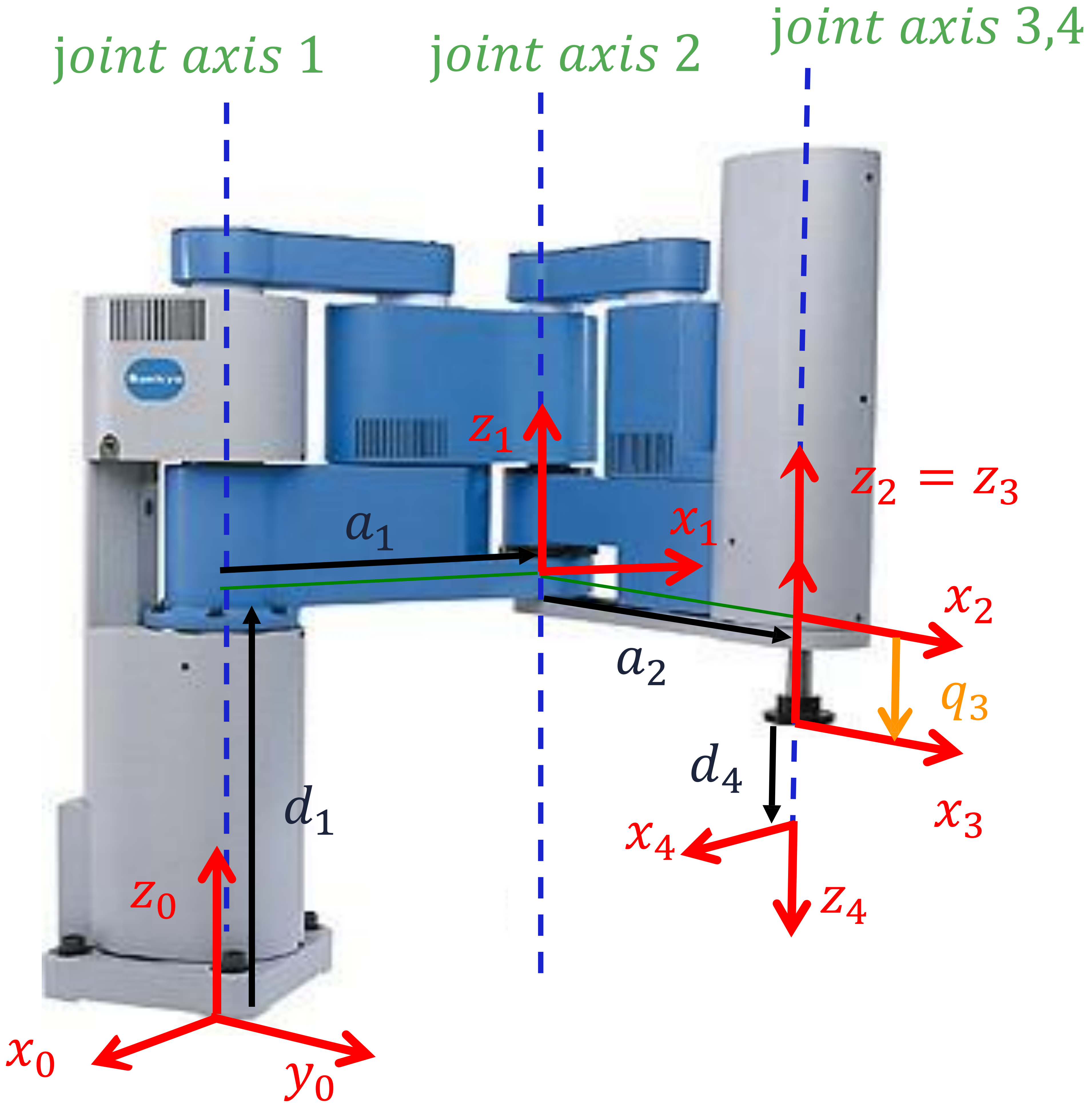
axes $\boldsymbol{y}_i$ for $i > 0$
are not shown

(nor needed; they form
right-handed frames)



Direct Kinematics (SCARA)

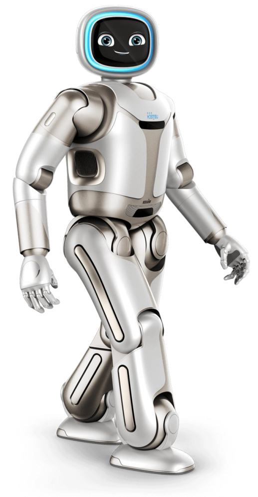
Step 4: DH table of parameters



i	α_i	a_i	d_i	θ_i
1	0	a_1	d_1	q_1
2	0	a_2	0	q_2
3	0	0	q_3	0
4	π	0	d_4	q_4

note that

- d_1 and d_4 could be set = 0
- $d_4 < 0$ (opposite to $\boldsymbol{z}_3$)
- also, $q_3 < 0$ in this configuration



Direct Kinematics (SCARA)

Step 5: DH transformation matrices

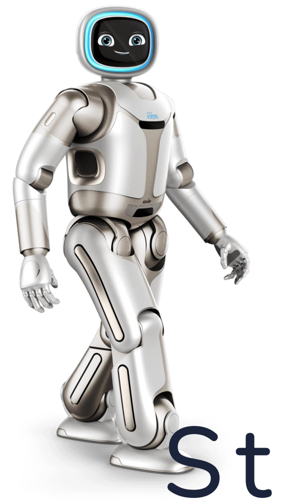
$$^{0}A_1(q_1) = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & a_1 c\theta_1 \\ s\theta_1 & c\theta_1 & 0 & a_1 s\theta_1 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{2}A_3(q_3) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{1}A_2(q_2) = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & a_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{3}A_4(q_4) = \begin{bmatrix} c\theta_4 & s\theta_4 & 0 & 0 \\ s\theta_4 & -c\theta_4 & 0 & 0 \\ 0 & 0 & -1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$q = (q_1, q_2, q_3, q_4) = (\theta_1, \theta_2, d_3, \theta_4)$$



Direct Kinematics (SCARA)

Step 6a: direct kinematics (homogeneous matrix $^{w}\boldsymbol{T}_{E}$ as product of the $^{i-1}\boldsymbol{A}_{i}(\boldsymbol{q}_{i})'\boldsymbol{s}$)

$$^{0}A_{2}(q_1,q_2)=\begin{bmatrix} c_{12} & -s_{12} & 0 & a_1c_1+a_2c_{12} \\ s_{12} & c_{12} & 0 & a_1s_1+a_2s_{12} \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}A_{3}(q_1,q_2,q_3)=\begin{bmatrix} c_{12} & -s_{12} & 0 & a_1c_1+a_2c_{12} \\ s_{12} & c_{12} & 0 & a_1s_1+a_2s_{12} \\ 0 & 0 & 1 & d_1+q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$(^{w}T_0 =^{4} T_E = I)$

$R(q_1,q_2,q_4)=[n \quad s \quad a]$ $\quad p=p(q_1,q_2,q_3)$

$$^{w}T_E = {}^{0}A_4(q_1,q_2,q_3,q_4)=\begin{bmatrix} c_{124} & s_{124} & 0 & a_1c_1+a_2c_{12} \\ s_{124} & -c_{124} & 0 & a_1s_1+a_2s_{12} \\ 0 & 0 & -1 & d_1+q_3+d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Direct Kinematics (SCARA)

Step 6b: direct kinematics (as task vector $\boldsymbol{r}$)

$$ {}^{w}T_E = {}^{0}A_4(q_1,q_2,q_3,q_4) = \begin{bmatrix} c_{124} & s_{124} & 0 & a_1c_1 + a_2c_{12} \\ s_{124} & -c_{124} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & -1 & d_1 + q_3 + d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

extract α_z from $R(q_1,q_2,q_4)$

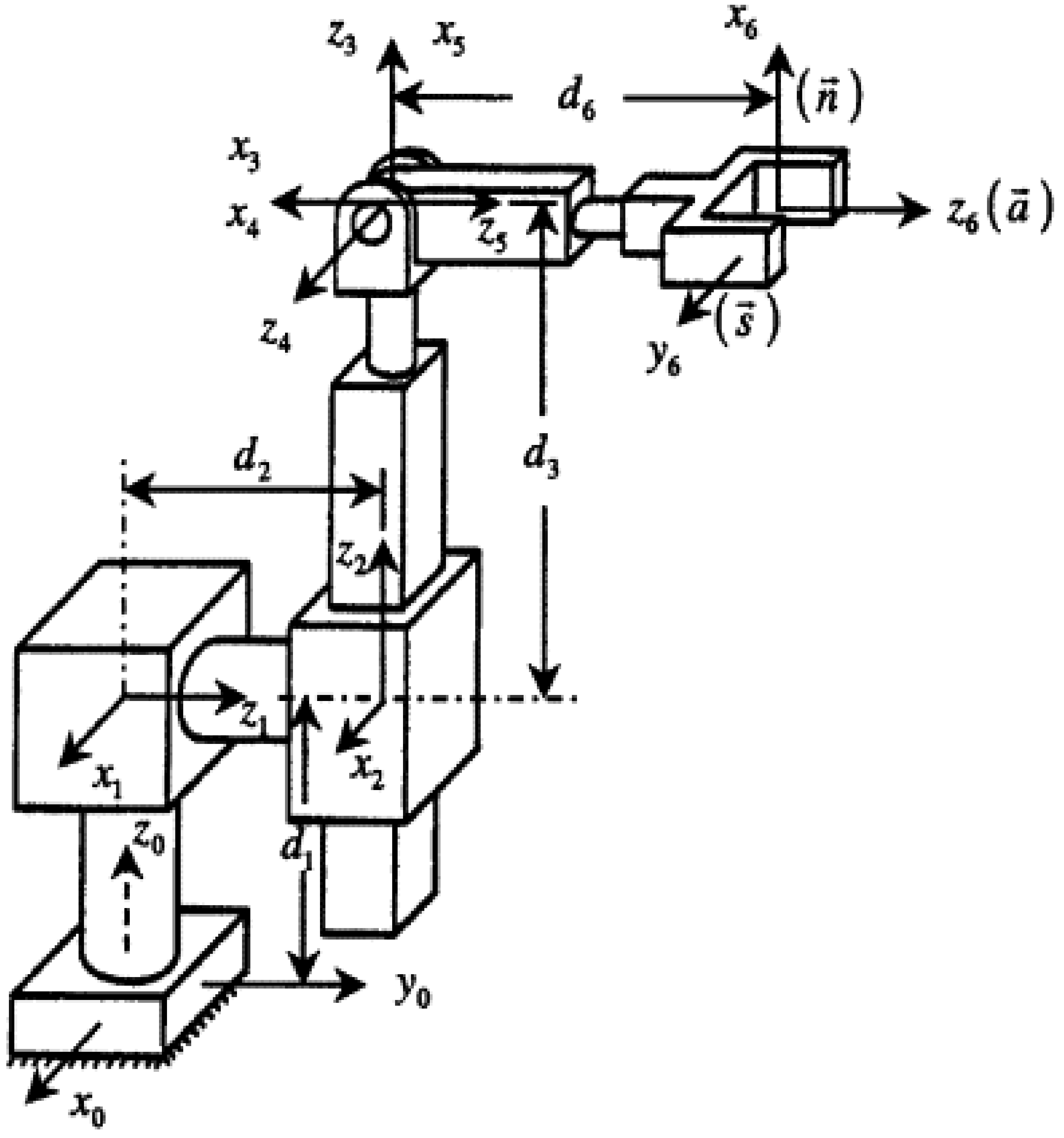
take $p \in \mathbb{R}^4$ as such from $p(q_1,q_2,q_3)$

$$ r = \begin{bmatrix} p_x \\ p_y \\ p_z \\ \alpha_z \end{bmatrix} = f_r(q) = \begin{bmatrix} a_1c_1 + a_2c_{12} \\ a_1s_1 + a_2s_{12} \\ d_1 + q_3 + d_4 \\ q_1 + q_2 + q_4 \end{bmatrix} \in \mathbb{R}^4 $$

Direct Kinematics (Stanford)

Stanford manipulator

- 6-dof: 2R-1P-3R (spherical wrist)

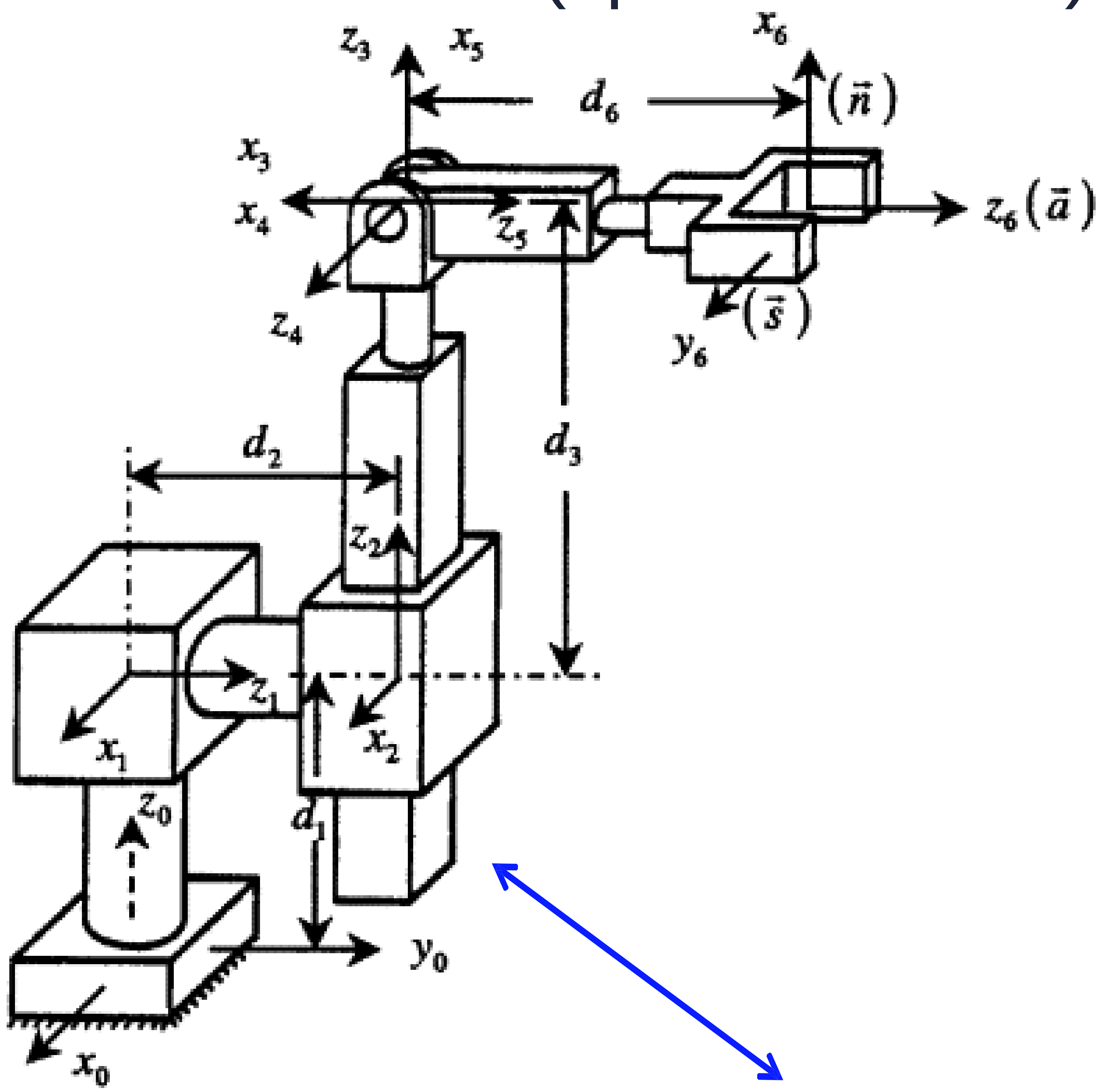


- robot with shoulder offset
- 'one possible' DH assignment of frames is shown
- determine the associated
  - table of DH parameters
  - homogeneous transformation matrices
  - direct kinematics
- write a program for computing the direct kinematics
  - numerically (Matlab), given a $\boldsymbol{q}$
  - symbolically (Mathematica, Maple, Symbolic Manipulation Toolbox of Matlab, …)

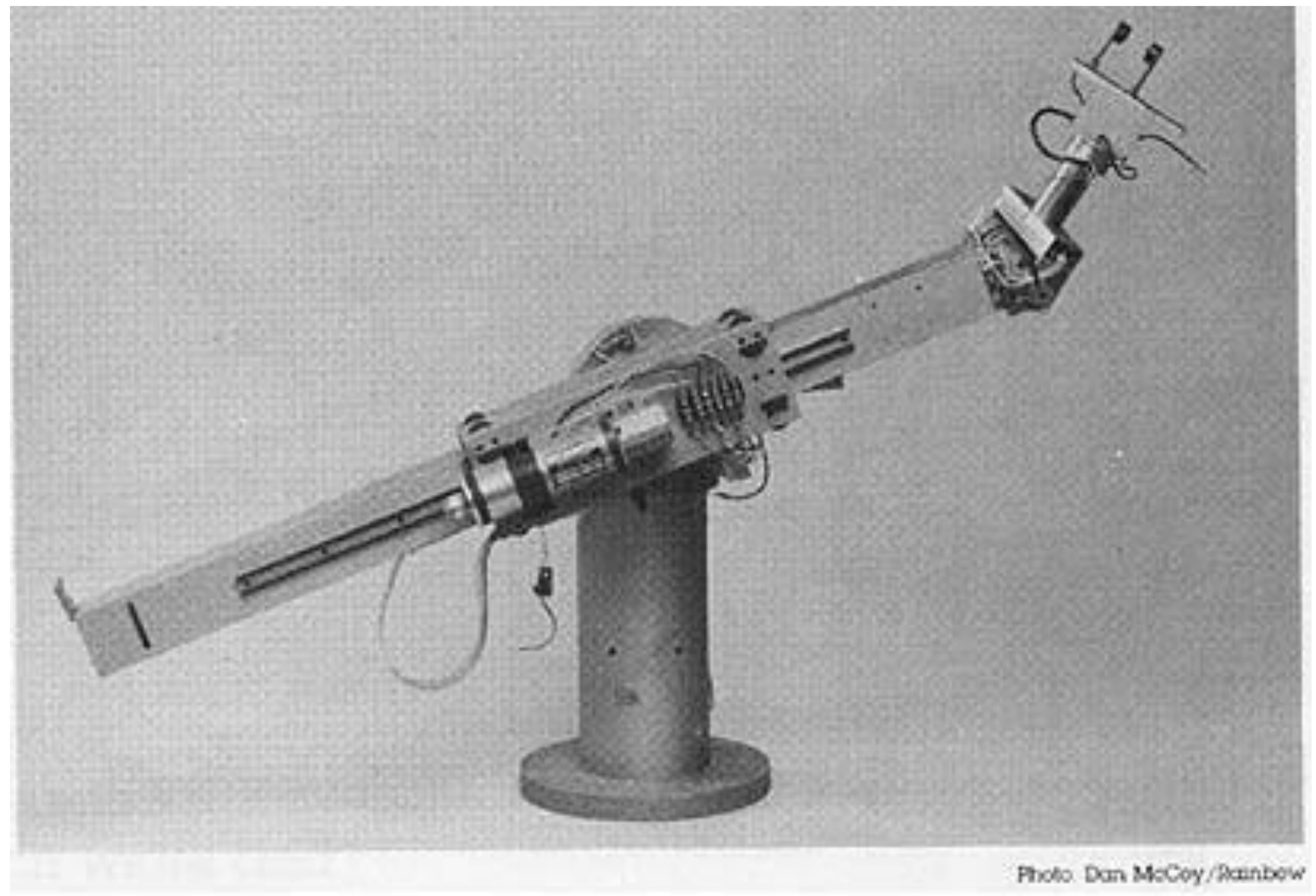
Direct Kinematics (Stanford)

DH table for Stanford manipulator

- 6-dof: 2R-1P-3R (spherical wrist)



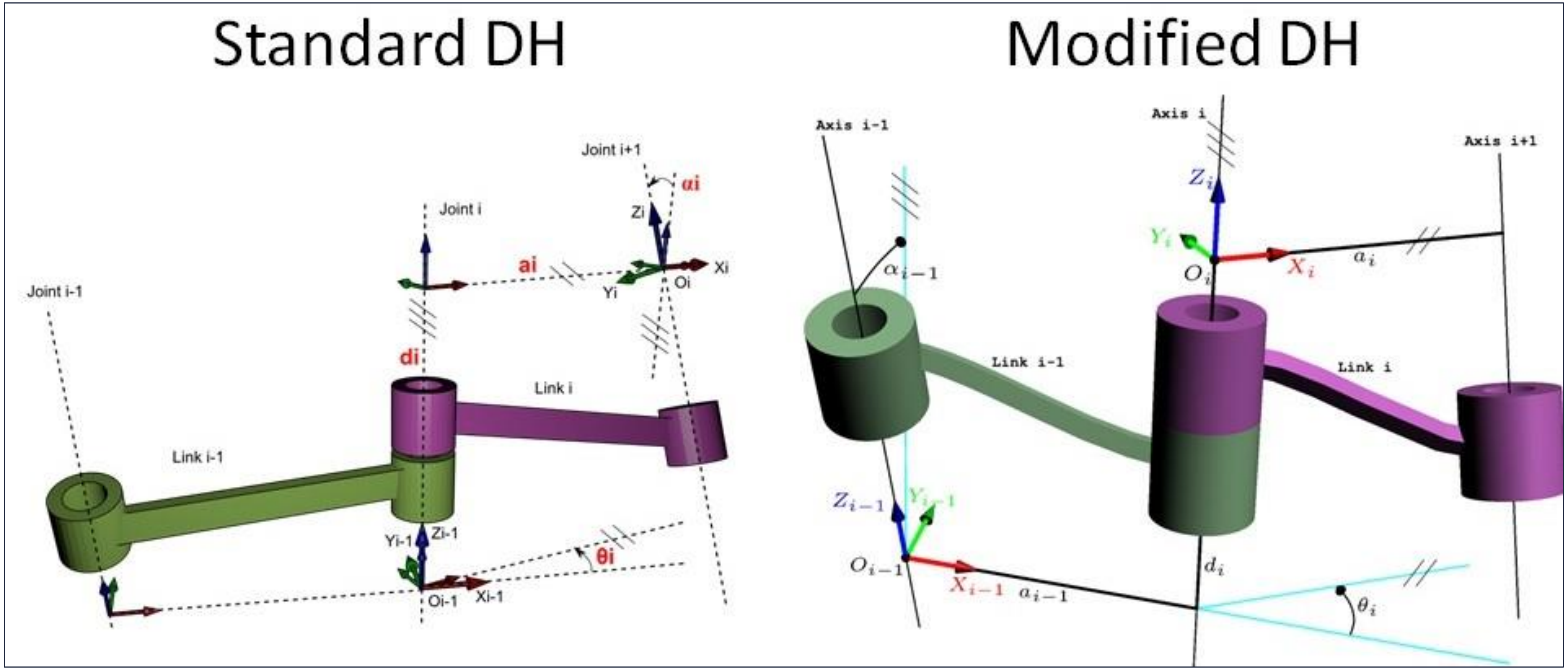
joint variables are in red, while their values in the robot configuration shown are in blue



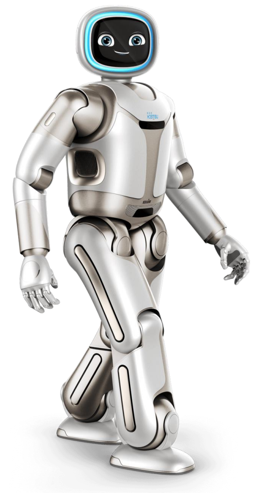
i	α_i	a_i	d_i	θ_i
1	$-\pi/2$	0	$d_1 > 0$	$q_1 = 0$
2	$\pi/2$	0	$d_2 > 0$	$q_2 = 0$
3	0	0	$q_3 > 0$	$-\pi/2$
4	$-\pi/2$	0	0	$q_4 = 0$
5	$\pi/2$	0	0	$q_5 = -\pi/2$
6	0	0	$d_6 > 0$	$q_6 = 0$

Modified DH

Do not be confused!



- Both are used in textbooks.
- We use Standard DH in ENGG5402 to keep the consistence.

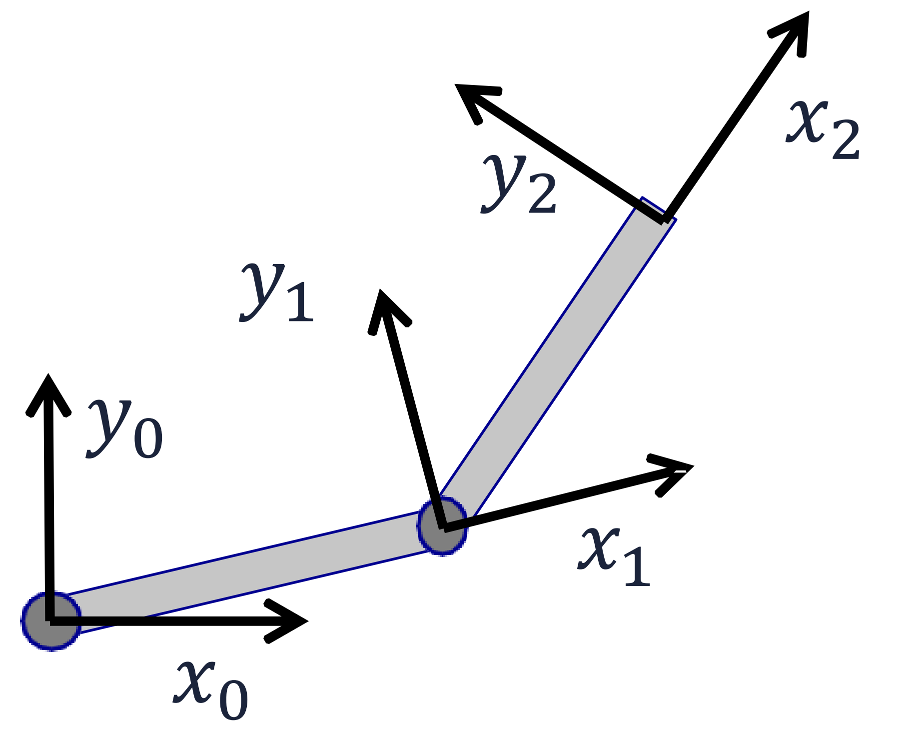


Modified DH

- a modified version introduced in J. Craig's book "Introduction to Robotics" (1986) and aligned for the indexing by Khalil and Kleinfinger (ICRA, 1986)
  - has z_i axis on joint i
  - $a_i \& \alpha_i$ =distance & twist angle from z_{i-1} to z_i, measured along & about x_{i-1}
  - $d_i \& \theta_i$ = distance & angle from x_{i-1} to x_i , measured along & about z_i
  - source of much confusion... if you are not aware of it (or don't mention it!)
  - convenient with link flexibility: a rigid frame at the base, another at the tip...

classical
(or distal)

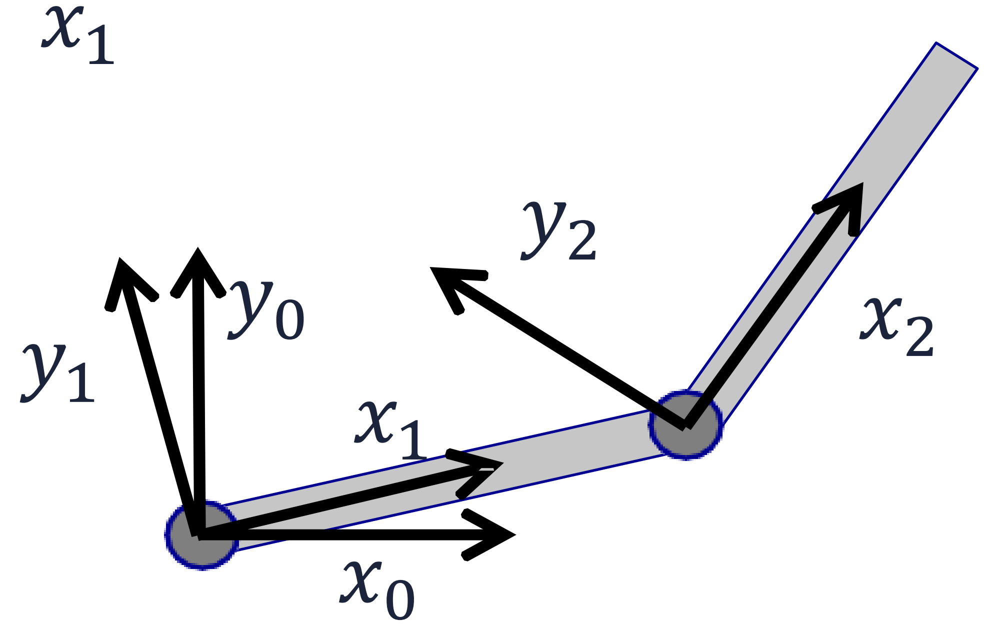
$$ {}^{i-1}A_i = \begin{bmatrix} c\theta_i & -c\alpha_i s\theta_i & s\alpha_i s\theta_i & a_i c\theta_i \\ s\theta_i & c\alpha_i c\theta_i & -s\alpha_i c\theta_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$



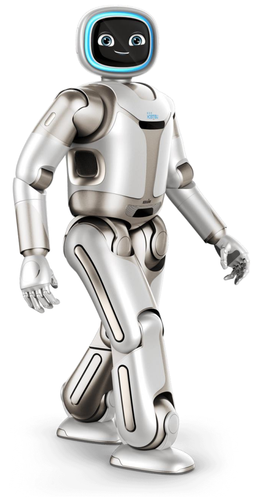
planar 2R example

modified
(or proximal)

$$ {}^{i-1}A_i^{mod} = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_i \\ c\alpha_i s\theta_i & c\alpha_i c\theta_i & -s\alpha_i & -d_i s\alpha_i \\ s\alpha_i s\theta_i & s\alpha_i c\theta_i & c\alpha_i & d_i c\alpha_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

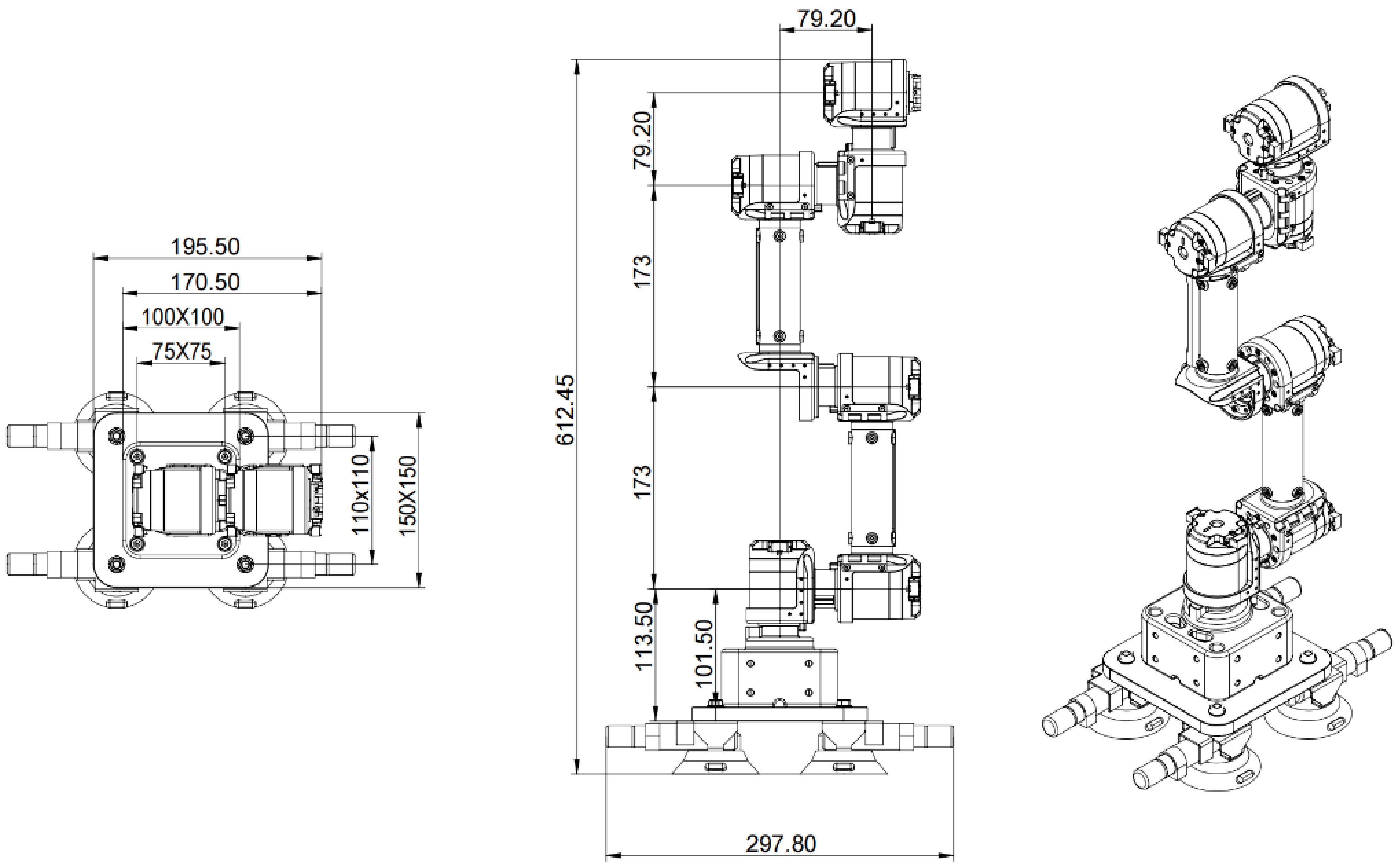


modified DH tends to place frames 'at the base' of each link



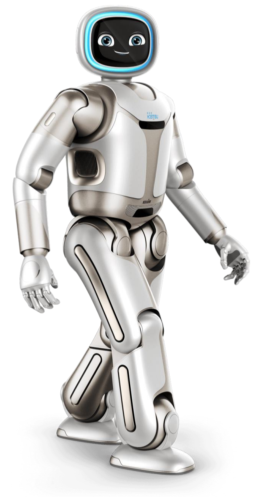
Robot Introduction for Projects

The robot model is Mintasca's GLUON-6L3



GLUON-6L3		
Specification	Maximum range of motion (mm)	425
	Degree of Freedom	6
Workspace Joint Limits	Joint 1 (°)	-140~140
	Joint 2 (°)	-90~90
	Joint 3 (°)	-140~140
	Joint 4 (°)	-140~140
	Joint 5 (°)	-140~140
	Joint 6 (°)	-360~360
Maximum Joint Speed	Joint 1 ($°/s$)	302
	Joint 2 ($°/s$)	302
	Joint 3 ($°/s$)	302
	Joint 4 ($°/s$)	302
	Joint 5 ($°/s$)	302
	Joint 6 ($°/s$)	302
Working Environment	Voltage (V)	42
	Watt (W)	Around 120
	Working Heat (°C)	10 - 50

Preparation



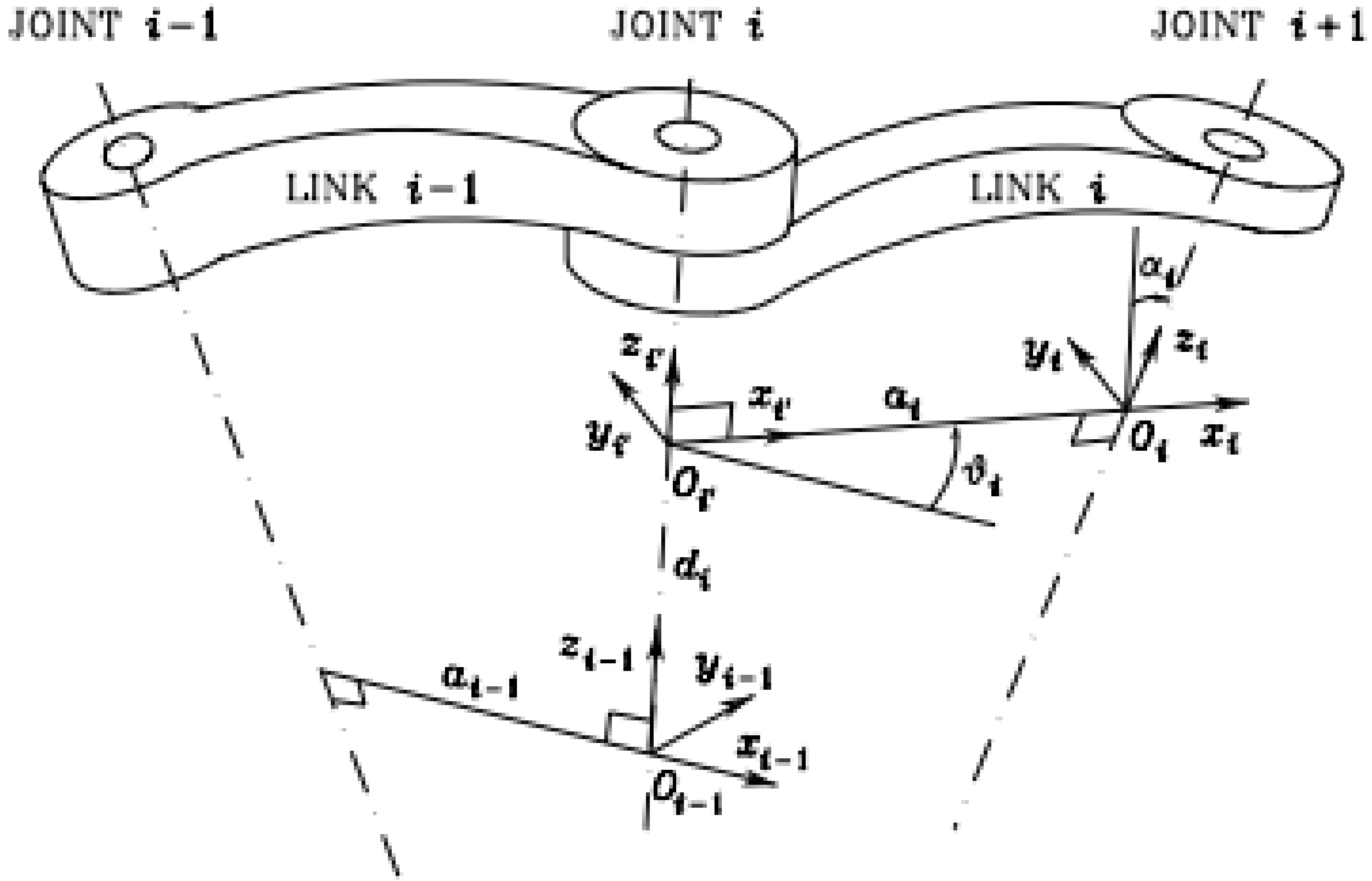
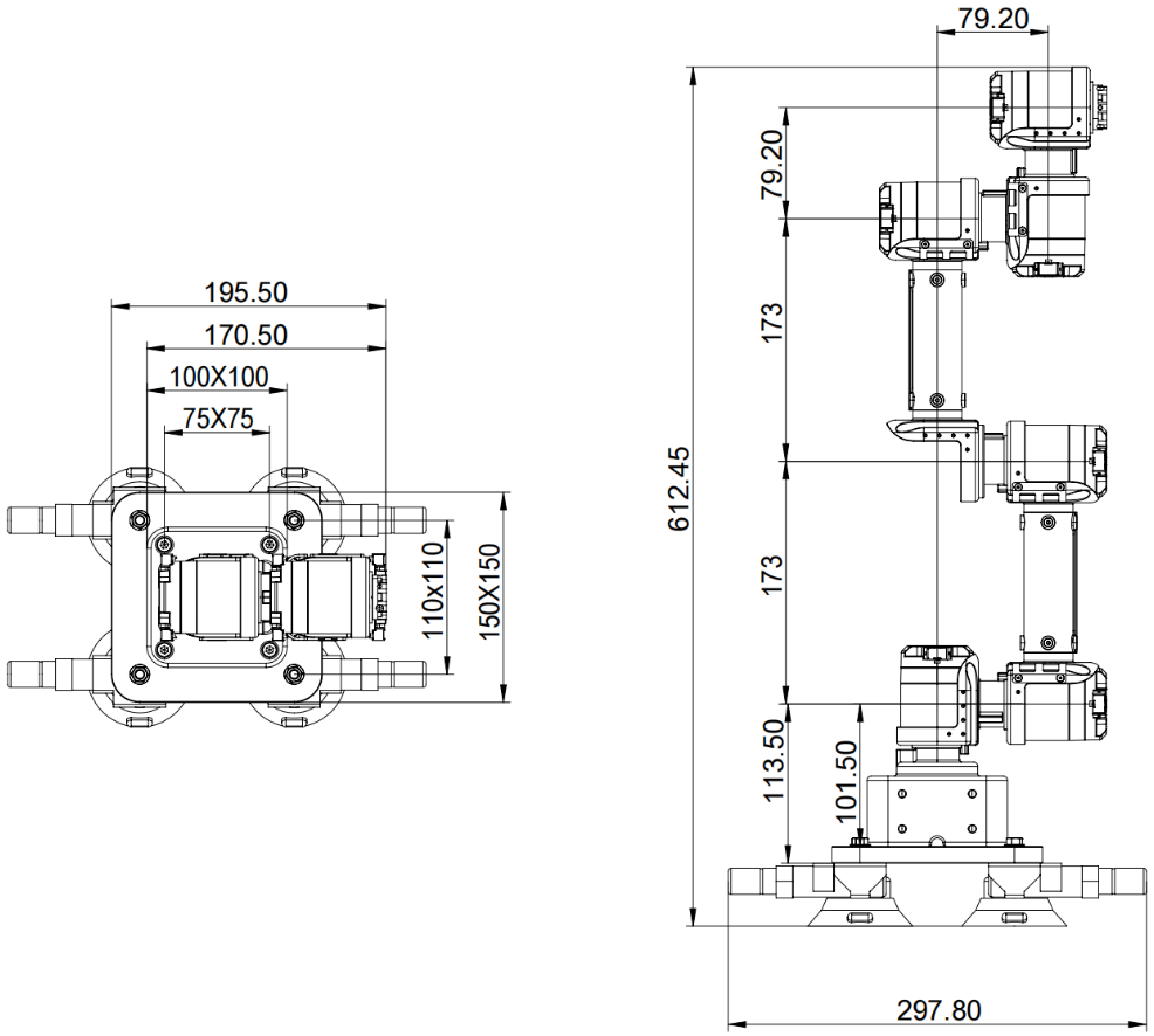
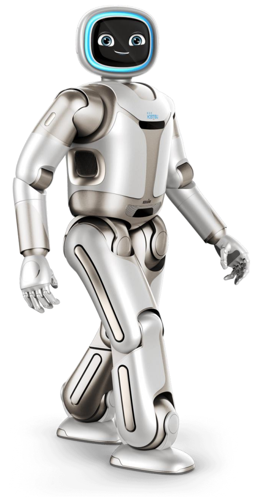


Fig. 2.16. Denavit–Hartenberg kinematic parameters

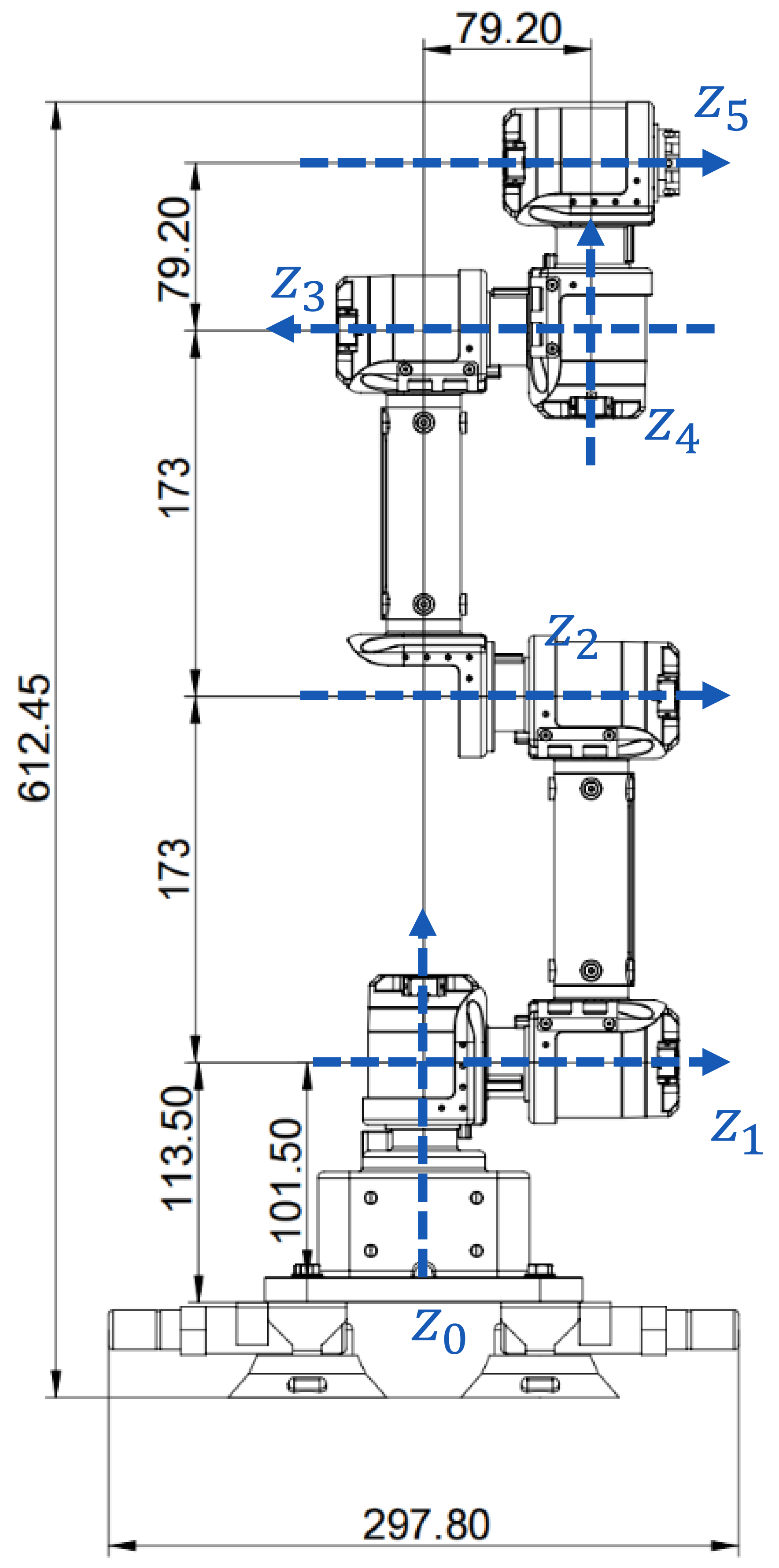
Note that the frame i, is located on joint i+1



The robot arm model diagram



Create Standard DH Table

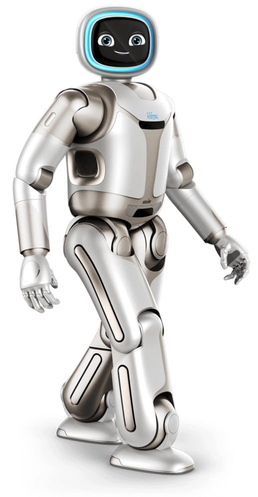


The method

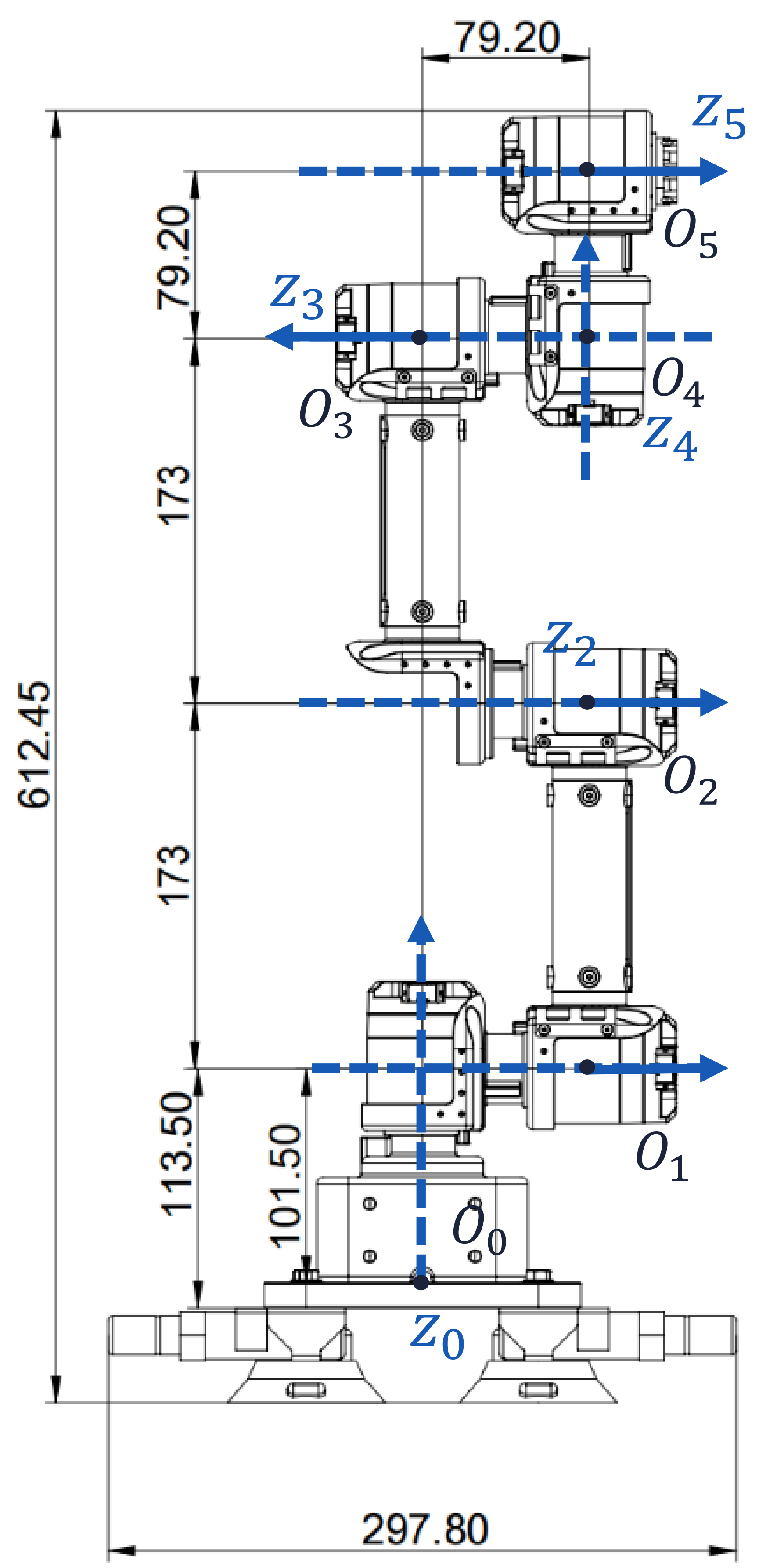
- Choose axis z_i along the axis of Joint i + 1.

Certain Cases

- For Frame 0, only the direction of axis z_0 is arbitrarily chosen as upwards; then O_0 and x_0 can be arbitrarily chosen.
- For Frame n, since there is no Joint n+ 1, z_n is not uniquely defined while x_n has to be normal to axis z_{n-1} . Typically, Joint n is revolute, and thus z_n can be aligned with the direction of z_{n-1} . The common normal between two lines is the line containing the minimum distance segment between the two lines.
- When two consecutive axes are parallel, the common normal between them is not uniquely defined.
- When two consecutive axes intersect, the positive direction of x_i is arbitrary.
- When Joint i is prismatic, only the direction of z_{i-1} is specified.



Create Standard DH Table

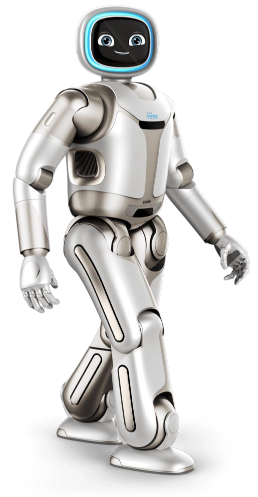


The method

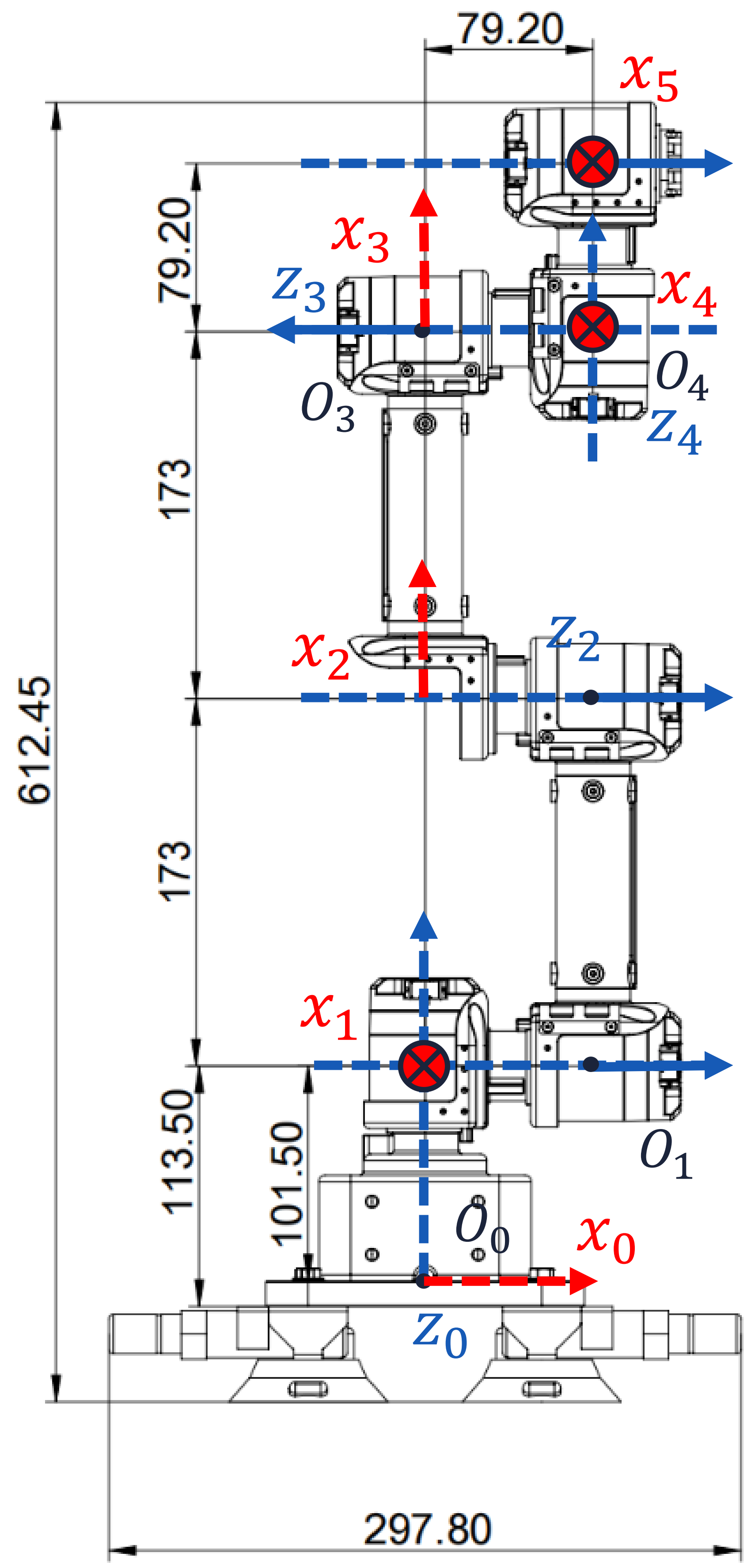
- Choose axis z_i along the axis of Joint i + 1.
- Find the origin O_i at the intersection of axis z_i with the common normal to axes z_{i-1} and z_i .

Certain Cases

- For Frame 0, only the direction of axis z_0 is arbitrarily chosen as upwards; then O_0 and x_0 can be arbitrarily chosen.
- For Frame n, since there is no Joint n+ 1, z_n is not uniquely defined while x_n has to be normal to axis z_{n-1} . Typically, Joint n is revolute, and thus z_n can be aligned with the direction of z_{n-1} . The common normal between two lines is the line containing the minimum distance segment between the two lines.
- When two consecutive axes are parallel, the common normal between them is not uniquely defined.
- When two consecutive axes intersect, the positive direction of x_i is arbitrary.
- When Joint i is prismatic, only the direction of z_{i-1} is specified.



Create Standard DH Table

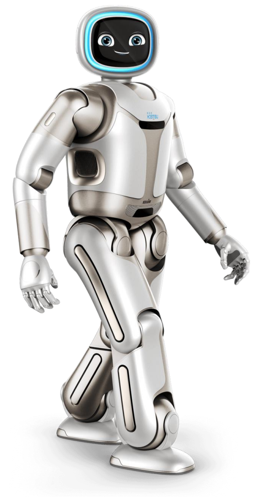


The method

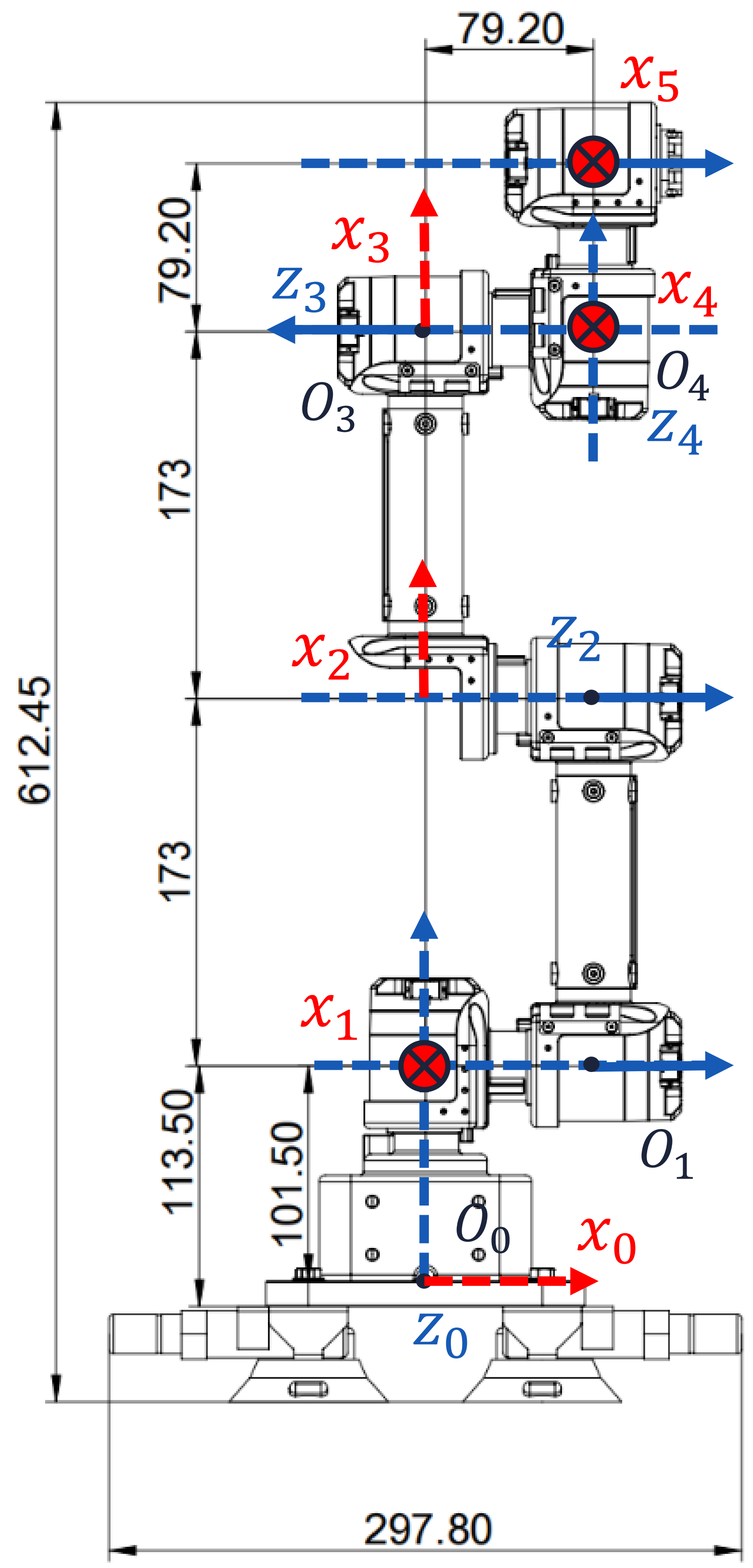
- Choose axis z_i along the axis of Joint i + 1.
- Find the origin O_i at the intersection of axis z_i with the common normal to axes z_{i-1} and z_i . Additionally, locate $O_{i'}$ at the intersection of the common normal with axis z_{i-1}.
- Choose axis x_i along the common normal to axis z_{i-1} and z_i with positive direction pointing towards Joint i + 1 from joint i.

Certain Cases

- For Frame 0, only the direction of axis z_0 is arbitrarily chosen as upwards; then O_0 and x_0 can be arbitrarily chosen.
- For Frame n, since there is no Joint n+ 1, z_n is not uniquely defined while x_n has to be normal to axis z_{n-1} . Typically, Joint n is revolute, and thus z_n can be aligned with the direction of z_{n-1} . The common normal between two lines is the line containing the minimum distance segment between the two lines.
- When two consecutive axes are parallel, the common normal between them is not uniquely defined.
- When two consecutive axes intersect, the positive direction of x_i is arbitrary.
- When Joint i is prismatic, only the direction of z_{i-1} is specified.



Create Standard DH Table

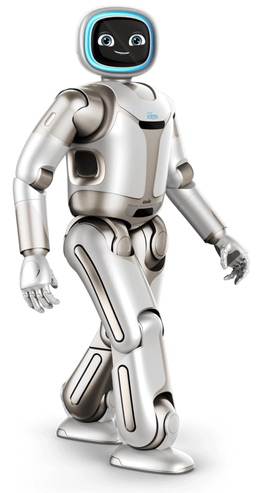


The method

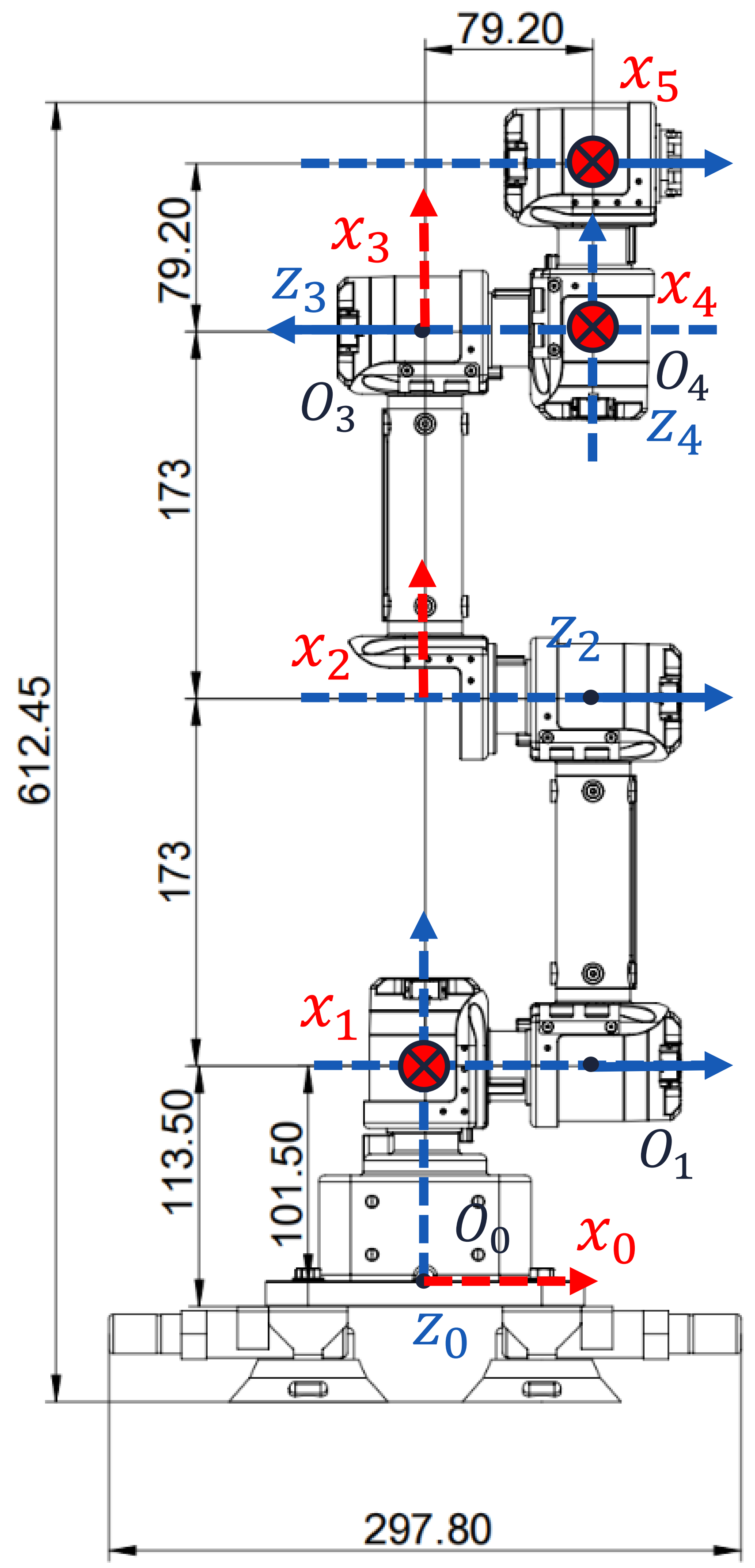
- Choose axis z_i along the axis of Joint i + 1.
- Find the origin O_i at the intersection of axis z_i with the common normal to axes z_{i-1} and z_i . Additionally, locate $O_{i'}$ at the intersection of the common normal with axis z_{i-1}.
- Choose axis x_i along the common normal to axis z_{i-1} and z_i with positive direction pointing towards Joint i + 1 from joint i.
- Axis z_i will be fixed as to complete a right-handed frame.

Certain Cases

- For Frame 0, only the direction of axis z_0 is arbitrarily chosen as upwards; then O_0 and x_0 can be arbitrarily chosen.
- For Frame n, since there is no Joint n+ 1, z_n is not uniquely defined while x_n has to be normal to axis z_{n-1} . Typically, Joint n is revolute, and thus z_n can be aligned with the direction of z_{n-1} . The common normal between two lines is the line containing the minimum distance segment between the two lines.
- When two consecutive axes are parallel, the common normal between them is not uniquely defined.
- When two consecutive axes intersect, the positive direction of x_i is arbitrary.
- When Joint i is prismatic, only the direction of z_{i-1} is specified.



Create Standard DH Table



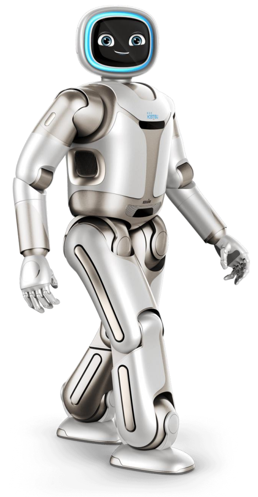
Constructing the kinematics graph

The kinematics graph of the Gluon EduBot arm should look like this. The parameters of the DH parameters are as follows:

- d_i, coordinate of O_i' along z_{i-1}
- θ_i, angle between axes x_{i-1} and x_i about axis z_{i-1} to be taken positive when rotation is made counter-clockwise
- a_i, distance between O_i and O_i'
- α_i, angle between axes z_{i-1} and z_i about axis x_i to be taken positive when rotation is made counter-clockwise

The DH table

Joints	$\boldsymbol{d}$ (m)	$\boldsymbol{\theta}\ (\boldsymbol{rad})$	$\boldsymbol{a}$ (m)	$\boldsymbol{\alpha}(\boldsymbol{rad})$
1	0.1015	$\pi/2$	0	$\pi/2$
2	0.0	$\pi/2$	0.173	0
3	0.0	0	0.173	π
4	-0.07920	$\pi/2$	0	$\pi/2$
5	0.07920	0	0	$\pi/2$
6	0	0	0	0



Demonstration

To visualize the forward kinematics, input a simple array of θ values with a known pose value in space.

$$\theta = \left[0, 0, \frac{\pi}{2}, 0, 0, \frac{\pi}{2}\right]$$

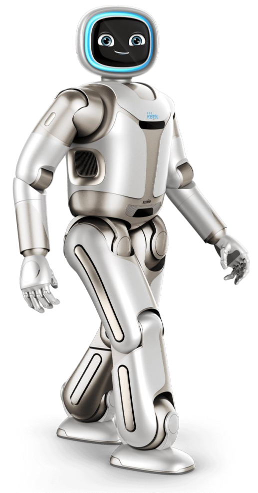
This θ value shows a 90° rotation on joint 3 and 6, hence the Edubot arm should bend into the paper with half of its height aligned with negative y-axis

$$\text{Pose Value} = \begin{bmatrix} 0.08011 \\ -0.2545 \\ 0.2795 \\ -1.5708 \\ -1.2250 \\ -1.7708 \end{bmatrix}$$

```
forward_kinematics = fk(robot_chain, [0, 0, pi/2, 0, 0, pi/2])
```

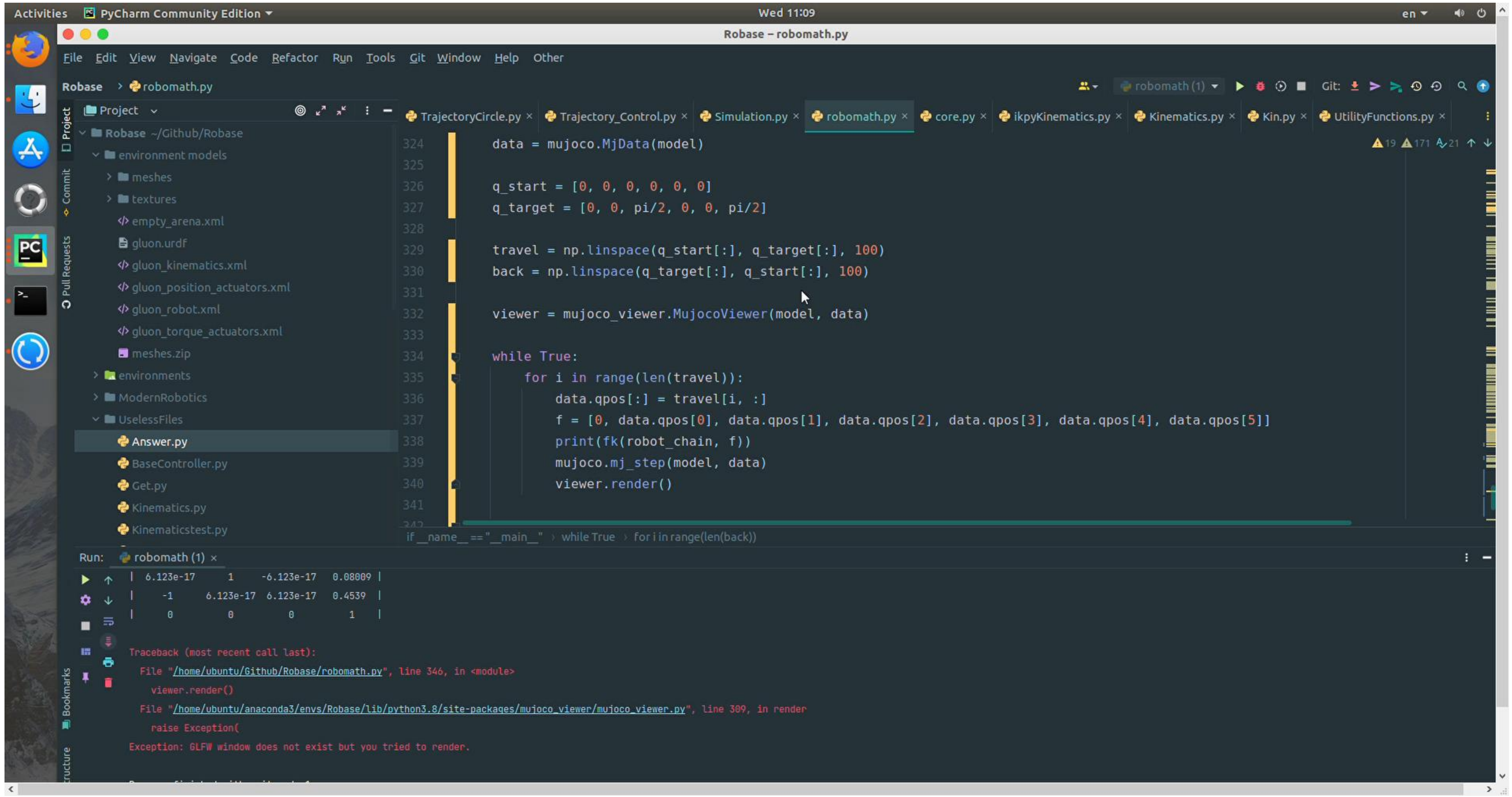
This function then gives a homogeneous transformation matrix of:

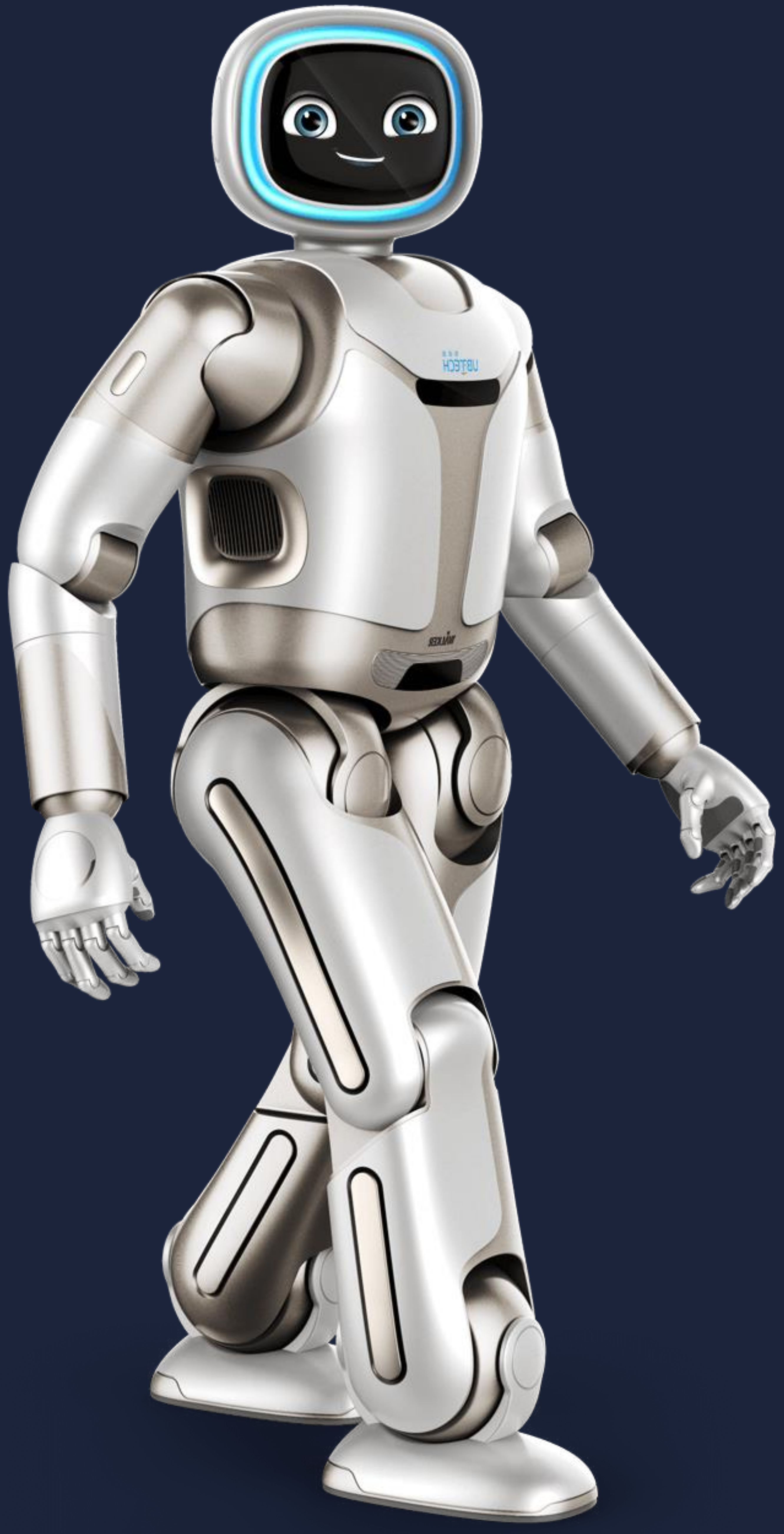
```
Forward kinematics
| -1.837e-16  -6.123e-17         1     0.08011 |
|      -1     -1.225e-16  -1.837e-16   -0.2545  |
|  1.225e-16        -1    -6.123e-17    0.2795  |
|       0           0           0          1    |
```



Demonstration

To further visualize and understand the forward kinematics derived from the DH parameters, a simulation can be run to check the values of θ





Q&A