香港中文大學
The Chinese University of Hong Kong

THE CHINESE UNIVERSITY OF HONG KONG

DEPARTMENT OF MECHANICAL & AUTOMATION ENGINEERING

MAEG5070 Nonlinear Control Systems

Assignment #9

by

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Problem 1

Consider the following system

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2 \\ \dot{x}_2 &= x_3 + \cos(x_1)u \\ \dot{x}_3 &= x_1 + x_2^2 + \lambda x_3 \\ y &= x_1 \end{aligned} \tag{1}$$

(a) For what values of λ is the system minimum phase? nonminimum phase?

(b) Assume a state feedback control law $u = k(x)$ is such that $\ddot{y}(t) + 4\dot{y}(t) + 2y(t) = 0$. Is the equilibrium point at the origin of the closed-loop system (locally) asymptotically stable for all $\lambda \in R$? Why or Why not?

Solution:

(a) The Jacobian linearization of the system at the origin is given by

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{2}$$

with

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & \lambda \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{3}$$

The transfer function of the above linear system is then given by

$$\begin{aligned} H(s) &= C(sI - A)^{-1} B \\ &= \frac{1}{s(s-1)(s-\lambda)+1} C \begin{bmatrix} s(s-\lambda) & s-\lambda & 1 \\ 1 & (s-1)(s-\lambda) & s-1 \\ s & 1 & s(s-1) \end{bmatrix} B \\ &= \frac{s-\lambda}{s(s-1)(s-\lambda)+1} \end{aligned} \tag{4}$$

The system has a zero at $s = \lambda$. Thus, it is minimum phase for all $\lambda < 0$ and it is nonminimum phase for all $\lambda \geq 0$.

(b) No. Since when $\lambda \geq 0$, the system is nonminimum phase at the origin, the equilibrium cannot be made locally asymptotically stable.

Problem 2

The motion equation of a single-link robot rigid-joint manipulator is given by

$$\ddot{y} + a\dot{y}\sin(y) = \beta(x)u \tag{5}$$

(a) Give the state space equation of Equation (5) with $x_1 = y$ and $x_2 = \dot{y}$.

(b) Assume $1 \le a \le 2$ and $0.5 < \beta(x) < 2.5$, using $\hat{a} = 1.5$ to design a sliding mode control law u such that

$$\frac{ds^2}{dt} \le -|s| \tag{6}$$

where $s = \dot{e} + 2e$ with $e = y - y_d$.

(c) Assume y_d is a unit step function, simulate your design on the closed-loop system consisting of the plant Equation (5) with $a = 2$ and the sliding mode control law with $\hat{a} = 1.5$. Illustrate the performance of your control law by plotting $y(t)$ and $y_d(t)$ in the same figure for $0 \le t \le 20$. Also, plot $u(t)$ for $0 \le t \le 20$.

(d) In the control law designed in part (b), replace $sgn(s)$ by $sat(s/0.2)$, and repeat part (c).

Hint: In simulation, you can let $F(x) = |\alpha(x) - \hat{\alpha}(x)|$ and $\beta(x) = 1$.

Solution:

(a) Let $x_1 = y$ and $x_2 = \dot{y}$, the state-space equation is

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -ax_2\sin x_1 + \beta(x)u \\ y = x_1 \end{cases} \tag{7}$$

(b) Define the estimate $\hat{\beta}$ of $\beta(x)$ by $\hat{\beta} = (b_{min}b_{max})^{1/2} = \frac{\sqrt{5}}{2}$ and let $b = (b_{max}/b_{min})^{1/2} = \sqrt{5}$.

Using $\hat{a} = 1.5$ yields that

$$\hat{\alpha} = -1.5x_2\sin x_1 \tag{8}$$

$$\Delta\alpha = \alpha(x) - \hat{\alpha} \Longrightarrow |\Delta\alpha| = |\alpha(x) - \hat{\alpha}| \le 0.5|x_2\sin x_1| \Longrightarrow F(x) = 0.5|x_2\sin x_1| \tag{9}$$

Therefore,

$$\hat{u} = -\hat{\alpha}(x) + \ddot{y}_d - \alpha_1\dot{e} \tag{10}$$

Because $s = \dot{e} + 2e$, $\alpha_1 = 2$. Therefore,

$$\hat{u} = 1.5x_2\sin x_1 + \ddot{y}_d - 2\dot{e} \tag{11}$$

Because the sliding mode control law u should satisfy

$$\frac{ds^2}{dt} \le -|s| \tag{12}$$

we can know that $\eta = \frac{1}{2}$. Therefore, we can let $\phi(x)$ be

$$\begin{aligned}\phi(x) &= b\left(F(x)+\eta\right)+(b-1)\left|\hat{u}\right| \\ &= \sqrt{5}\left(0.5\left|x_2\sin x_1\right|+\frac{1}{2}\right)+\left(\sqrt{5}-1\right)\left|1.5x_2\sin x_1+\ddot{y}_d-2\dot{e}\right| \\ &= \sqrt{5}\left(0.5\left|x_2\sin x_1\right|+\frac{1}{2}\right)+\left(\sqrt{5}-1\right)\left|1.5x_2\sin x_1+\ddot{y}_d-2\left(\dot{y}-\dot{y}_d\right)\right| \\ &= \sqrt{5}\left(0.5\left|x_2\sin x_1\right|+\frac{1}{2}\right)+\left(\sqrt{5}-1\right)\left|1.5x_2\sin x_1+\ddot{y}_d-2\left(x_2-\dot{y}_d\right)\right|\end{aligned} \tag{13}$$

Hence, we can design a sliding mode control law u as follows:

$$\begin{aligned}u &= \hat{\beta}^{-1}\left[\hat{u}-\phi(x)\,sgn(s)\right] \\ &= \frac{2}{\sqrt{5}}\left[1.5x_2\sin x_1+\ddot{y}_d-2\dot{e}-\phi(x)\,sgn(s)\right] \\ &= \frac{2}{\sqrt{5}}\left[1.5x_2\sin x_1+\ddot{y}_d-2\left(x_2-\dot{y}_d\right)-\phi(x)\,sgn(s)\right]\end{aligned} \tag{14}$$

where

$$\phi(x) = \sqrt{5}\left(0.5\left|x_2\sin x_1\right|+\frac{1}{2}\right)+\left(\sqrt{5}-1\right)\left|1.5x_2\sin x_1+\ddot{y}_d-2\left(x_2-\dot{y}_d\right)\right| \tag{15}$$

$$s = \dot{e}+2e = \left(\dot{y}-\dot{y}_d\right)+2\left(y-y_d\right) = \left(x_2-\dot{y}_d\right)+2\left(x_1-y_d\right) \tag{16}$$

The MATLAB shown below is used to simulate the performance of the designed controller.

```
clc; clf; clear all;
%% sgn part
[t,x] = ode45('Q9_2_Systemsgn',[0,20],[0 0]);
phi = 0.5+0.5*abs(x(:,2).*sin(x(:,1)));
y_d = 1;
y_ddot = 0;
y_dddot = 0;
y = x(:,1);
ydot = x(:,2);
s = ydot-y_ddot+2*(y-y_d);
u = -phi.*sgn(s)+1.5*x(:,2).*sin(x(:,1))+y_dddot-2*(ydot-y_ddot);
figure(1);
hold on;
plot(t, y,'color',[0.667 0.667 1],'LineWidth',2.5);
plot(t, y_d+t*0,'color',[1 0.5 0],'LineWidth',2.5);
hold off;
grid on;
legend('$y\left(t\right)$','$y_d\left(t\right)$','interpreter','latex');
xlabel('$t, \ (\mathrm{s})$','interpreter','latex');
ylabel('$\theta, \ (\mathrm{rad})$','interpreter','latex');
a = get(gca,'XTickLabel');
set(gca,'XTickLabel',a,'FontName','Times','fontsize',12);
```

```
set(gca,’position’,[0.15 0.20 0.6 0.6]);
set(gcf,’position’,[100 100 800 600]);
set(gcf,’renderer’,’painters’);
filename = "Q9-2-yyd-sgn"+".pdf";
saveas(gcf,filename);
figure(2);
plot(t, u,’color’,[0.667 0.667 1],’LineWidth’,2.5);
grid on;
xlabel(’$t, \ (\mathrm{s})$’,’interpreter’,’latex’);
ylabel(’$u, \ (\mathrm{N\cdot m})$’,’interpreter’,’latex’);
a = get(gca,’XTickLabel’);
set(gca,’XTickLabel’,a,’FontName’,’Times’,’fontsize’,12);
set(gca,’position’,[0.15 0.20 0.6 0.6]);
set(gcf,’position’,[100 100 800 600]);
set(gcf,’renderer’,’painters’);
filename = "Q9-2-u-sgn"+".pdf";
saveas(gcf,filename);
%% sat part
[t,x] = ode45(’Q9_2_Systemsat’,[0,20],[0 0]);
phi = 0.5+0.5*abs(x(:,2).*sin(x(:,1)));
y_d = 1;
y_ddot = 0;
y_dddot = 0;
y = x(:,1);
ydot = x(:,2);
s = ydot-y_ddot+2*(y-y_d);
u = -phi.*sgn(s)+1.5*x(:,2).*sin(x(:,1))+y_dddot-2*(ydot-y_ddot);
figure(3);
hold on;
plot(t, y,’color’,[0.667 0.667 1],’LineWidth’,2.5);
plot(t, y_d+t*0,’color’,[1 0.5 0],’LineWidth’,2.5);
hold off;
grid on;
legend(’$y\left(t\right)$’,’$y_d\left(t\right)$’,’interpreter’,’latex’);
xlabel(’$t, \ (\mathrm{s})$’,’interpreter’,’latex’);
ylabel(’$\theta, \ (\mathrm{rad})$’,’interpreter’,’latex’);
a = get(gca,’XTickLabel’);
set(gca,’XTickLabel’,a,’FontName’,’Times’,’fontsize’,12);
set(gca,’position’,[0.15 0.20 0.6 0.6]);
set(gcf,’position’,[100 100 800 600]);
set(gcf,’renderer’,’painters’);
filename = "Q9-2-yyd-sat"+".pdf";
saveas(gcf,filename);
figure(4);
plot(t, u,’color’,[0.667 0.667 1],’LineWidth’,2.5);
grid on;
```

```
xlabel('$t, \ (\mathrm{s})$','interpreter','latex');
ylabel('$u, \ (\mathrm{N\cdot m})$','interpreter','latex');
a = get(gca,'XTickLabel');
set(gca,'XTickLabel',a,'FontName','Times','fontsize',12);
set(gca,'position',[0.15 0.20 0.6 0.6]);
set(gcf,'position',[100 100 800 600]);
set(gcf,'renderer','painters');
filename = "Q9-2-u-sat"+".pdf";
saveas(gcf,filename);
```

where the codes for the system representation are shown below:

```
function xd = Q9_2_Systemsgn(t,x)
    xd(1) = x(2);
    phi = 0.5+0.5*abs(x(2)*sin(x(1)));
    y_d = 1;
    y_ddot = 0;
    y_dddot = 0;
    y = x(1);
    ydot = x(2);
    s = ydot-y_ddot+2*(y-y_d);
    u = -phi*sgn(s)+1.5*x(2)*sin(x(1))+y_dddot-2*(ydot-y_ddot);
    xd(2) = -2*x(2)*sin(x(1))+u;
    xd = xd';
end
```

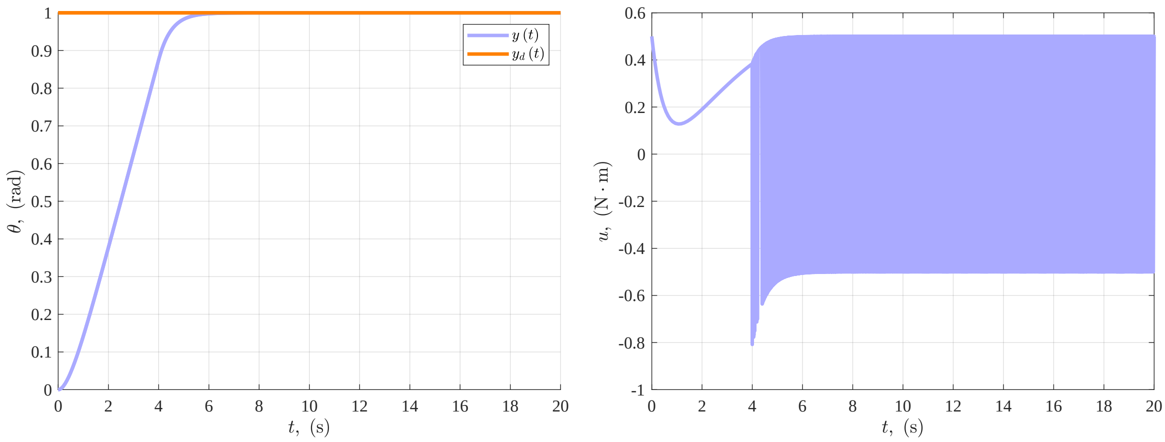
and the sgn function is designed as follows:

```
function y = sgn(s)
    if s == 0
        y = 0;
    else
        y = s./abs(s);
    end
end
```

The simulation results are shown in Figure 1.

(c) The codes for the controller are changed for sat function as shown below:

```
function xd = Q9_2_Systemsat(t,x)
    xd(1) = x(2);
    phi = 0.5+0.5*abs(x(2)*sin(x(1)));
    y_d = 0*t+1;
    y_ddot = 0;
    y_dddot = 0;
    y = x(1);
    ydot = x(2);
```



(a) $y(t)$ and $y_d(t)$ v.s. t.

(b) u v.s. t.

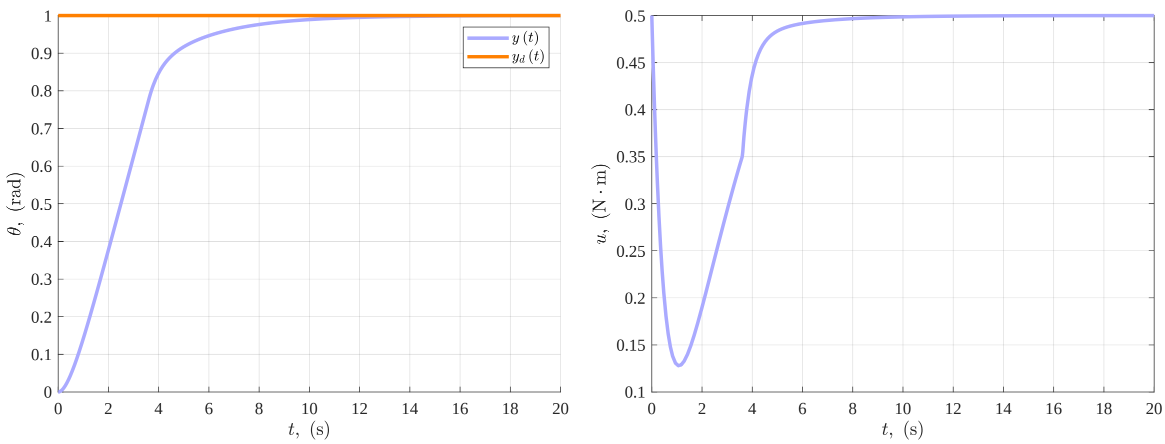
Figure 1: Simulation results for the controller with sgn.

```
    s = ydot-y_ddot+2*(y-y_d);
    u = -phi*sat(s)+1.5*x(2)*sin(x(1))+y_dddot-2*(ydot-y_ddot);
    xd(2) = -2*x(2)*sin(x(1))+u;
    xd = xd';
end
```

and the sat function is designed as follows:

```
function y = sat(s)
    if abs(s) < 0.2
        y = s;
    elseif s < 0
        y = -1;
    else
        y = 1;
    end
end
```

The simulation results are shown in Figure 2.



(a) $y(t)$ and $y_d(t)$ v.s. t.

(b) u v.s. t.

Figure 2: Simulation results for the controller with sat.

Problem 3

Consider the motion equation of a single-link robot rigid-joint manipulator given in Equation (5) where $\beta(x) = 1$ for all x.

(a) Assume $a = 1.5$, $y_d = 2\sin t$ and $s = \dot{e} + 3e$. Design a control law of the form (8.7) of the lecture note with $k = 2$ and simulate the performance of your control law by plotting $y(t)$ and $y_d(t)$ in the same figure for $0 \le t \le 20$. Also, plot $u(t)$ for $0 \le t \le 20$.

(b) Assume the actual value of $a = 2$. Use the same control law as the one in Part (i) to simulate the performance of your control law by plotting $y(t)$ and $y_d(t)$ in the same figure for $0 \le t \le 20$. Also, plot $u(t)$ for $0 \le t \le 20$.

(c) Assume a is unknown, put the system in the form (8.3) of the lecture note and identify a_0, a_1, a_2 and f_1, f_2.

(d) Design an adaptive control law of the form (8.10) and (8.12) of the lecture note with $\gamma_i = 3$. Assume the actual value of $a = 2.5$, respectively. Simulate the performance of your control law by plotting $y(t)$ and $y_d(t)$ in the same figure for $0 \le t \le 20$. Also, plot $u(t)$ and $\hat{a}_i(t)$ for $0 \le t \le 20$.

Hint: Note Part (iv) of Remark 8.1.

Solution:

(a) Because $\beta(x) = 1$, $a_0 = 1$. Consider the control law,

$$u = a_0 f_0(x,t) - ks + \sum_{i=1}^{m} a_i f_i(x,t) \tag{17}$$

where $k = 2$, and

$$f_0(x,t) = \ddot{y}_d - \alpha_1 \dot{e} \tag{18}$$

Because $s = \dot{e} + 3e$, $\alpha_1 = 3$. Therefore, the designed control law for the system is as follows:

$$\begin{aligned} u &= \ddot{y}_d - 3\dot{e} - 2s + 1.5\dot{y}\sin(y) \\ &= \ddot{y}_d - 3(x_2 - \dot{y}_d) - 2((x_2 - \dot{y}_d) + 3(x_1 - y_d)) + 1.5x_2\sin x_1 \end{aligned} \tag{19}$$

The MATLAB shown below is used to simulate the performance of the designed controller.

```
clc; clf; clear all;
%% Q9-3-a
[t,x] = ode45('Q9_3_a_System',[0,20],[0 0]);
y_d = 2*sin(t);
y_ddot = 2*cos(t);
y_dddot = -2*sin(t);
```

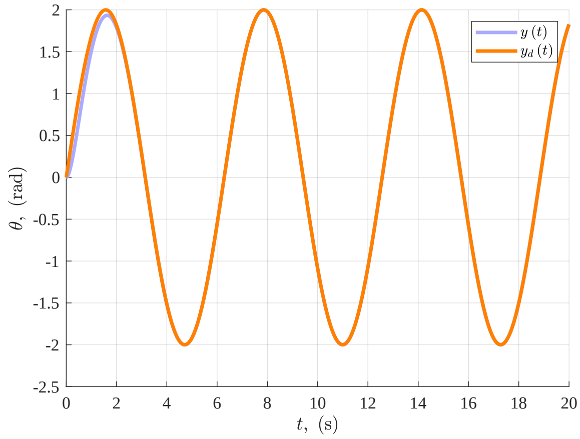
```
y = x(:,1);
ydot = x(:,2);
s = ydot-y_ddot+3*(y-y_d);
a0 = 1;
f0 = y_dddot-3*(ydot-y_ddot);
u = a0*f0-2*s+1.5*x(:,2).*sin(x(:,1));
figure(1);
hold on;
plot(t, y,'color',[0.667 0.667 1],'LineWidth',2.5);
plot(t, y_d+t*0,'color',[1 0.5 0],'LineWidth',2.5);
hold off;
grid on;
legend('$y\left(t\right)$','$y_d\left(t\right)$','interpreter','latex');
xlabel('$t, \ (\mathrm{s})$','interpreter','latex');
ylabel('$\theta, \ (\mathrm{rad})$','interpreter','latex');
a = get(gca,'XTickLabel');
set(gca,'XTickLabel',a,'FontName','Times','fontsize',12);
set(gca,'position',[0.15 0.20 0.6 0.6]);
set(gcf,'position',[100 100 800 600]);
set(gcf,'renderer','painters');
filename = "Q9-3-a-yyd"+".pdf";
saveas(gcf,filename);
figure(2);
plot(t, u,'color',[0.667 0.667 1],'LineWidth',2.5);
grid on;
xlabel('$t, \ (\mathrm{s})$','interpreter','latex');
ylabel('$u, \ (\mathrm{N\cdot m})$','interpreter','latex');
a = get(gca,'XTickLabel');
set(gca,'XTickLabel',a,'FontName','Times','fontsize',12);
set(gca,'position',[0.15 0.20 0.6 0.6]);
set(gcf,'position',[100 100 800 600]);
set(gcf,'renderer','painters');
filename = "Q9-3-a-u"+".pdf";
saveas(gcf,filename);
%% Q9-3-b
[t,x] = ode45('Q9_3_b_System',[0,20],[0 0]);
y_d = 2*sin(t);
y_ddot = 2*cos(t);
y_dddot = -2*sin(t);
y = x(:,1);
ydot = x(:,2);
s = ydot-y_ddot+3*(y-y_d);
a0 = 1;
f0 = y_dddot-3*(ydot-y_ddot);
u = a0*f0-2*s+1.5*x(:,2).*sin(x(:,1));
figure(3);
```

```
hold on;
plot(t, y,'color',[0.667 0.667 1],'LineWidth',2.5);
plot(t, y_d+t*0,'color',[1 0.5 0],'LineWidth',2.5);
hold off;
grid on;
legend('$y\left(t\right)$','$y_d\left(t\right)$','interpreter','latex');
xlabel('$t, \ (\mathrm{s})$','interpreter','latex');
ylabel('$\theta, \ (\mathrm{rad})$','interpreter','latex');
a = get(gca,'XTickLabel');
set(gca,'XTickLabel',a,'FontName','Times','fontsize',12);
set(gca,'position',[0.15 0.20 0.6 0.6]);
set(gcf,'position',[100 100 800 600]);
set(gcf,'renderer','painters');
filename = "Q9-3-b-yyd"+".pdf";
saveas(gcf,filename);
figure(4);
plot(t, u,'color',[0.667 0.667 1],'LineWidth',2.5);
grid on;
xlabel('$t, \ (\mathrm{s})$','interpreter','latex');
ylabel('$u, \ (\mathrm{N\cdot m})$','interpreter','latex');
a = get(gca,'XTickLabel');
set(gca,'XTickLabel',a,'FontName','Times','fontsize',12);
set(gca,'position',[0.15 0.20 0.6 0.6]);
set(gcf,'position',[100 100 800 600]);
set(gcf,'renderer','painters');
filename = "Q9-3-b-u"+".pdf";
saveas(gcf,filename);
```

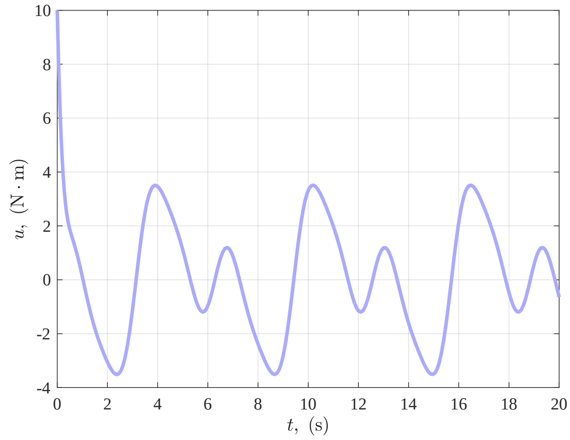
where the codes for the system representation are shown below:

```
function xd = Q9_3_a_System(t,x)
    xd(1) = x(2);
%     phi = 0.5+0.5*abs(x(2)*sin(x(1)));
    y_d = 2*sin(t);
    y_ddot = 2*cos(t);
    y_dddot = -2*sin(t);
    y = x(1);
    ydot = x(2);
    s = ydot-y_ddot+3*(y-y_d);
    a0 = 1;
    f0 = y_dddot-3*(ydot-y_ddot);
    u = a0*f0-2*s+1.5*x(2)*sin(x(1));
    xd(2) = -1.5*x(2)*sin(x(1))+u;
    xd = xd';
end
```

The simulation results are shown in Figure 3.



(a) $y(t)$ and $y_d(t)$ v.s. t.



(b) u v.s. t.

Figure 3: Simulation results for the controller with for the adaptive control with $k = 2$.

(b) The codes for the system representation are changed as shown below:

```
function xd = Q9_3_b_System(t,x)
    xd(1) = x(2);
%     phi = 0.5+0.5*abs(x(2)*sin(x(1)));
    y_d = 2*sin(t);
    y_ddot = 2*cos(t);
    y_dddot = -2*sin(t);
    y = x(1);
    ydot = x(2);
    s = ydot-y_ddot+3*(y-y_d);
    a0 = 1;
    f0 = y_dddot-3*(ydot-y_ddot);
    u = a0*f0-2*s+1.5*x(2)*sin(x(1));
    xd(2) = -2*x(2)*sin(x(1))+u;
    xd = xd';
end
```

The simulation results are shown in Figure 4.

(c)

$$a_0 y^{(n)} + \sum_{i=1}^{m} a_i f(x,t) = u \tag{20}$$

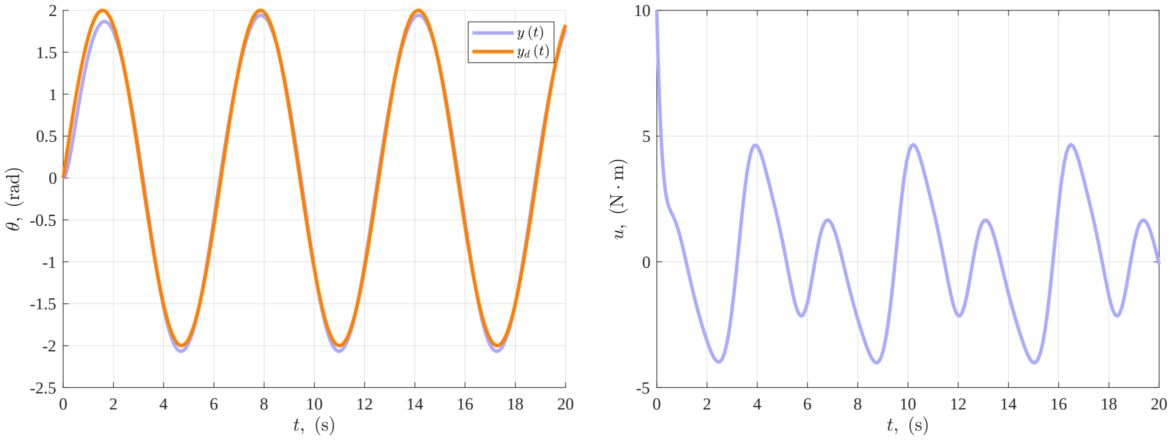
Here, $a_0 = 1$, $a_1 = a$, $a_2 = 0$, $f_1(x) = x_2 \sin x_1$, $f_2(x) = 0$

(d)

$$\begin{aligned} u &= a_0 f_0(x,t) - ks + \textstyle\sum_{i=1}^{m} \hat{a}_i f_i(x,t) \\ \dot{\hat{a}}_i &= -\gamma_i sgn(a_0)\, s f_i, i = 1, \cdots, m \end{aligned} \tag{21}$$

Because $\gamma_i = 3$ and $a_0 = 1$, Equation (21) can be simplified into

$$\begin{aligned} u &= (\ddot{y}_d - 3(x_2 - \dot{y}_d)) - 2((x_2 - \dot{y}_d) + 3(x_1 - y_d)) + \hat{a}_1 x_2 \sin x_1 \\ \dot{\hat{a}}_1 &= -3((x_2 - \dot{y}_d) + 3(x_1 - y_d)) x_2 \sin x_1 \end{aligned} \tag{22}$$



(a) $y(t)$ and $y_d(t)$ v.s. t. **(b)** u v.s. t.

Figure 4: Simulation results for the controller with for the adaptive control with $k = 2$ and the actual value of $a = 2$.

The Simulink as shown in Figure 5 is used to simulate the performance of the designed controller.

And we use the following code to plot the results:

```
clear all; clc;
figg1 = openfig('Q9-3-d-ahat.fig','reuse');
grid on;
xlabel('$t, \mathrm{\ \left(s\right)}$','interpreter','latex');
ylabel('$\hat{a}$', 'interpreter','latex');
% legend('$y_d$', '$y$', 'interpreter','latex','Location','southeast');
title('');
a = get(gca,'XTickLabel');
set(gca,'XTickLabel',a,'FontName','Times','fontsize',12);
set(gca,'position',[0.15 0.20 0.6 0.6]);
set(gcf,'position',[100 100 800 600]);
set(gcf,'renderer','painters');
filename = "Q9-3-d-ahat"+".pdf";
saveas(gcf,filename);
close(figg1);
figg2 = openfig('Q9-3-d-u.fig','reuse');
grid on;
xlim([0.07 20]);
ylim([-5 6]);
xlabel('$t, \ (\mathrm{s})$','interpreter','latex');
ylabel('$u, \ (\mathrm{N\cdot m})$','interpreter','latex');
% legend('$y_d$', '$y$', 'interpreter','latex','Location','southeast');
title('');
a = get(gca,'XTickLabel');
set(gca,'XTickLabel',a,'FontName','Times','fontsize',12);
set(gca,'position',[0.15 0.20 0.6 0.6]);
```

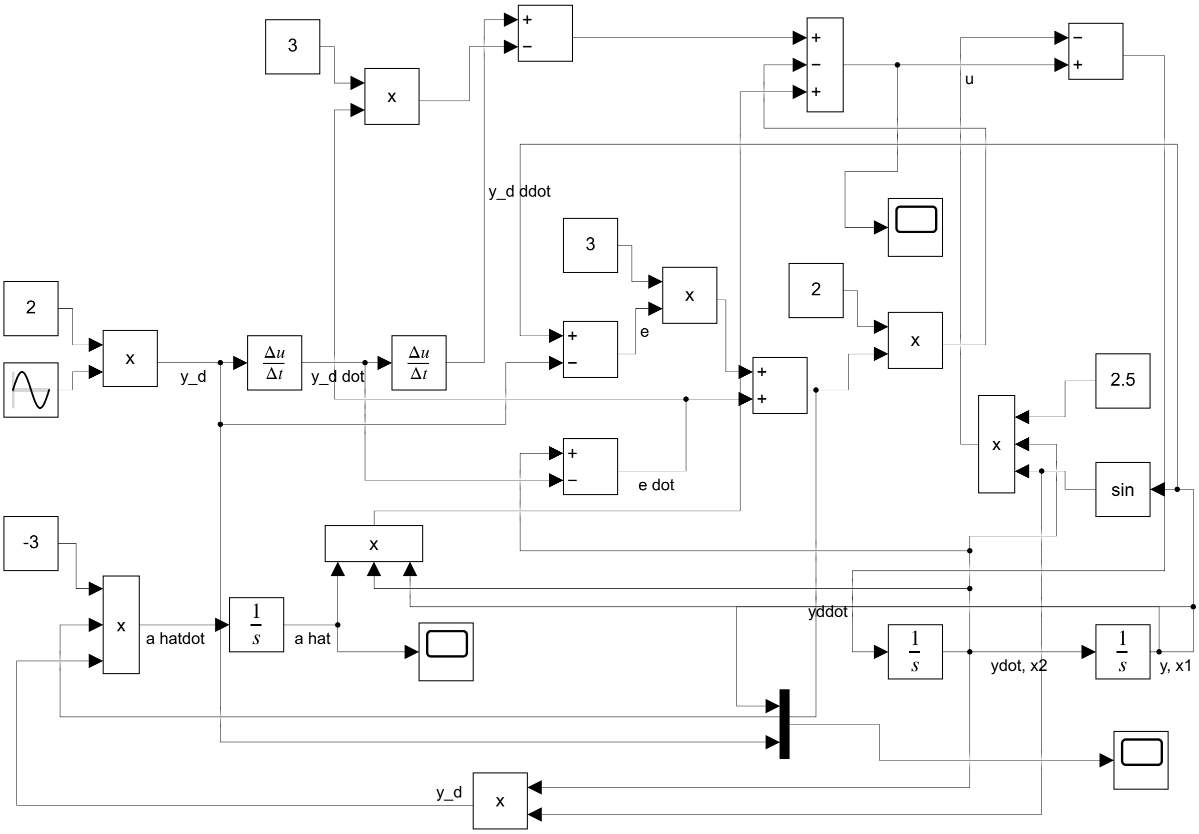
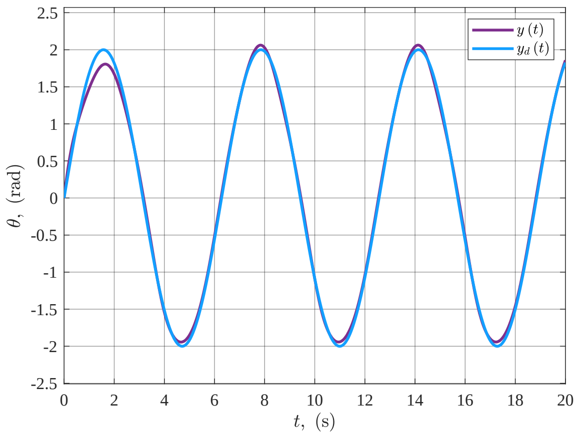


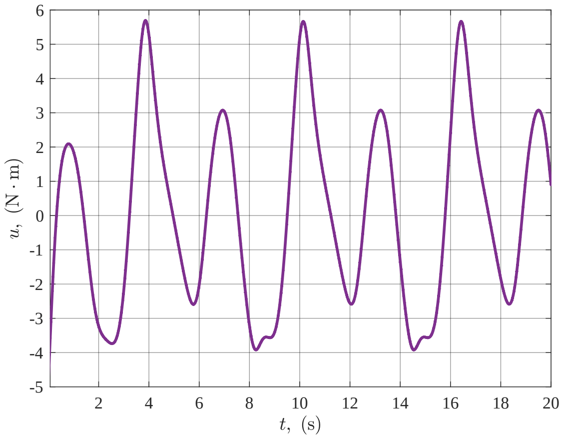
Figure 5: Block diagram for the system.

```
set(gcf,'position',[100 100 800 600]);
set(gcf,'renderer','painters');
filename = "Q9-3-d-u"+".pdf";
saveas(gcf,filename);
close(figg2);
figg3 = openfig('Q9-3-d-yyd.fig','reuse');
grid on;
legend('$y\left(t\right)$','$y_d\left(t\right)$','interpreter','latex');
xlabel('$t, \ (\mathrm{s})$','interpreter','latex');
ylabel('$\theta, \ (\mathrm{rad})$','interpreter','latex');
title('');
a = get(gca,'XTickLabel');
set(gca,'XTickLabel',a,'FontName','Times','fontsize',12);
set(gca,'position',[0.15 0.20 0.6 0.6]);
set(gcf,'position',[100 100 800 600]);
set(gcf,'renderer','painters');
filename = "Q9-3-d-yyd"+".pdf";
saveas(gcf,filename);
close(figg3);
```

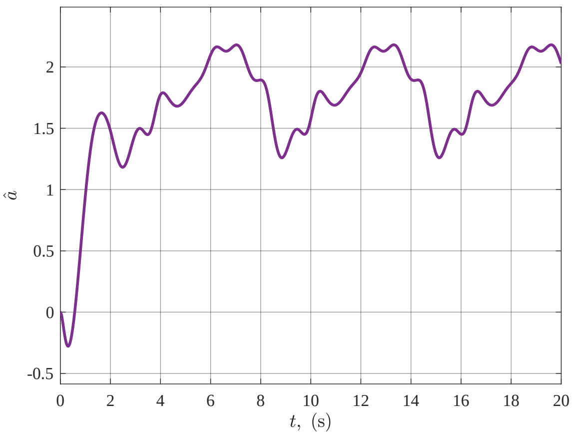
The simulation results are shown in Figure 6.



(a) $y(t)$ and $y_d(t)$ v.s. t.



(b) u v.s. t.



(c) $\hat{a}$ v.s. t.

Figure 6: Simulation results for the controller with for the adaptive control law of the form (8.10) and (8.12) of the lecture note.