MAEG4070 Engineering Optimization

Lecture 1 Introduction: What is Optimization?

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Course Setup

Welcome to the course on Engineering Optimization, with an introduction of its application in energy systems!

Basic administrative details:

- **Instructor**:
  - Prof. Yue CHEN yuechen@mae.cuhk.edu.hk (FYB 604C)
  - Office hour: 13:00-14:00, every Friday (online)
  - English, Cantonese, Mandarin

- **Teaching assistants**:
  - DENG Zhiyu zydeng@mae.cuhk.edu.hk (ERB113)
  - INNERO Gino ginnero@mae.cuhk.edu.hk (AB1 205)
  - JIN Liuchao lcjin@mae.cuhk.edu.hk (ERB 201)
  - LIU Jingjun jingjunliu@mae.cuhk.edu.hk (AB1 2F)
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Time for tutorial?

Posted on: Thursday, September 1, 2022 10:42:27 AM HKT

Dear MAEG4070 students,

Please select your time available for tutorial via the following link before Sep 7.
https://forms.gle/zseCDZk6qdcqJFWFA

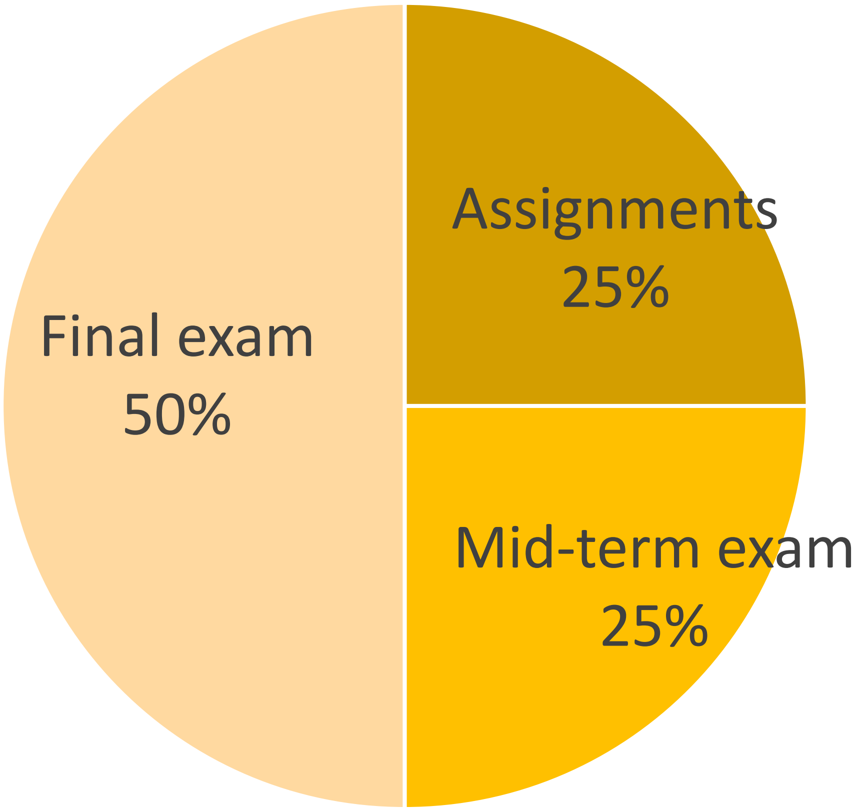
The tutorial will be given by TAs, starting from week 2.

Thanks!

Your time available for tutorial

- [ ] Tuesday 13:30 - 14:15
- [ ] Wednesday 10:30 - 11:15
- [ ] Wednesday 12:30 - 13:15
- [ ] Thursday 14:30 - 15:15
- [ ] Friday 11:30 - 12:15

Course Grading Scheme



- Due days for assignments will be specified.
- Assignments will carry a **50 % penalty** if handed in late.
- No credit will be given to assignments which are more than **three days late**.
- *Copying is **strictly prohibited***.

- No make-up exam is given unless you get approval from the Director of Registry Services for permission for absence.
- If not permitted, a zero grade in that exam will be given.

Perquisite and reference

Assuming work with knowledge of:

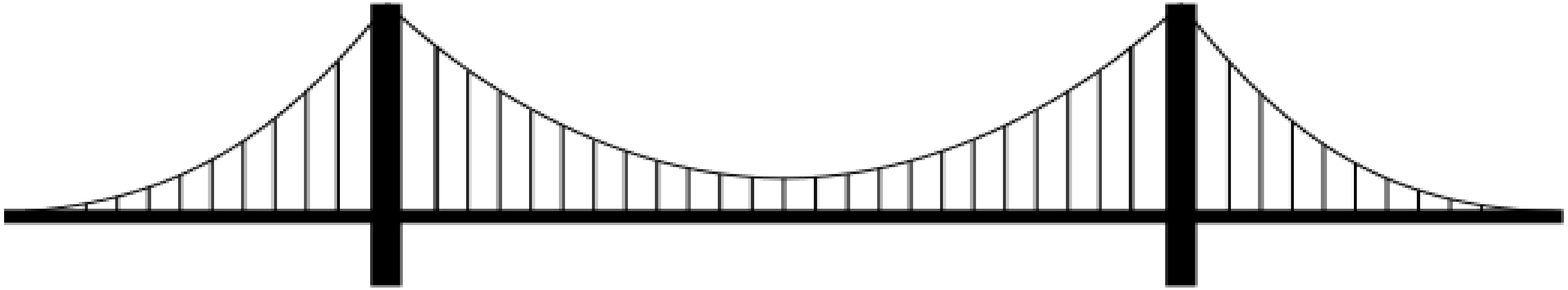
- Calculus, Linear Algebra
- Formal mathematical thinking
- Programming (Matlab, Python, …)

If you fall short on any one of these things, **don't worry**!

Reference:

- S. Boyd, L. Vandenberghe. Convex optimization. Cambridge university press, 2004.
- R. Sioshansi, A. J. Conejo. Optimization in Engineering: Models and Algorithms. Springer, 2017.
- S. S. Rao. Engineering optimization: theory and practice. John Wiley & Sons, 2019.

Orientation of this course



Basic models:
- Linear programming
- Convex optimization
- Integer programming
- ……

Algorithms & Solvers:
- Lagrange / Dual Methods
- Linearization approaches
- CPLEX, MOSEK, …

Engineering problems:
- Scheduling
- Machine Learning
- Market Analysis
- …..

A course bridges *mathematical models* and *engineering applications*
A course combining *math* and *programming*

Pictures from Google image

What's optimization?

Different people may have different understanding about *optimization*

- Listing all possibilities and pick up the best?
- qualitative suggestions that lead to a better product design?
- trial and error?

Translate  into $P: \min_{x \in \mathcal{X}} f(x)$

Conceptual idea → Optimization problem

A **quantitative** and systematic methodology to search for the **best design** among numerous possibilities while **satisfying given constraints**.

Where can we see optimization?

As a matter of fact, any rational human behavior could be modelled as an optimization problem (intentionally or subconsciously) almost all the time.

Demand & supply of goods →  → **Minimize** Travel time

warehouse placement Location of warehouses

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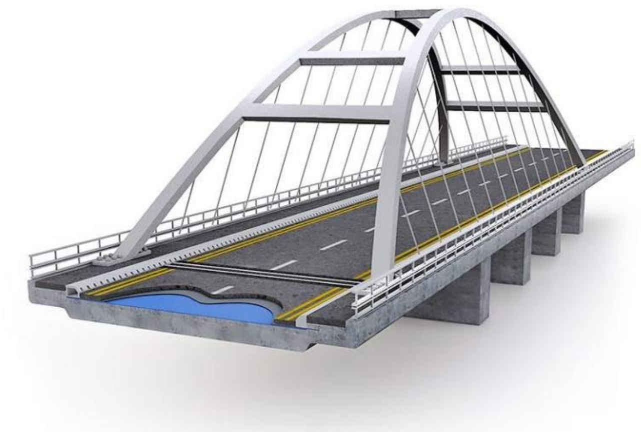
Where can we see optimization?

Parameters, Plan



Maximize
Load Bearing

bridge construction

Detailed Design

Historical stock prices, economy

Maximize
Returns

stock market

Stock Portfolio

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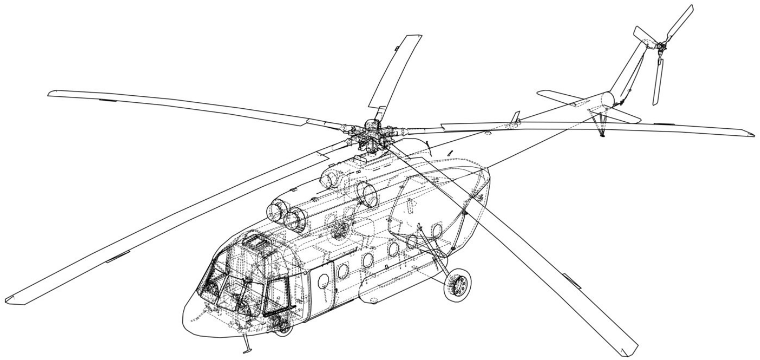
Where can we see optimization?

Parameters, Use plan

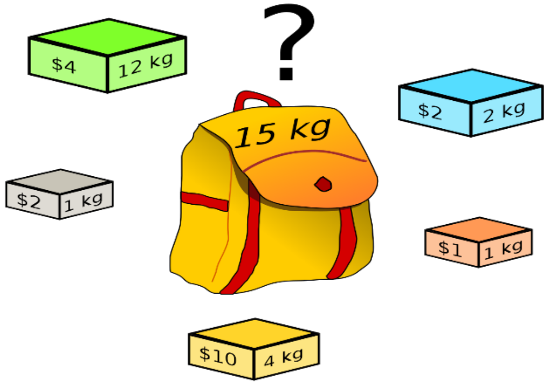


Minimize
Weight

Airplane design

Design

List of items



Maximize
Total value

knapsack problem

Number of items

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Successful Applications in Real-world

Organization	Application	Cost Saving
AT&T	Location planning of telephone selling centers	$ 406 million
79 electric utilities	Purchasing and energy management	$ 125 million
Delta Air Line	Aircraft scheduling	$ 100 million
SNCF	Train scheduling	$ 15 million

- Not all engineering problems can be quantitatively modelled as an optimization and solved with current technologies.
- With the development of more advanced mathematics and computational tools, we can model and simulate more sophisticated process, behaviors, etc.
- **The application of optimization is increasingly practical, significant, and worthwhile!**

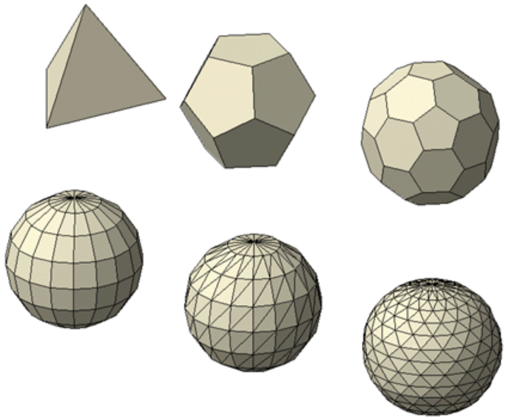
History of optimization

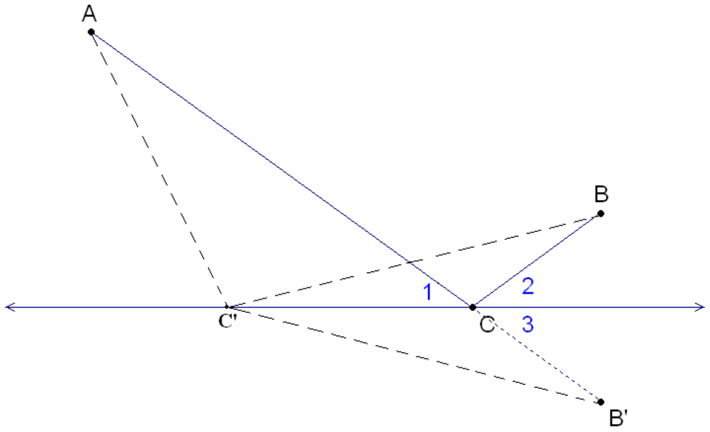
Greek mathematicians considers geometrical optimization problems.

Euclid
- the minimal distance between a point and a line.
- the greatest area among the rectangles with given total length of edges

Zenodorus
- A sphere encloses the greatest volume for a given surface area

Heron
- light travels between two points through the path with shortest traveling time when reflected at or refracted through a boundary





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History of optimization

The Earliest Optimization Approach: Calculus

- where $\frac{df(x)}{dx} = 0$ gives a maximum or minimum.
- Pierre De Fermat & Joseph-Louis Lagrange: calculus-based formulae for identifying optima
- Isaac Newton & Johann C.F. Gauss: iterative methods to search for an optimum.

Classic Optimization approaches

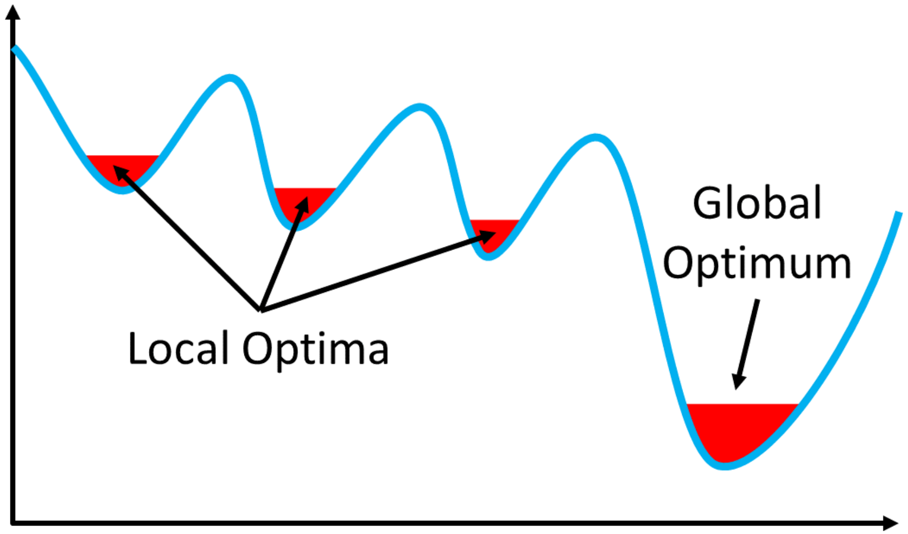
- Leonid Kantorovich: Linear Programming, 1939
- George Dantzig: the Simplex Method, 1947
- John von Neumann: **the Theory of Duality**, 1947
- The **Karush-Kuhn-Tucker (KKT) condition**, 1939 and 1951 by two separate groups

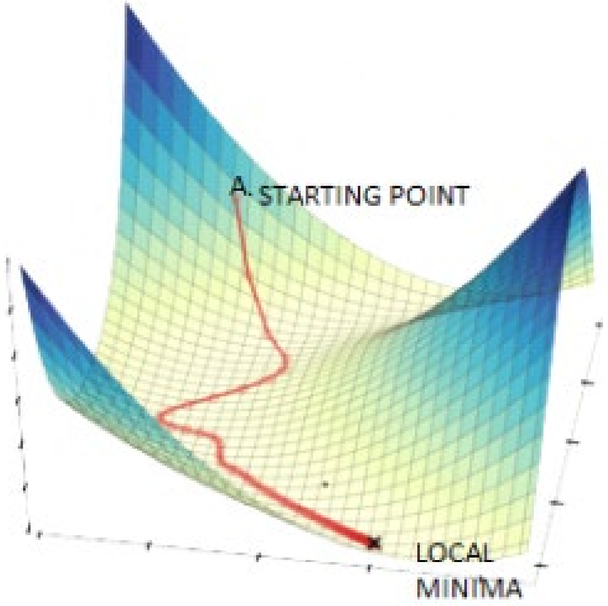
- Models: Unconstrained/Constrained optimization, single-/multi-objective optimization, etc.
- Algorithms: the steepest descent method, Newton's method, the penalty method, etc.

History of optimization

First Generation Optimization: Local, Iterative, and Gradient-based

- Considers a *local optimal*
- Iterative methods: based on the location of the previously explored point
- Reliance on gradients or higher order derivatives
- Cons: hard to explain why optimal is optimal, limited computing power





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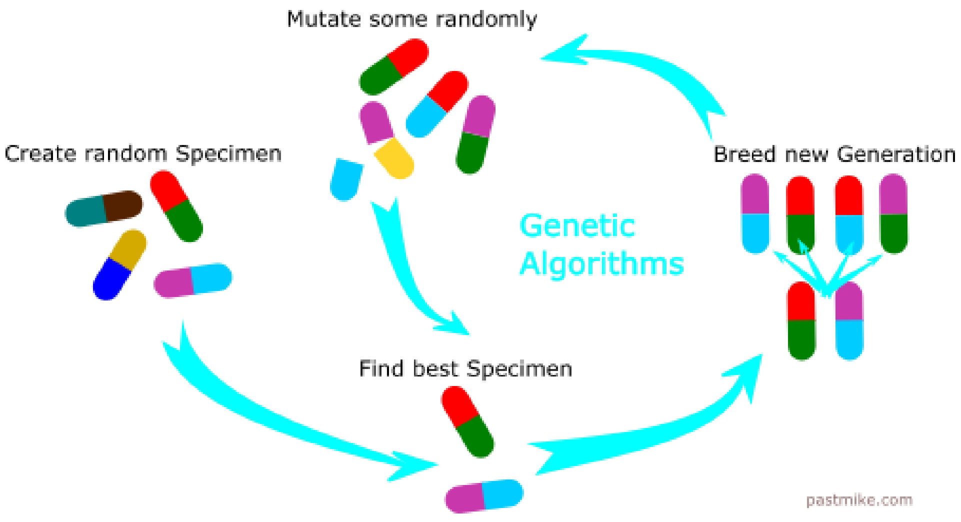
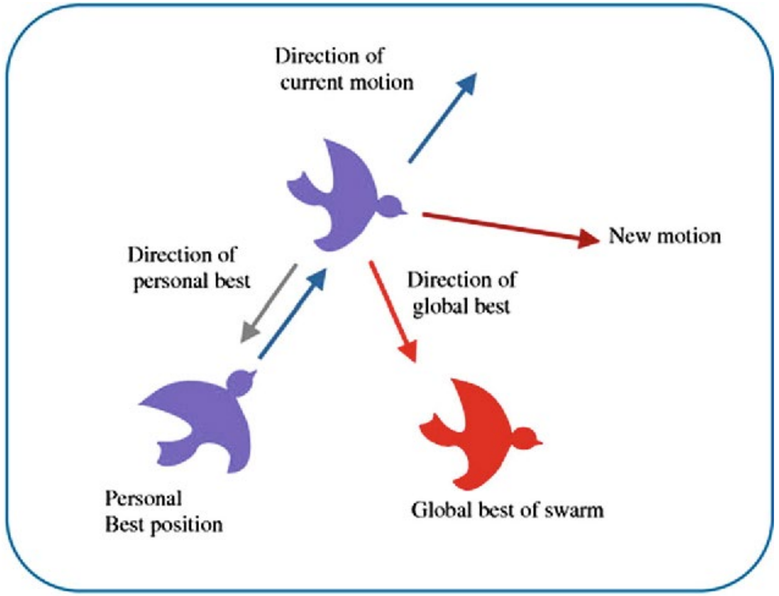
History of optimization

Second Generation Optimization: The Metaheuristic Approach

- Genetic Algorithms, “survival of the fittest”; Particle Swarm Optimization
- Pros: global optimization approaches; do not require gradients; support parallel computation.
- Cons: an enormous amount of trial points; the computation time could be hours or days.



Genetic algorithms



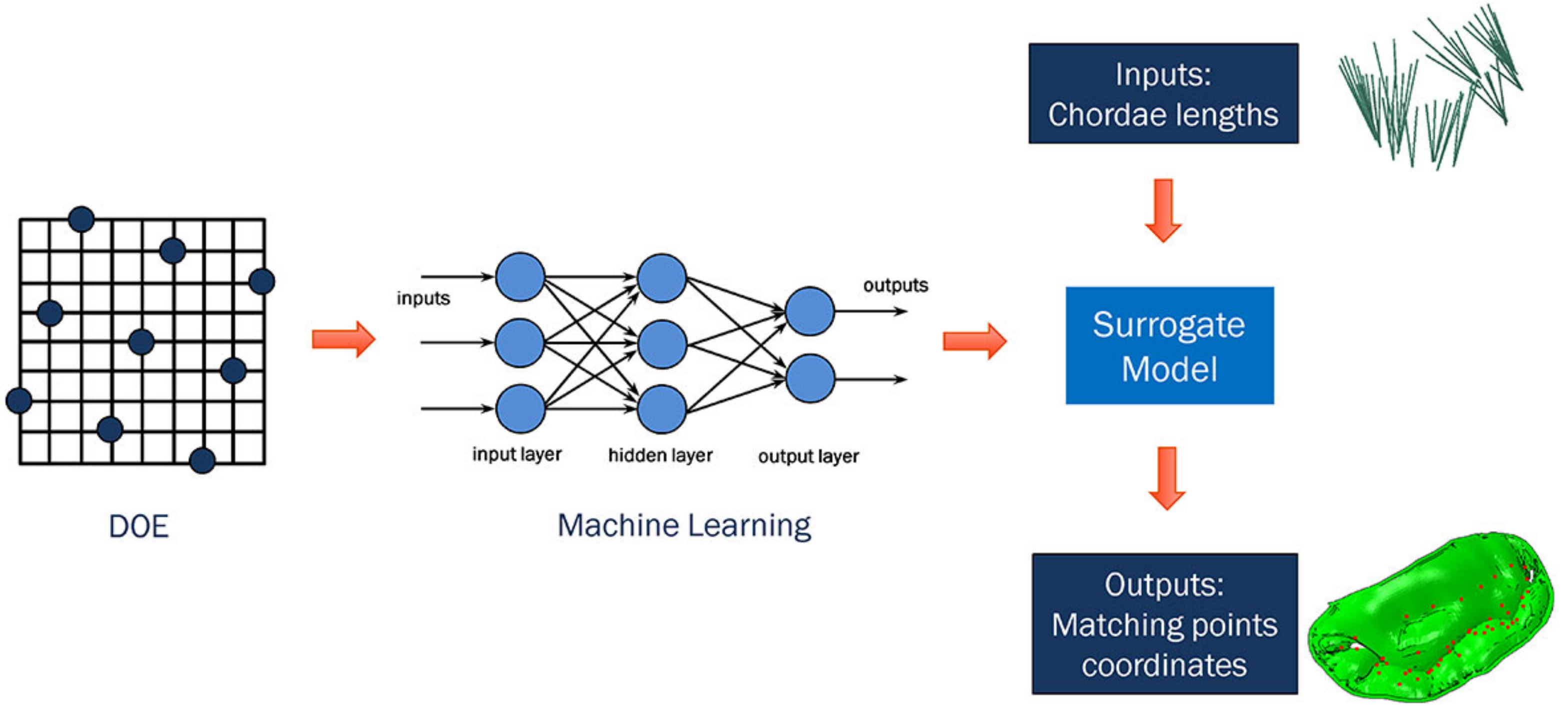
Particle swarm optimization

Pictures from Google image

History of optimization

Third Generation Optimization: The AI-Based Approach

- Also called surrogate-based optimization
- Pros: support parallel computation, do not need gradients, use fewer design trials than before, offer insight and knowledge about the design problem
- Cons: curse-of-dimensionality (Richard E. Bellman)



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Basic concept

$$P: \min_{x \in \mathcal{X}} f(x)$$

- **Variable**: x - entities and parameters that a designer can change
- **Objective**: $f(x)$ - the goals that a designer wants to achieve
- **Constraints**: $\mathcal{X}$ – technical restrictions that a product or process must satisfy; or can be called *feasible region*; x is a feasible point if $x \in \mathcal{X}$.

An optimization is either:

- **Infeasible**, where $\mathcal{X} = \Phi$
- **Unbounded**, where $\mathcal{X} \neq \Phi$, but $f(x)$ can go to infinitely negative (unbounded below, also called "optimum/minimum not attained") or infinitely positive (unbounded above, usually treated as an infeasible point) when x varies in $\mathcal{X}$
- **Feasible**, where $\mathcal{X} \neq \Phi$, and the value $f(x)$ is finite.

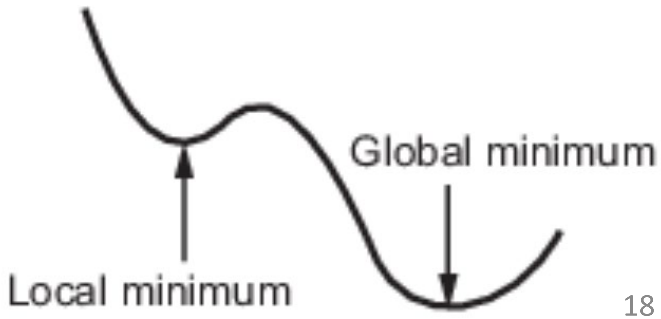
Basic concept

Global optimum. Let $f(x)$ be the objective function, $\mathcal{X}$ be the feasible region, and $x_0 \in \mathcal{X}$. Then x_0 is the global optimum if and only if $f(x) \geq f(x_0), \forall x \in \mathcal{X}$.

Local optimum. Let $f(x)$ be the objective function, $\mathcal{X}$ be the feasible region , and $x_0 \in \mathcal{X}$. If there is a neighborhood of x_0 with radius $\varepsilon > 0$:

$$\mathcal{N}_\epsilon(x_0) = \{x \mid ||x - x_0|| < \epsilon\}$$

Such that $\forall x \in \chi \cap N_\varepsilon(x_0)$, we have $f(x) \geq f(x_0)$. Then x_0 is a local optimum.



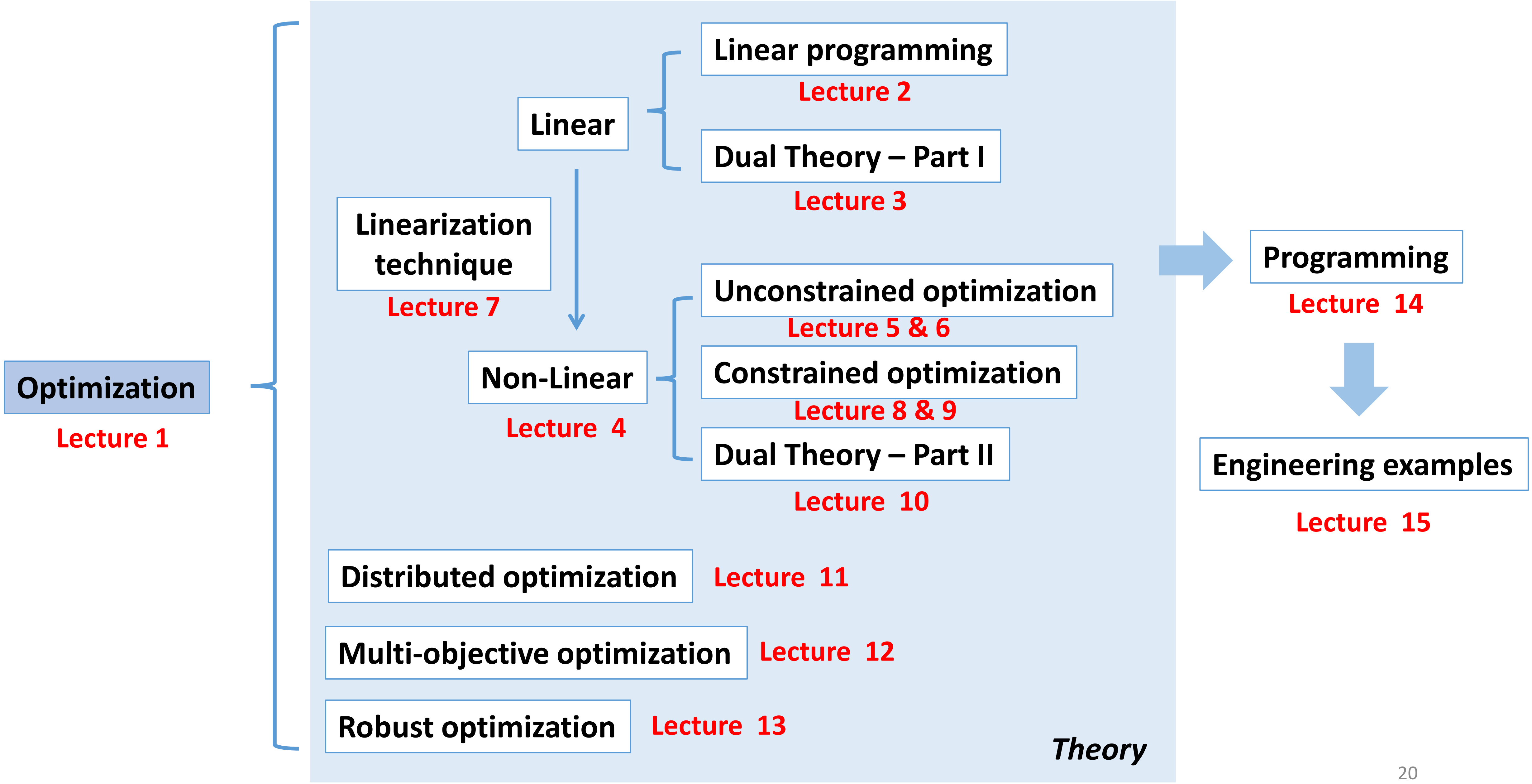
Classification of optimization problems

Problem:
- Constrained v.s. nonconstrained
- Single-objective v.s. multi-objective
- Single-level v.s. multi-level
- Deterministic v.s. uncertain

Response:
- Linear v.s. nonlinear
- Convex v.s. nonconvex
- Smooth v.s. nonsmooth

Variable:
- Continuous v.s. discrete (integer/binary)

Content of this course (tentative)



Procedure of applying optimization in engineering

In general, applying optimization in engineering follows five steps:

Step 1: Propose optimization problems, collect relevant data and information

Step 2: Establish mathematical model: determine the variables, the objective function and constraints

Step 3: Analyze the model and select the appropriate optimization method

Step 4: Solve: obtain the optimal solution via programming

Step 5: Test and implementation of optimal solution

Pictures from Google image

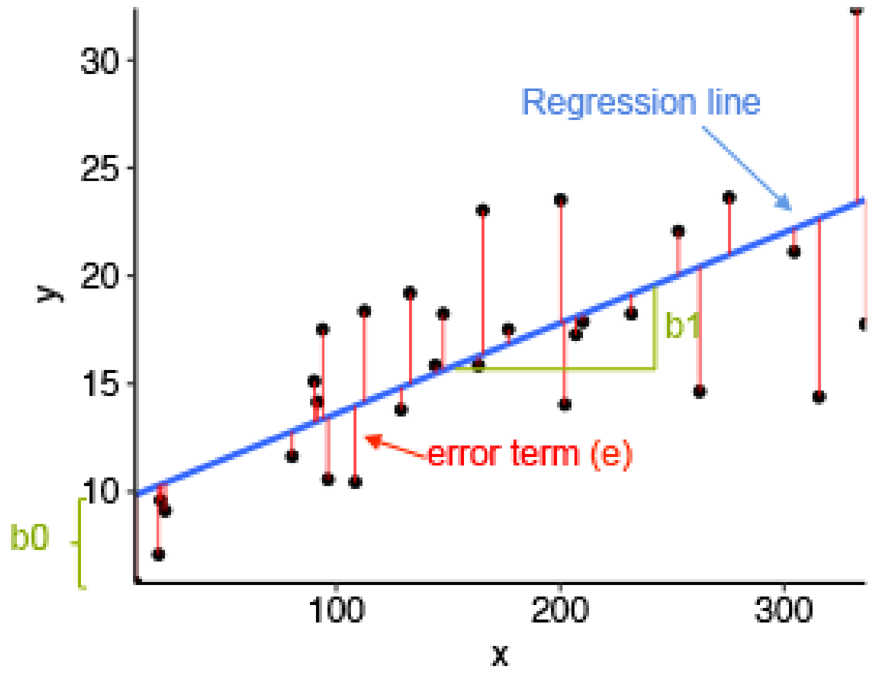
Example-1 Parametric Regression

Given a data set $\{y_i, x_i\}, \forall i = 1, \dots, N$
we assume that the relationship between the dependent variable y and the regressor x is linear.

$$y = b_0 + b_1 x + \varepsilon$$

We want to minimize the sum of square error.

$$\min_{b_0, b_1} \sum_{i=1}^{N} (y_i - b_0 - b_1 x_i)^2$$



Example-2 Production planning

A company has some resources to produce three products (denoted as A, B, C). Each product consumes a different mix of resources, and there will be a profit from selling the product. The endowment of resources and its relationship with products are:

	A	B	C	Endowment
Steel	3	4	2	600
Wood	2	1	2	400
Label	1	3	3	300
Machine	1	4	4	200
Profit	2	4	3	

Question: How to maximize the total profit?

Example-2 Production planning

Variables: Denote x_1, x_2, x_3 be the production of products A, B, C, respectively.

Objective: How to maximize the total profit?

Constraints: do not violate the resource endowment

$$
\begin{aligned}
\max_{x_1,x_2,x_3} \quad & 2x_1 + 4x_2 + 3x_3 \\
\text{s.t.} \quad & 3x_1 + 4x_2 + 2x_3 \leq 600 \\
& 2x_1 + x_2 + 3x_3 \leq 400 \\
& x_1 + 3x_2 + 3x_3 \leq 300 \\
& x_1 + 4x_2 + 4x_3 \leq 200 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

Endowment limits

Production must larger than 0

Example-3 Transportation

A building materials company has three cement factories A_1, A_2, A_3 and four dealers B_1, B_2, B_3, B_4. Its output, sales volume and freight are shown in the table below

	B_1	B_2	B_3	B_4	Production
A_1	8	7	3	2	2000
A_2	4	7	5	1	10000
A_3	2	4	9	6	4000
Sales	3000	2000	4000	5000	

Question: How to minimize the freight?

Example-3 Transportation

Variables: Denote x_{ij} as the quantity from factory A_i to dealer B_j

Constraints: Production ability limits; selling requirement

Objective: Minimize the total freight

$$\begin{aligned}\min_{x_{ij},\forall i,j}\ & 8x_{11}+7x_{12}+3x_{13}+2x_{14}+4x_{21}+7x_{22}\\ & +5x_{23}+x_{24}+2x_{31}+4x_{32}+9x_{33}+6x_{34}\\ \text{s.t. } & x_{11}+x_{12}+x_{13}+x_{14}\le 2000\\ & x_{21}+x_{22}+x_{23}+x_{24}\le 10000\\ & x_{31}+x_{32}+x_{33}+x_{34}\le 4000\\ & x_{11}+x_{21}+x_{31}\ge 3000\\ & x_{12}+x_{22}+x_{32}\ge 2000\\ & x_{13}+x_{23}+x_{33}\ge 4000\\ & x_{14}+x_{24}+x_{34}\ge 5000\\ & x_{ij}\ge 0, i=1,2,3; j=1,2,3,4\end{aligned}$$

Production ability

Sells requirement

Quantity must larger than 0

Example-4 Location problem

Suppose there are $j = 1, \ldots, n$ markets whose location is (a_j, b_j). Market j needs q_j product. We plan to build m warehouse whose capacities are $c_i, i = 1, \ldots, m$. How to choose the location of these warehouses so that the total cost is minimized?

$$\min_{x_i, y_i, w_{ij}} \quad \sum_{j=1}^{n} \sum_{i=1}^{m} w_{ij} \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} w_{ij} \leq c_i, \forall i = 1, .., m$$

$$\sum_{i=1}^{m} w_{ij} = q_j, \forall j = 1, ..., n$$

$$w_{ij} \geq 0, i = 1, ..., m, j = 1, ..., n$$

Thanks!