ENGG 5403 / MAEG 5725

Design Problem No. 2: An HDD Servo System

Prepared by

Ben M. Chen
Department of Mechanical and Automation Engineering
Chinese University of Hong Kong

Declaration

Please sign the following and turn in this page together with your project report.

This page must be signed to receive a grade for this design project.

I have not given nor received any aid from others on this design project.

____

Signed

You should refer to the problem description on an HDD servo system design in the appendix.

* * * * * *

Using either H_2 or H_∞ control techniques to design appropriate measurement feedback control laws that meet all the design specifications specified in the problem.

Follow the following procedure to generate parameters p and q:

1. Write your CUHK 10-digit student ID (SID)
2. Let S = the last 5 digits of your SID. For example, if your SID is 1155154321, then S = 54321.
3. Run the following program in MATLAB:

```
for i = 1 : S
    p = rand;
    q = rand;
end
```

4. Replace the system in Equation (11.1) with the following system data:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ (6+p)\times 10^7 \end{bmatrix} \left(\text{sat}(u) + w_{\text{in}} \right) \tag{11.1$'$}$$

5. The overshoot of the actual actuator output should be less than $(5-q)$% instead of 5% as that in the appendix.

Other system data and design specifications remain unchanged.

Show all the detailed design and calculation procedure and simulate your design using MATLAB and/or SIMULINK. Give all the necessary plots that show the evidence of your design.

Appendix

A Benchmark Problem on an HDD Servo System Design

Adopted from the monograph

B. M. Chen, T. H. Lee, K. Peng and V. Venkataramanan

Hard Disk Drive Servo Systems

2nd Edition, Springer, New York, 2006

11

A Benchmark Problem

Before ending this book, we post in this chapter a typical HDD servo control design problem. The problem has been tackled in the previous chapters using several design methods, such as PID, RPT, CNF, PTOS and MSC control. We feel that it can serve as an interesting and excellent benchmark example for testing other linear and nonlinear control techniques.

We recall that the complete dynamics model of a Maxtor (Model 51536U3) hard drive VCM actuator can be depicted as in Figure 11.1:

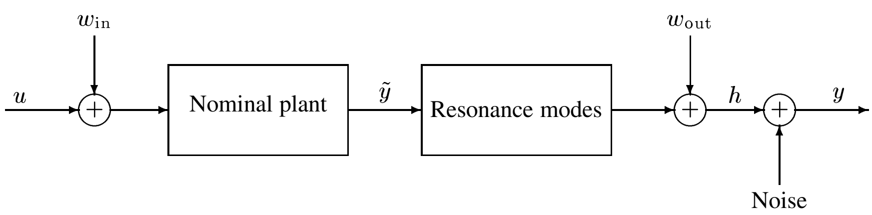


Figure 11.1. Block diagram of the dynamical model of the hard drive VCM actuator

The nominal plant of the HDD VCM actuator is characterized by the following second-order system:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 6.4013 \times 10^7 \end{bmatrix} \left(\text{sat}(u) + w_{\text{in}} \right) \tag{11.1}$$

>>> refer to Page 3 to set up your own system data <<<

and

$$\tilde{y} = [\,1 \quad 0\,]\, x \tag{11.2}$$

where the control input u is limited within ± 3 V and w_{in} is an unknown input disturbance with $|w_{\text{in}}| \leq 3$ mV. For simplicity and for simulation purpose, we assume that the unknown disturbance $w_{\text{in}} = -3$ mV. The measurement output available for control, *i.e.* y (in µm), is the measured displacement of the VCM R/W head and is given by

$$y = \left[\prod_{i=1}^{4} G_{\mathrm{r},i}(s) \right] \tilde{y} + w_{\mathrm{out}} + \text{Noise} \tag{11.3}$$

where the transfer functions of the resonance modes are given by

$$\left. \begin{aligned} G_{\mathrm{r},1}(s) &= \frac{0.912s^2 + 457.4s + 1.433(1+\delta)\times 10^8}{s^2 + 359.2s + 1.433(1+\delta)\times 10^8} \\ G_{\mathrm{r},2}(s) &= \frac{0.7586s^2 + 962.2s + 2.491(1+\delta)\times 10^8}{s^2 + 789.1s + 2.491(1+\delta)\times 10^8} \\ G_{\mathrm{r},3}(s) &= \frac{9.917(1+\delta)\times 10^8}{s^2 + 1575s + 9.917(1+\delta)\times 10^8} \\ G_{\mathrm{r},4}(s) &= \frac{2.731(1+\delta)\times 10^9}{s^2 + 2613s + 2.731(1+\delta)\times 10^9} \end{aligned} \right\} \tag{11.4}$$

with $-20\% \le \delta \le 20\%$ represents the variation of the resonance modes of the actual actuators whose resonant dynamics change from time to time and also from disk to disk in a batch of million drives. Note that many new hard drives in the market nowadays might have resonance modes at much higher frequencies (such as those for the IBM microdrives studied in Chapter 9). But, structurewise, they are almost the same. The output disturbance (in µm), which is mainly the repeatable runouts, is given by

$$w_{\mathrm{out}} = 0.1\sin(110\pi t) + 0.05\sin(220\pi t) + 0.02\sin(440\pi t) + 0.01\sin(880\pi t) \tag{11.5}$$

and the measurement noise is assumed to be a zero-mean Gaussian white noise with a variance $\sigma_{\mathrm{n}}^2 = 9\times 10^{-6}$ (µm)^2.

The problem is to design a controller such that when it is applied to the VCM actuator system, the resulting closed-loop system is asymptotically stable and the actual displacement of the actuator, *i.e.* h, tracks a reference $r = 1$ µm. The overall design has to meet the following specifications:

1. the overshoot of the actual actuator output is less than 5%; >>> refer to Page 3 for your own spec.
2. the mean of the steady-state error is zero;
3. the gain margin and phase margin of the overall design are, respectively ,greater than 6 dB and 30°; and
4. the maximum peaks of the sensitivity and complementary sensitivity functions are less than 6 dB.

The results of Chapter 6 show that the 5% settling times of our design using the CNF control technique are, respectively, 0.80 ms in simulation and 0.85 ms in actual hardware implementation. We note that the simulation result can be further improved if we do not consider actual hardware constraints in our design. For example, the

CNF control law given below meets all design specifications and achieves a 5% settling time of 0.68 ms. It is obtained by using the toolkit of [55] under the option of the pole-placement method with a damping ratio of 0.1 and a natural frequency of 2800 rad/sec together with a diagonal matrix $W = \text{diag}\{1.5,\ 0.01,\ 2 \times 10^{-10}\}$. The dynamic equation of the control law is given by

$$\begin{pmatrix} \dot{x}_\text{i} \\ \dot{x}_\text{v} \end{pmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -4000 \end{bmatrix} \begin{pmatrix} x_\text{i} \\ x_\text{v} \end{pmatrix} - \begin{bmatrix} -10 \\ 1.6 \times 10^7 \end{bmatrix} y + \begin{bmatrix} 0 \\ 6.4013 \times 10^7 \end{bmatrix} \text{sat}(\tilde{u}) - \begin{pmatrix} 10r \\ 0 \end{pmatrix} \tag{11.6}$$

$$\begin{aligned} \tilde{u} = \rho(e) \, [0.61237 \quad 0.049449 \quad 8.4622 \times 10^{-5}] \begin{pmatrix} x_\text{i} \\ y - r \\ x_\text{v} + 4000y \end{pmatrix} \\ - [1.2248 \quad 0.12335 \quad 1.0310 \times 10^{-5}] \begin{pmatrix} x_\text{i} \\ y - r \\ x_\text{v} + 4000y \end{pmatrix} \end{aligned} \tag{11.7}$$

where

$$\rho(e) = -2.65 \left| \text{e}^{-0.7|e|} - \text{e}^{-0.7} \right| \tag{11.8}$$

and

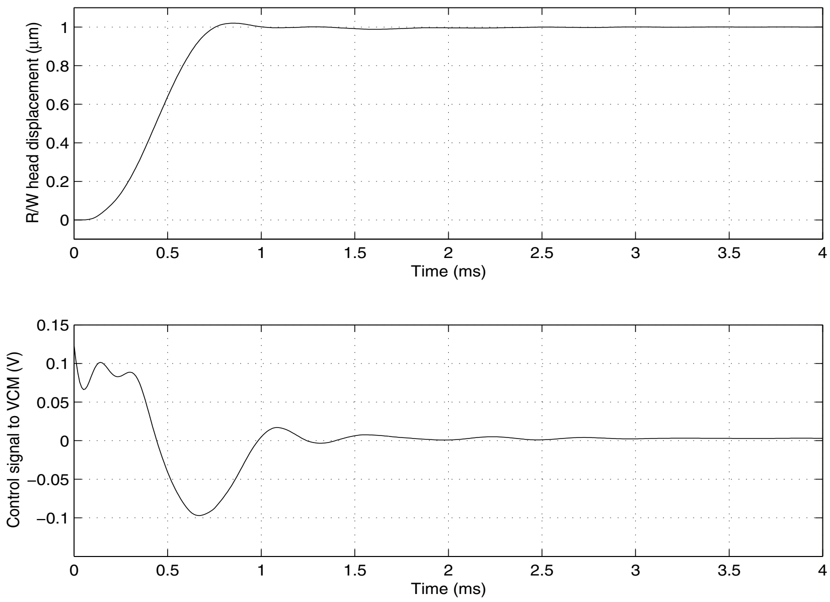
$$u = G_\text{notch}(s) \cdot \tilde{u} \tag{11.9}$$

with $G_\text{notch}(s)$ being given as in Equation 6.9.(see also its expression at the bottom of this page)

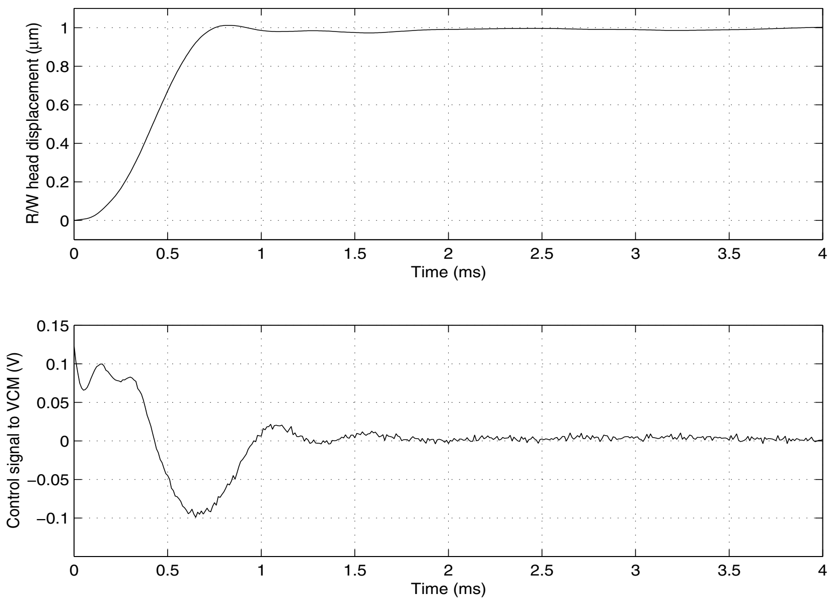
The simulation results obtained with $\delta = 0$ given in Figures 11.2 to 11.4 show that all the design specifications have been achieved. In particular, the resulting 5% settling time is 0.68 ms, the gain margin is 7.85 dB and the phase margin is 44.7°, and finally, the maximum values of the sensitivity and complementary sensitivity functions are less than 5 dB. The overall control system can still produce a satisfactory result and satisfy all the design specifications by varying the resonance modes with the value of δ changing from -20% to 20%.

Nonetheless, we invite interested readers to challenge our design. Noting that for the track-following case, *i.e.* when $r = 1$ µm, the control signal is far below its saturation level. Because of the bandwidth constraint of the overall system, it is not possible (and not necessary) to utilize the full scale of the control input to the actuator in the track-following stage. However, in the track-seeking case or equivalently by setting a larger target reference, say $r = 500$ µm, the very problem can serve as a good testbed for control techniques developed for systems with actuator saturation. Interested readers are referred to Chapter 7 for more information on track seeking of HDD servo systems.

$$\begin{aligned} G_\text{notch}(s) = \left(\frac{s^2 + 238.8s + 1.425 \times 10^8}{s^2 + 2388s + 1.425 \times 10^8} \right) \times \left(\frac{s^2 + 314.2s + 2.467 \times 10^8}{s^2 + 3142s + 2.467 \times 10^8} \right) \\ \times \left(\frac{s^2 + 628.3s + 9.87 \times 10^8}{s^2 + 12570s + 9.87 \times 10^8} \right) \end{aligned} \tag{6.9}$$

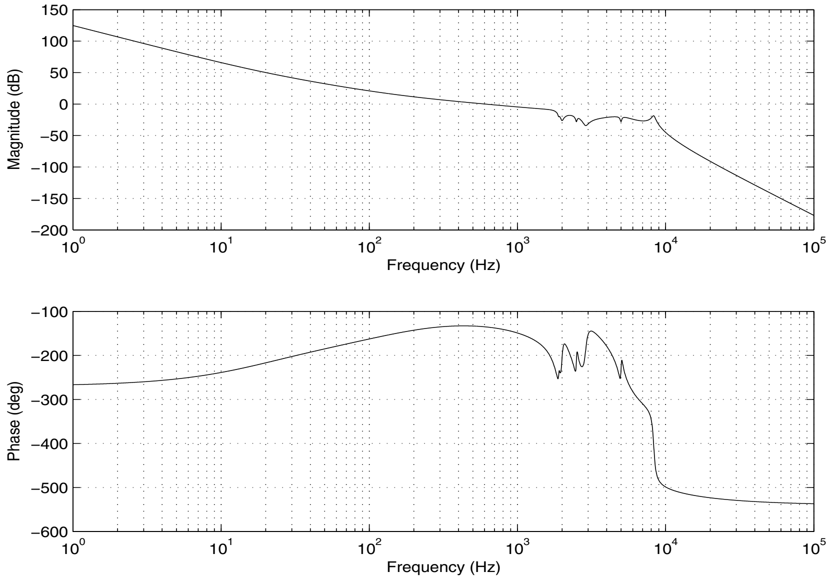


(a) h and u for the system without output disturbance and noise

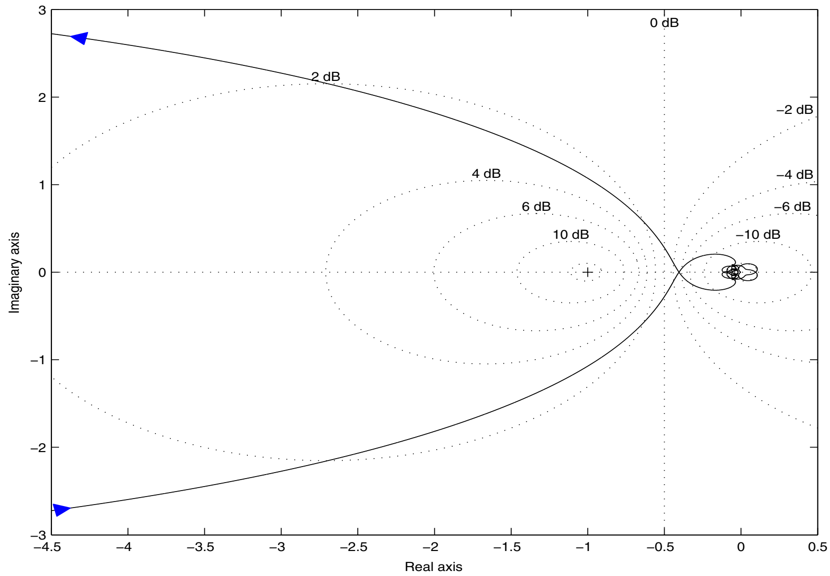


(b) h and u for the system with output disturbance and noise

Figure 11.2. Output responses and control signals of the CNF control system



(a) Bode plot



(b) Nyquist plot

Figure 11.3. Bode and Nyquist plots of the CNF control system

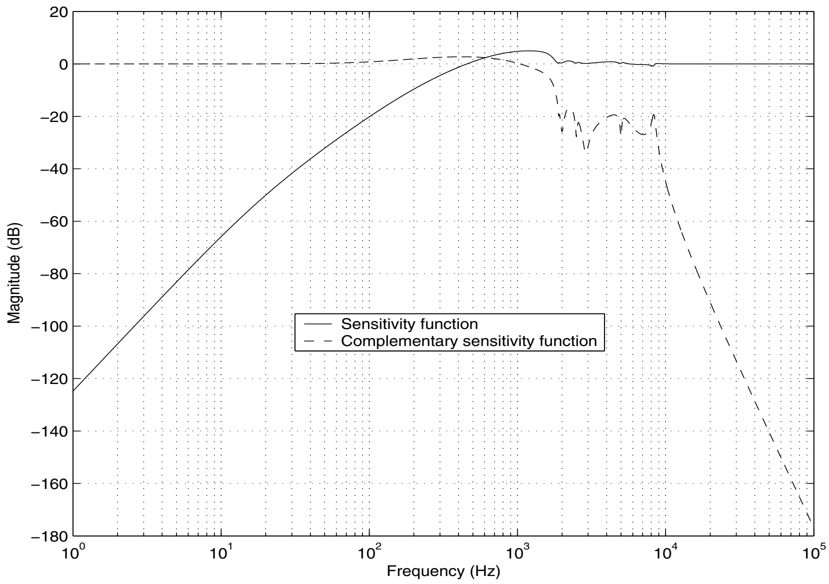


Figure 11.4. Sensitivity and complementary sensitivity functions with the CNF control