MAEG4070 Engineering Optimization

Solution 4

1. The Lagrange function is

$$\begin{aligned} L(x,\lambda) &= 6x_1^2 + 7x_2^2 + \lambda(3x_1 + 7x_2 - 10) \\ &= \underbrace{6x_1^2 + 3\lambda x_1}_{L_1(x_1,\lambda)} + \underbrace{7x_2^2 + 7\lambda x_2}_{L_2(x_2,\lambda)} - 10\lambda \end{aligned}$$

The updates are

$$\begin{aligned} x_1^{k+1} &= \text{argmin}_{x_1} L_1(x_1,\lambda) = -\frac{\lambda^k}{4} \\ x_2^{k+1} &= \text{argmin}_{x_2} L_2(x_2,\lambda) = -\frac{\lambda^k}{2} \\ \lambda^{k+1} &= \lambda^k + \alpha(3x_1^{k+1} + 7x_2^{k+1} - 10) \end{aligned}$$

Iteration 1:

$$x_1^1 = -\frac{0}{4} = 0$$

$$x_2^1 = -\frac{0}{2} = 0$$

$$\lambda^1 = 0 + 0.1 \times (3 \times 0 + 7 \times 0 - 10) = -1$$

Iteration 2:

$$x_1^2 = \frac{1}{4} = 0.25$$

$$x_2^2 = \frac{1}{2} = 0.5$$

$$\lambda^2 = -1 + 0.1 \times (3 \times 0.25 + 7 \times 0.5 - 10) = -1.575$$

2. Both f_1 and f_2 are minimize
   (1) $f_1(A) > f_1(B)$, $f_2(A) > f_2(B)$, A is dominated by B, or B dominates A.
   (2) $f_1(B) = f_1(C)$, $f_2(B) < f_2(C)$, C is dominated by B, or B dominates C.
   (3) $f_1(A) > f_1(C)$, $f_2(A) = f_2(C)$, A is dominated by C, or C dominates A.

3. $x^* = (1,2)^T$ is not a robust feasible solution. The robust optimization is equivalent to

$$\begin{aligned} \min_{x_1,x_2} \quad & 3x_1 + 4x_2 \\ \text{s.t.} \quad & x_1 + x_2 \le 2 \\ & x_1 + x_2 \le 3 \\ & 2x_1 + x_2 \le 2 \\ & 2x_1 + x_2 \le 3 \end{aligned}$$

The last constraint is violated.