

Homework 1

Due: Sep 26, 2022

1. Transform the following LP into its standard form and write down the compact form. (30 points)

$$\begin{aligned} \max_{x_1, x_2} \quad & 3x_1 + x_2 \\ \text{s.t.} \quad & 2x_1 + |x_2 - 3| \leq 10 \\ & x_1 + 5x_2 \geq 3 \\ & x_1 \geq 1 \end{aligned}$$

2. Determine the optimal solution of the following LP using Graphical method. (20 points)

$$\begin{aligned} \max_{x_1, x_2} \quad & 3x_1 - 2x_2 \\ \text{s.t.} \quad & 2x_1 - x_2 \leq 3 \\ & x_1 + 2x_2 \leq 8 \\ & x_1 - 4x_2 \leq 4 \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

3. Write down the dual problem of (20 points)

$$\begin{aligned} \min_{x_1, x_2, x_3} \quad & 2x_1 + 3x_3 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 \geq 2 \\ & x_1 - 2x_3 \leq 10 \\ & x_2 + 2x_3 = 5 \\ & x_1 \geq 0, x_3 \leq 0 \end{aligned}$$

4. Prove that the set $S = \{(x_1, x_2) | 3x_1 - x_2 \geq 1, x_1 + x_2 \leq 1\}$ is convex. (15 points)
5. Is the function $f(x_1, x_2) = x_1^2 - 4x_1x_2 + 4x_2^2 + 3x_1 + x_2$ convex? Why? (15 points)