$m\ddot{x} + c\dot{x} + kx = F(t)$

$\Rightarrow x(t) = x_h(t) + x_p(t)$: total response

where $x_h(t)$ is the solution of the homogeneous eq.

$$m\ddot{x} + c\dot{x} + kx = 0$$

represents transient response

free vibration dies out with time

$x_p(t)$ is the particular solution

represents steady-state response

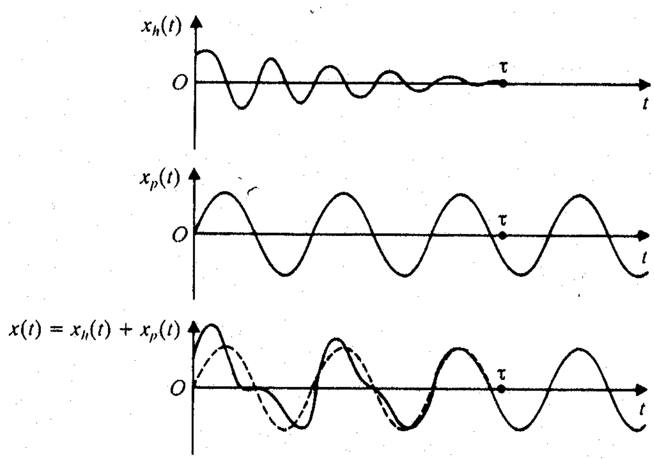
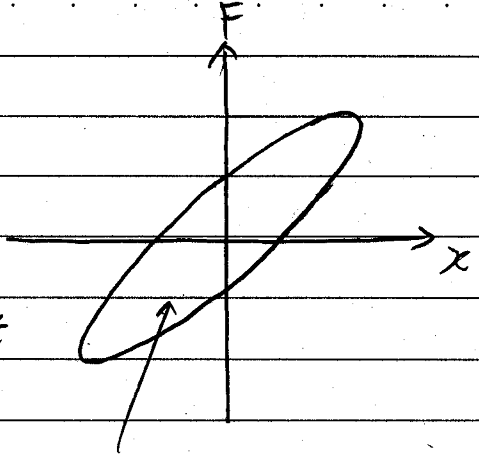


FIGURE 3.2 Homogenous, particular, and general solutions of Eq. (3.1) for an underdamped case.

- Energy Dissipated in Viscous Damping

$$E_d = \oint F\,dx = \int_0^{\frac{2\pi}{\omega}} F\dot{x}\,dt$$

$$= \int_0^{\frac{2\pi}{\omega}} F_0 \cos\omega t\left[-\omega X \sin(\omega t-\phi)\right]dt$$



$$\Rightarrow \boxed{E_d = \pi c \omega X^2} \qquad (30)$$

E_d = Area of Hysteresis Loop

Impulse Responsese

$$\text{Impulse} = F\Delta t = m\dot{x}_2 - m\dot{x}_1 \tag{1}$$

$$\tilde{F} = \int_t^{t+\Delta t} F\,dt \tag{2}$$

Unit impulse

$$\tilde{f} = \lim_{\Delta t \to 0} \int_t^{t+\Delta t} F\,dt = 1 \tag{3}$$

Consider

$$m\ddot{x} + c\dot{x} + kx = 0$$

for an underdamped system,

$$x(t) = e^{-\zeta\omega_n t}\left\{x_0 \cos\omega_d t + \frac{v_0 + \zeta\omega_n x_0}{\omega_d}\sin\omega_d t\right\} \tag{4}$$

If the mass is at rest before the unit impulse is applied at $t=0$,
($x=\dot{x}=0$ for $t<0$ or at $t=0^-$)

$$\text{Impulse} = \tilde{f} = 1 = m\dot{x}(t=0) - m\dot{x}(t=0^-)$$

$$= m\dot{x}(0). \tag{5}$$

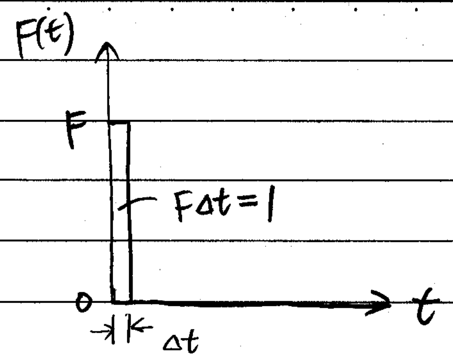
ICs:

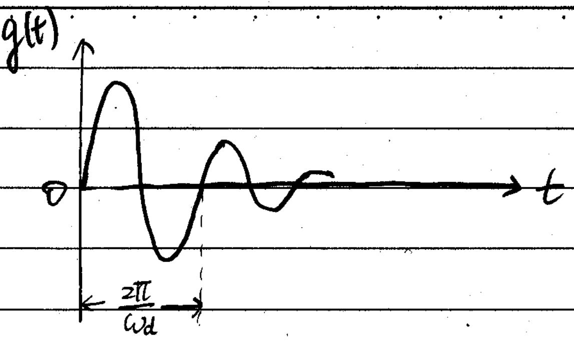
$$x(0) = x_0 = 0$$
$$\dot{x}(0) = v_0 = \frac{1}{m} \tag{6}$$

Sub. into eq.(4),

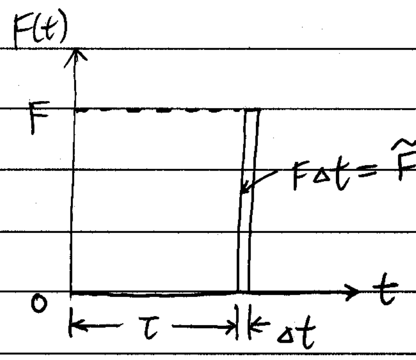
$$x(t) = g(t) = \frac{1}{m\omega_d} e^{-\zeta\omega_n t}\sin\omega_d t \tag{7}$$

: impulse response function

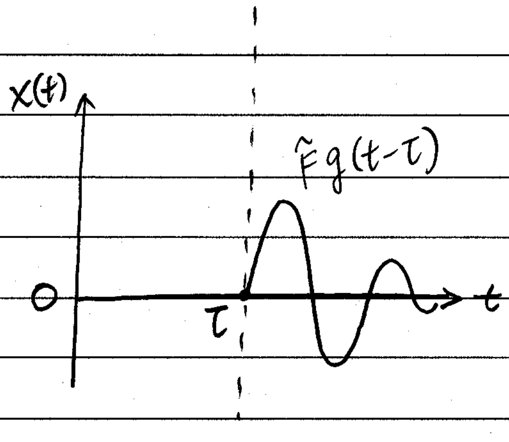




If the impulse $\tilde{F}$ is applied at an arbitary time $t = \tau$,

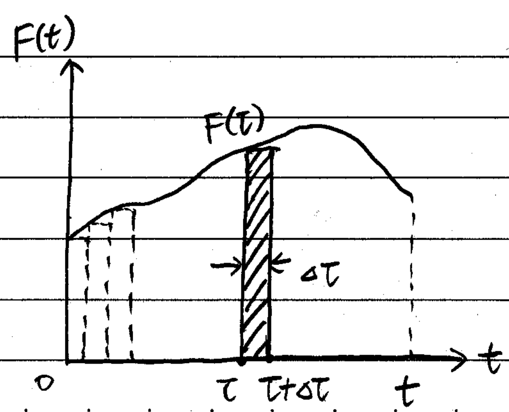


the displacement x at a later time t due to <u>that</u> impulse at time τ, is given by



$$x(t) = \tilde{F}\, g(t-\tau) \qquad (8)$$

<u>Convolution Integral</u>



For an arbitrary forcing function $F(t)$,

the response at t due to the impulse $\tilde{F} = F(\tau)\Delta\tau$:

$$\Delta x(t) = F(\tau)\Delta\tau\, g(t-\tau) \qquad (9)$$

The total response at time t as the result of all prior impulses applied,

$$x(t) \simeq \Sigma F(\tau)\, g(t-\tau)\, \Delta\tau \qquad (10)$$

Letting $\Delta\tau \to 0$,

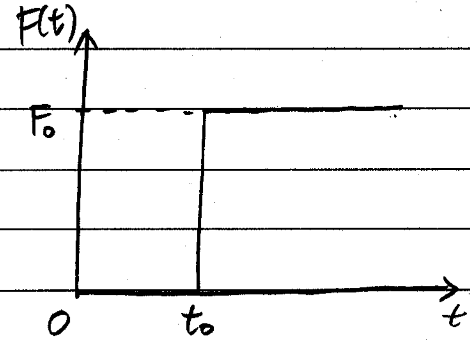
$$x(t) = \int_0^t F(\tau)\, g(t-\tau)\, d\tau \qquad (11)$$

By substituting Eq. (7) into Eq. (11),

Convolution (Duhamel) Integral

$$x(t) = \frac{1}{m\omega_d}\int_0^t F(\tau)\, e^{-\xi\omega_n(t-\tau)} \sin\omega_d(t-\tau)\, d\tau \qquad (12)$$

• Step Response

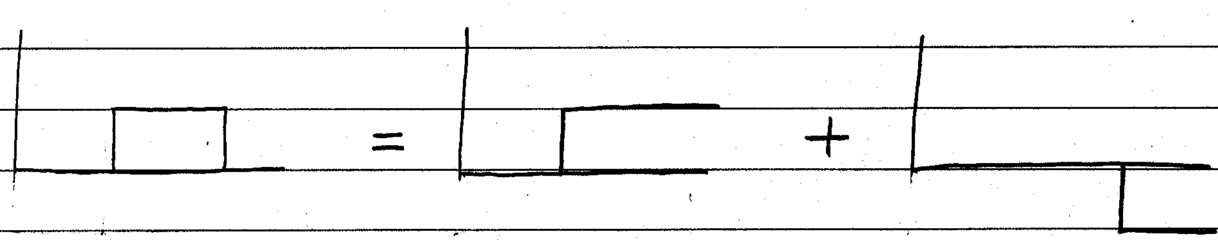


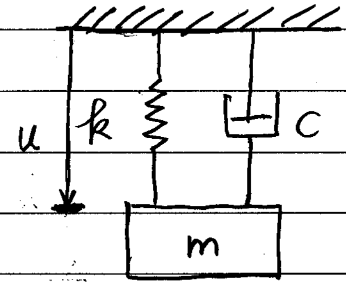
$$x(t) = \frac{F_0}{m\omega_d}\int_{t_0}^t e^{-\xi\omega_n(t-\tau)} \sin\omega_d(t-\tau)\, d\tau$$

$$= \frac{F_0}{k}\left[1 - \frac{e^{-\xi\omega_n(t-t_0)}}{\sqrt{1-\xi^2}} \cos\{\omega_d(t-t_0)-\phi\}\right]$$

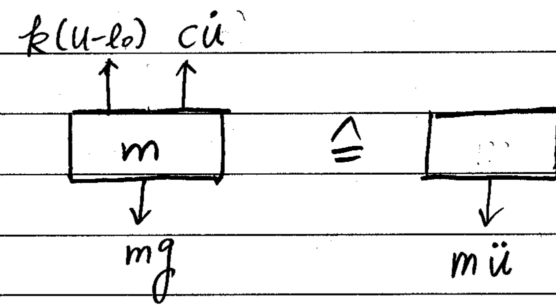
where $\phi = \tan^{-1}\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)$

• Pulse





l_0 : unstretched length of spring



$$m\ddot{u} = mg - k(u - l_0) - c\dot{u}$$

$$\Rightarrow \; m\ddot{u} + c\dot{u} + ku = mg + kl_0$$

find static equilibrium $(\dot{u} = \ddot{u} = 0)$

$$u_{eq} = \frac{mg + kl_0}{k} = \frac{mg}{k} + l_0 > l_0$$

define $\quad x = u - u_{eq} \;, \quad \dot{x} = \dot{u} \;, \quad \ddot{x} = \ddot{u}$

$$\Rightarrow \; m\ddot{x} + c\dot{x} + k(x + u_{eq}) = mg + kl_0$$

or $\quad m\ddot{x} + c\dot{x} + kx + k\left(\frac{mg + kl_0}{k}\right) = mg + kl_0$

$$\therefore \; m\ddot{x} + c\dot{x} + kx = 0$$