

## Homework 2

**Due: Oct 17, 2022**

1. If a firm invests  $K$  units of capital and  $L$  units of labor for production, it can produce  $K^{\frac{1}{4}}L^{\frac{1}{4}}$  units of the product. Suppose the prices for capital and labor are all 30. The firm wants to produce at least 10 units of the product with the lowest cost. Please write down the optimization problem the company solves. (20 points)

2. Determine the optimal solution(s) of (25 points)

$$\min_{x_1, x_2} f(x) = \frac{1}{3}x_1^3 + \frac{1}{3}x_2^3 - x_2^2 - x_1$$

Are they maximum or minimum points? Are they locally optimal or globally optimal?  
Are they strictly optimal or relatively optimal?

3. Apply Newton method to the following problem, and write down the first two iterations with initial point as  $x^{(0)} = (0,1)^T$ . (25 points)

$$\min_{x_1, x_2} (x_1 - 1)^3 + 2x_2^2$$

4. Try to linearize the following terms:

(1)  $z = 5xy, x \in [3,10], y \in \{0,1\}$  (15 points)

(2)  $y = \max\{x_1, x_2, x_3\}, x_1 \in [1,10], x_2 \in [2,8], x_3 \in [3,12]$  (15 points)