Homework 3

Due: Nov 14, 2022

1. Check if point $x^* = \left(\frac{3}{2}, \frac{5}{2}\right)^T$ is an KKT point of the following optimization problem: (20 points)

$$\max_{x_1, x_2} \quad -(x_1 - 4)^2 - (x_2 - 4)^2$$
$$\text{s.t. } x_1 + x_2 \le 4$$
$$x_1 + 3x_2 \le 9$$

2. Consider the following optimization problem:

$$\min_{x_1, x_2} \quad (x_1 - 1)^2 + (x_2 + 1)^2$$
$$\text{s.t. } -x_1 + 2x_2 \ge 5$$

(1) Is it a convex optimization problem? (10 points)
(2) Determine the optimal solution using Graphical method. (15 points)
(3) Determine the optimal solution based on KKT condition. (15 points)
(4) What's its dual problem? (15 points)

3. (1) Prove that the set $S = \{(x_1, x_2)|2x_1 + x_2 \le 4\}$ is convex. (10 points)
   (2) Determine a point $(x_1, x_2) \in S$ that has the smallest distance to the point $(4,4)$. (15 points)

4. A game (open question. The statistics will be used for a case study in our next lecture 😊): Each students names an integer between 0 and 100. The student who names the integer closest to 2/3 of the average integer wins the game! So the integer you choose is _____??