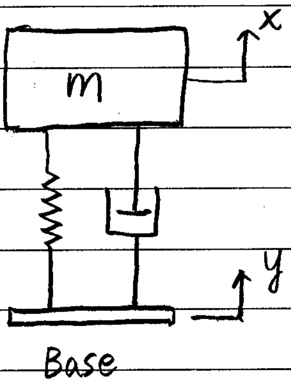
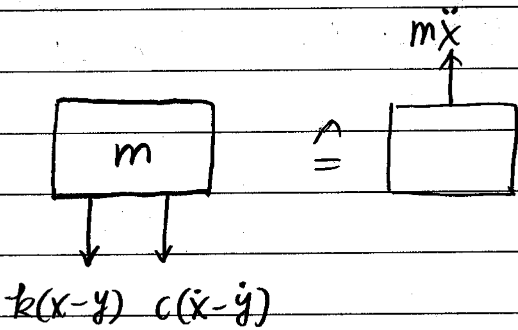
Base Excitation





Assume $x=0$ ($y=0$) at static equilibrium,

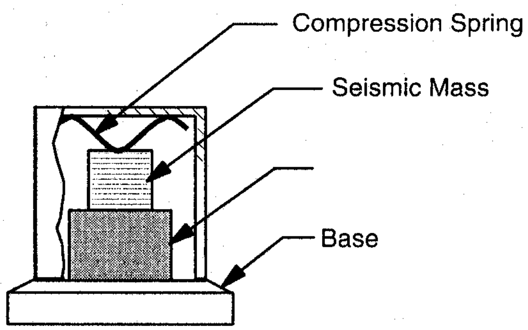
Using Newton's 2nd Law:

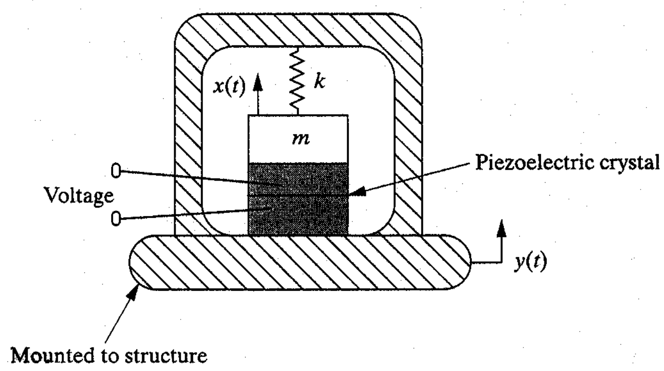
$$m\ddot{x} = -k(x-y) - c(\dot{x}-\dot{y})$$

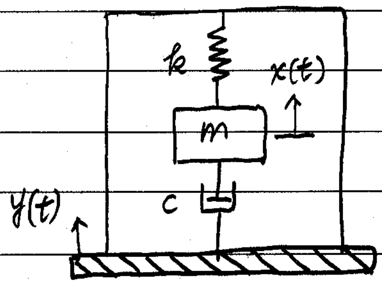
$$\Rightarrow \quad m\ddot{x} + c(\dot{x}-\dot{y}) + k(x-y) = 0$$

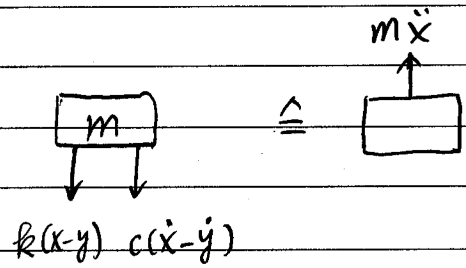
or $$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$

Piezoelectric Accelerometer









$y(t)$ = motion of structure

$$m\ddot{x} = -c(\dot{x}-\dot{y}) - k(x-y) \qquad (1)$$

The motion of the accelerometer mass relative to the base, denoted $z(t)$,

$$z(t) = x(t) - y(t) \qquad (2)$$

Eq. (1) becomes

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y} \qquad (3)$$

Assume $y = Y \cos \omega t$ (4)

Eq. (3) becomes

$$m\ddot{z} + c\dot{z} + kz = m\omega^2 Y \cos \omega t \quad (5)$$

The steady-state solution is given by

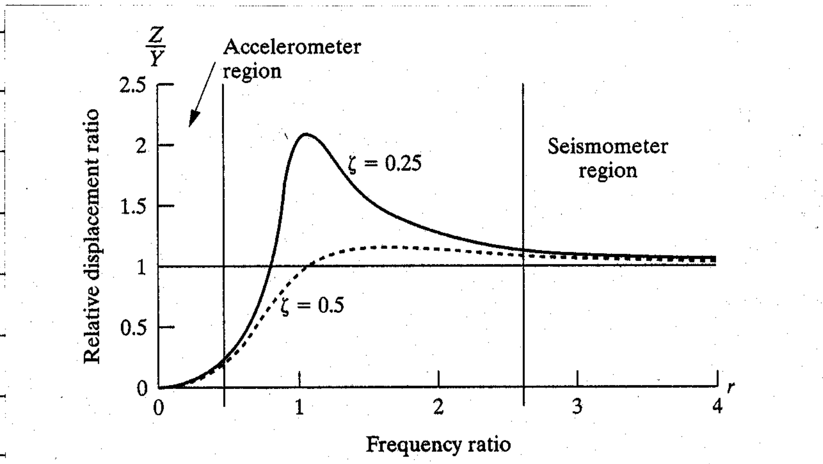
$$z(t) = Z \cos(\omega t - \phi) \quad (6)$$

where $Z = \dfrac{m\omega^2 Y}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} = \dfrac{r^2 Y}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$ (7)

$$\phi = \tan^{-1}\left(\frac{c\omega}{k - m\omega^2}\right) = \tan^{-1}\left(\frac{2\zeta r}{1 - r^2}\right) \quad (8)$$

$$r = \frac{\omega}{\omega_n} \quad ; \quad \zeta = \frac{c}{2m\omega_n}$$

$$\frac{Z}{Y} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \quad (9)$$



For larger values of r ($r \geq 3$),

$$\frac{Z}{Y} \simeq 1 \quad \text{or} \quad Z \simeq Y$$

- relative displacement and the displacement of the base have the same amplitude
- can be used to measure harmonic base displacement

<u>Seismometer</u>: instrument with low natural frequency compared to the excitation frequency

Rewrite eq. (6),

$$\omega_n^2 z(t) = \frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} \underbrace{\omega^2 Y \cos(\omega t - \phi)}_{-\ddot{y}(t)} \qquad (10)$$

$$\Rightarrow \underbrace{-\omega_n^2 z(t)}_{\text{measured acceleration}} = \frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} \underbrace{\ddot{y}(t)}_{\text{true acceleration}} \qquad (11)$$

$$\therefore \frac{\text{Measured value}}{\text{True value}} = \frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} \qquad (12)$$

For small values of r,

$$\frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} \simeq 1 \qquad (13)$$

Eq. (11) becomes

$$-\omega_n^2 z(t) \simeq \ddot{y}(t) \qquad (14)$$

- the relative position $z(t)$ is proportional to the base acceleration

- $\zeta = 0.7$, the accelerometer can be used in the range

$$0 < \frac{\omega}{\omega_n} < 0.4$$

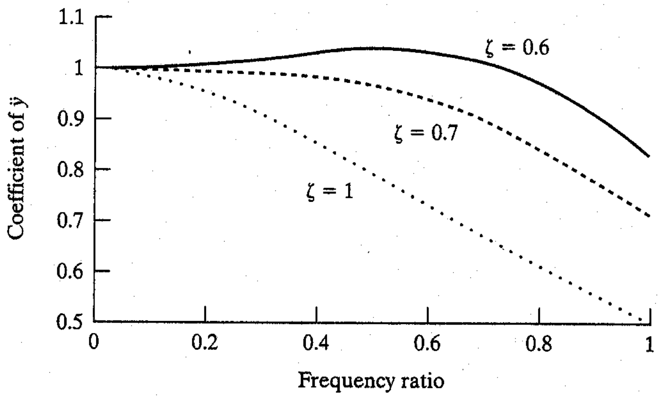


Figure 2.24 Effect of damping on the constant of proportionality between base acceleration and the relative displacement (voltage) for an accelerometer.

Accelerometer : a high-natural-frequency instrument that measures the acceleration of a vibrating body.

e.g., a mass resting on a piezoelectric ceramic crystal,
$\omega_n \simeq 8 \times 10^4$ Hz (Kistler 8694)
up to 16,000 Hz can be measured ($0 < \frac{\omega}{\omega_n} < 0.2$)

By the Laplace transform method,

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y} \quad \text{for Piezoelectric Accelerometer}$$

$$(ms^2 + cs + k)\, Z(s) = -ms^2\, Y(s)$$

$$\frac{Z(s)}{Y(s)} = \frac{-ms^2}{ms^2 + cs + k} \quad : \text{Transfer Function}$$

by letting $s = i\omega$

$$\frac{Z(i\omega)}{Y(i\omega)} = \frac{m\omega^2}{-m\omega^2 + ci\omega + k} = \frac{m\omega^2}{(k - m\omega^2) + ic\omega}$$

: Frequency Response Function (FRF)

$$\left|\frac{Z(i\omega)}{Y(i\omega)}\right| = \frac{m\omega^2}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$