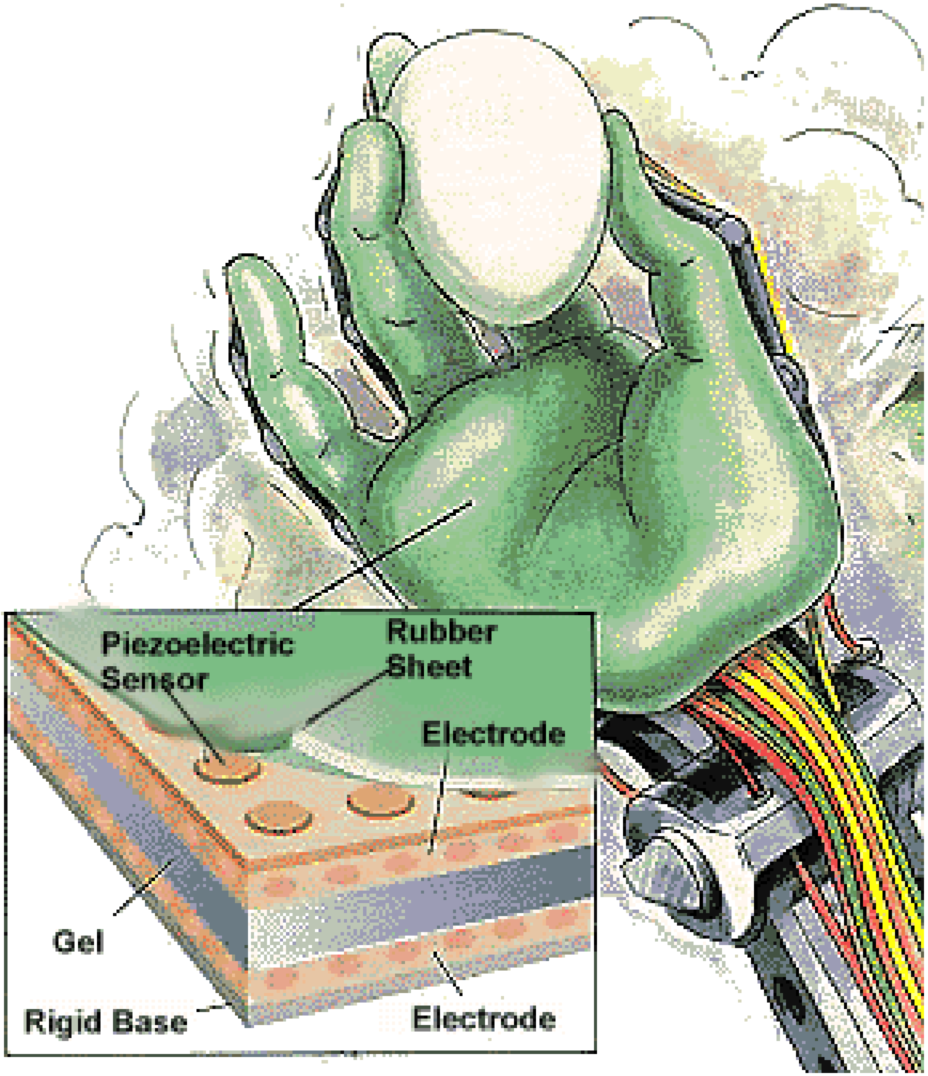
Piezoelectric
Sensor
Rubber
Sheet
Electrode
Gel
Rigid Base
Electrode

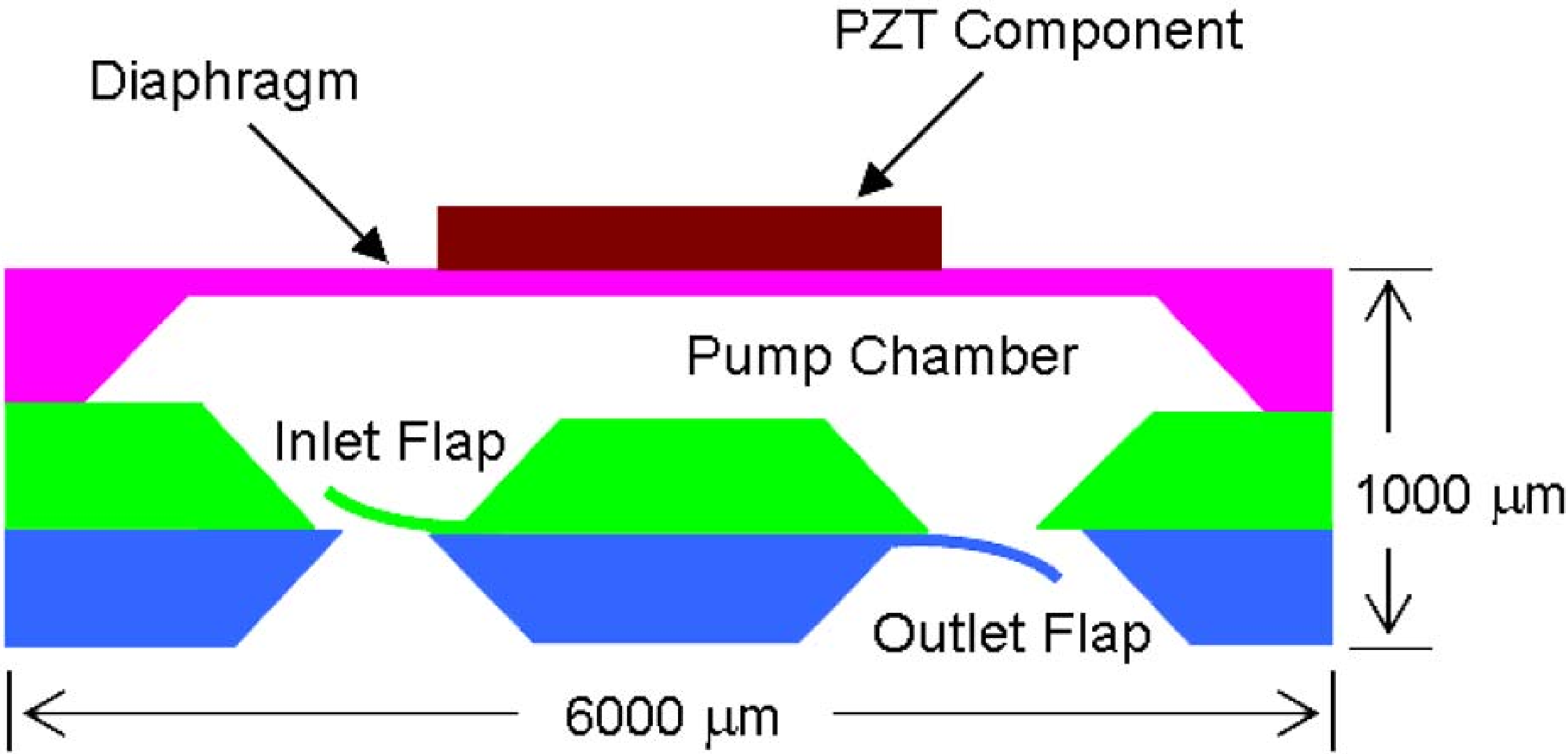


**Figure 1:** Sketch of micropump cross-section. Alternating voltage causes the PZT component to expand and contract along the horizontal direction. This induces a bending stress on the diaphragm, which in turn pumps the fluid through the chamber.

- Piezoelectric Actuators/Sensors

Constitutive equations:

where piezoelectric constants

impermittivity constants for constant T

elasticity matrix for constant D

The “strain” constants *d*

- A measure of the strain produced by an applied electric field (The Motor Effect)
- A measure of the short circuit charge density to the applied stress (The Generator Effect)

The “voltage” constants *g*

– A measure of the electric field (open circuit) by an applied mechanical stress

$$d = \frac{\text{strain developed}}{\text{applied electric field}}$$

$$= \frac{\text{short circuit charge density}}{\text{applied mechanical stress}}$$

$$g = \frac{\text{open circuit electric field}}{\text{applied mechanical stress}}$$

$$= \frac{\text{strain developed}}{\text{applied charge density}}$$

- Actuators need high “strain” constants *d*
- Sensors need high “voltage” constants *g*

# PIEZOCERAMIC APPLICATION DATA

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## INTRODUCTION

When a piezoceramic element is stressed electrically by a voltage, its dimensions change. When it is stressed mechanically by a force, it generates an electric charge. If the electrodes are not short-circuited, a voltage associated with the charge appears.

A piezoceramic is therefore capable of acting as either a sensing or transmitting element, or both. Since piezoceramic elements are capable of generating very high voltages, they are compatible with today's generation of solid-state devices — rugged, compact, reliable, and efficient.

The following text describes the terminology of piezoceramics and the relationship among variables for functional applications.

found on the finished element (Figure 1b). When the mechanical stress or strain is shear, the subscript 5 is used in the second place.

Piezoelectric coefficients with double subscripts link electrical and mechanical quantities. The first subscript gives the direction of the electrical field associated with the voltage applied, or the charge produced. The second subscript gives the direction of the mechanical stress or strain.

Several piezoceramic material constants may be written with a "superscript" which specifies either a mechanical or electrical boundary condition. The superscripts are T, E, D and S, signifying:

T = constant stress
   = mechanically free

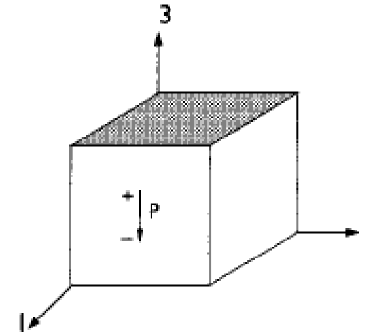



Figure 1a

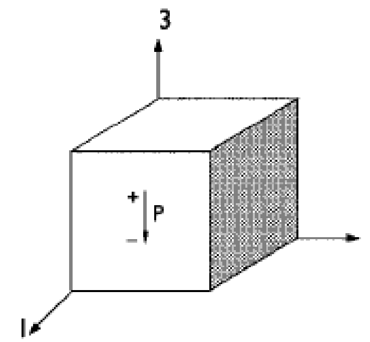



Figure 1b

## RELATIONSHIPS

Relationships between applied forces and the resultant responses depend upon: the piezoelectric properties of the ceramic; the size and shape of the piece; and the direction of the electrical and mechanical excitation.

To identify directions in a piezoceramic element, three axes are used. These axes, termed 1, 2 and 3, are analogous to X,Y and Z of the classical three dimensional orthogonal set of axes (Figure 1a)

The polar or 3 axis is taken parallel to the direction of polarization within the ceramic. This direction is established during manufacturing by a high DC voltage that is applied between a pair of electroded faces to activate the material. The polarization vector "P" is represented by an arrow pointing from the positive to the negative poling electrode. In shear operations, these poling electrodes are later removed and replaced by electrodes deposited on a second pair of faces. In this event, the 3 axis is not altered, but is then parallel to the electroded faces

E = constant field = short circuit
D = constant electrical displacement
= open circuit
S = constant strain
= mechanically clamped

As an example, $K_3^T$ expresses the relative dielectric constant (K), measured in the polar direction (3) with no mechanical clamping applied.

## "d" CONSTANT

The piezoelectric constants relating the mechanical strain produced by an applied electric field are termed the strain constants, or the "d" coefficients. The units may then be expressed as meters per meter, per volts per meter (meters per volt).

$$d = \frac{\text{strain developed}}{\text{applied electric field}}$$

It is useful to remember that large $d_{ij}$ constants relate to large mechanical displacements which are usually sought in motional transducer devices. Conversely, the coefficient may be viewed as relating the charge collected

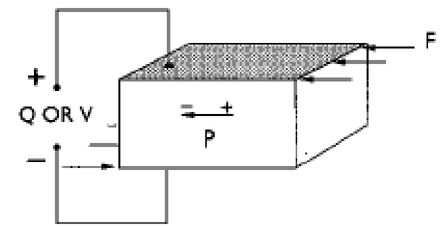



Figure 2a

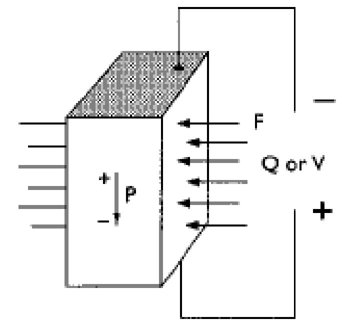



Figure 2b

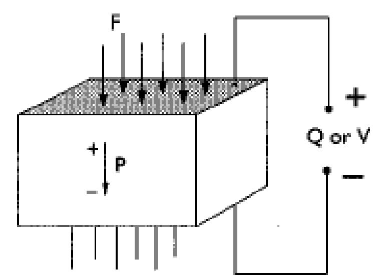



Figure 2c

on the electrodes, to the applied mechanical stress. $d_{33}$ applies when the force is in the 3 direction (along the polarization axis) and is impressed on the same surface on which the charge is collected (Figure 2a). $d_{31}$ applies when the charge is collected on the same surface as before, but the force is applied at right angles to the polarization axis (Figure 2b).

The subscripts in $d_{15}$ indicate that the charge is collected on electrodes which are at right angles to the original poling electrodes and that the applied mechanical stress is shear (Figure 2c.)

The units for the $d_{ij}$ coefficients are commonly expressed as coulombs/square meter per newton/square meter.

$$d = \frac{\text{short circuit charge density}}{\text{applied mechanical stress}}$$

by the thickness of ceramic between electrodes. A "33" subscript indicates that the electric field and the mechanical stress are both along the polarization axis. (Figure 2a.) A "31" subscript signifies that the pressure is applied at right angles to the polarization axis, but the voltage appears on the same electrodes as in the "33" case. (Figure 2b.)

A "15" subscript implies that the applied stress is shear and that the resulting electric field is perpendicular to the polarization axis. (Figure 2c.)

High $g_{ij}$ constants favor large voltage output, and are sought after for sensors.

Although the g coefficient are called voltage coefficients, it is also correct to say the $g_{ij}$ is the ratio of strain developed over the applied charge density with units of meters per meter over coulombs per square meter.

by the thickness separating the electrodes. Units are expressed in farads.

$$C = \frac{K \epsilon_0 A}{t}$$

K3 is related to the capacitance between the original poling electrodes. K1 is related to the capacitance between the second pair of electrodes applied after removal of the poling electrodes for the purposes of shear excitation.

At frequencies far below resonance, piezoelectric ceramic transducers are fundamentally capacitors. Consequently, the voltage coefficients $g_{ij}$ are related to the charge coefficients $d_{ij}$ by the dielectric constant $K_i$ as, in a capacitor, the voltage V is related to the charge Q by the capacitance C.

The equations are:

When the force that is applied is distributed over an area which is fully covered by electrodes (even if that is only a portion of the total electrode) the units of area cancel from the equation and the coefficient may be expressed in terms of change per unit force, coulombs per newton. To view the $d_{ij}$ coefficients in this manner is useful when charge generators are contemplated, e.g., accelerometers.

## "g" CONSTANT

The piezoelectric constants relating the electric field produced by a mechanical stress are termed the voltage constants, or the "g" coefficients. The units may then be expressed as volts/meter per newton/square meter.

$$g = \frac{\text{open circuit electric field}}{\text{applied mechanical stress}}$$

Output voltage is obtained by multiplying the calculated electric field by the thickness of the ceramic between electrodes. A "g" constant may also be expressed as meters/meter (strain) per coulombs/square meter (charge density) — that is, the strain developed over coulombs per square meter.

$$g = \frac{\text{strain developed}}{\text{applied charge density}}$$

## DIELECTRIC CONSTANTS

The relative dielectric constant is the ratio of the permittivity of the material, $\epsilon$, to the permittivity of free space, $\epsilon_0$, in the unconstrained condition, i.e., well below the mechanical resonance of the part.

$$K = \frac{\text{permittivity of material}}{\text{permittivity of free space}} = \frac{\epsilon}{\epsilon_0}$$

## CAPACITANCE

Whereas the relative dielectric constant is strictly a material property, the capacitance is a quantity dependent on the type of material and its dimensions. Capacitance is calculated by multiplying the relative dielectric constant by the permittivity of free space ($\epsilon_0 = 8.9 \times 10^{-12}$ farads/meter) and electrode surface area, and then dividing

The equations are:

$$Q = CV$$

$$d_{33} = K_3\ \epsilon_0\ g_{33}$$

$$d_{31} = K_3\ \epsilon_0\ g_{31}$$

$$d_{15} = K_3\ \epsilon_0\ g_{15}$$

At resonance, the dielectric constant will be reduced by the factor $(1-k^2)$ where k is the coupling coefficient of the mode in question.

## COUPLING COEFFICIENTS

Electromechanical coupling $k_{33}$, $k_{31}$, kp, and $k_{15}$ describe the conversion of energy by the ceramic element from electrical to mechanical form or vice versa. The ratio of the stored converted energy of one kind (mechanical or electrical) to the input energy of the second kind (electrical or mechanical) is defined as the square of the coupling coefficient.

$$k = \sqrt{\frac{\text{mechanical energy stored}}{\text{electrical energy applied}}}$$

or

$$k = \sqrt{\frac{\text{electrical energy stored}}{\text{mechanical energy applied}}}$$

Subscripts denote the relative directions of electrical and mechanical quantities and the kind of motion involved. They can be associated with vibratory modes of certain simple transducer shapes; $k_{33}$ is appropriate for a long thin bar, electroded on the ends, and polarized along the length, and vibrating in a simple length expansion and contraction. $k_{31}$ relates to a long thin bar, electroded on a pair of long faces, polarized in thickness, and vibrating in simple length expansion and contraction. $k_p$ signifies the coupling of electrical and mechanical energy in a thin round disc, polarized in thickness and vibrating in radial expansion and contraction. $k_{15}$ describes the energy conversion in a thickness shear vibration. Since these coefficients are energy ratios, they are dimensionless.

## YOUNG'S MODULUS

As with all solids, piezoelectric ceramics have mechanical stiffness usually newtons/square meter.

It should be clearly understood that the piezoceramic properties described above are defined for ideal shapes measured under ideal mechanical and electrical boundary conditions. When put to use under practical device operating conditions, the predicted performance is approached but seldom realized. Non-ideal shapes and non-ideal boundary conditions contribute to transduction losses due to such things as standing waves, interfering vibrational modes, pseudo-clamping, stray electric and dielectric resistances. Since the possibilities are infinite, the designer must evaluate each component under the use conditions for which it is intended.

## DENSITY

The ratio of the mass to volume in the material, expressed in kg/m$^3$

$$\rho = \frac{\text{mass}}{\text{volume}}$$

## AGING RATE

Aging is the attempt of the ceramic to change back to its original state prior to polarization. Aging of piezoelectric ceramics is a logarithmic function with time. The aging rate defines change in the material parameters per decade of time, i.e., 1-10 days, 5-50 days, etc.

## PERFORMANCE

Deflection and Force: Piezoelectric actuators are usually specified in terms of their free deflection and blocked force. Free deflection, X(f), refers to displacement at a given voltage level without the actuator working against any external load. Blocked force, F(b), refers to the force exerted at a given voltage level when the actuator is not allowed to move. Since the force at maximum deflection is zero, all other values of simultaneous displacement and force (for a given voltage level) are determined by a line drawn between these points on a force versus deflection diagram as shown below. In

## YOUNG'S MODULUS

As with all solids, piezoelectric ceramics have mechanical stiffness properties described as Young's Modulus. Young's Modulus is the ratio of stress (force per unit area) to strain (change in length per unit length).

$$Y = \frac{\text{stress}}{\text{strain}}$$

Because mechanical stressing of the ceramic produces an electrical response which opposes the resultant strain, the effective Young's Modulus with electrodes short circuited is lower than with the electrodes open circuited. In addition, the stiffness is different in the 3 direction from that in the 1 or 2 direction. Therefore, in expressing such quantities both direction and electrical conditions must be specified. $Y^E_{33}$ is the ratio of stress to strain in the 3 direction at constant field E (electrodes shorted). $Y^D_{33}$ is the equivalent with the electrodes open circuited. $Y^E_{11}$ and $Y^D_{11}$ are the moduli in the 1 or 2 direction. $Y^E_{55}$ and $Y^D_{55}$ are the ratios of shear stress to shear strain. Units are

The ratio of the mass to volume in the material, expressed in kg/m$^3$

$$\rho = \frac{\text{mass}}{\text{volume}}$$

## DISSIPATION FACTOR

A measure of the dielectric losses in the material-defined as the tangent of the loss angle or the ratio of parallel resistance to the parallel reactance, expressed in percent.

## MECHANICAL (Qm)

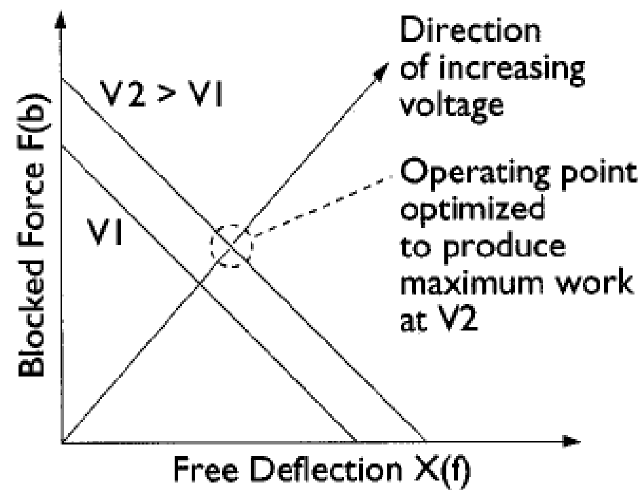
The ratio of reactance to resistance in the equivalent series circuit representing the mechanical vibrating resonant system. The shape of the part affects the value.

## CURIE TEMPERATURE

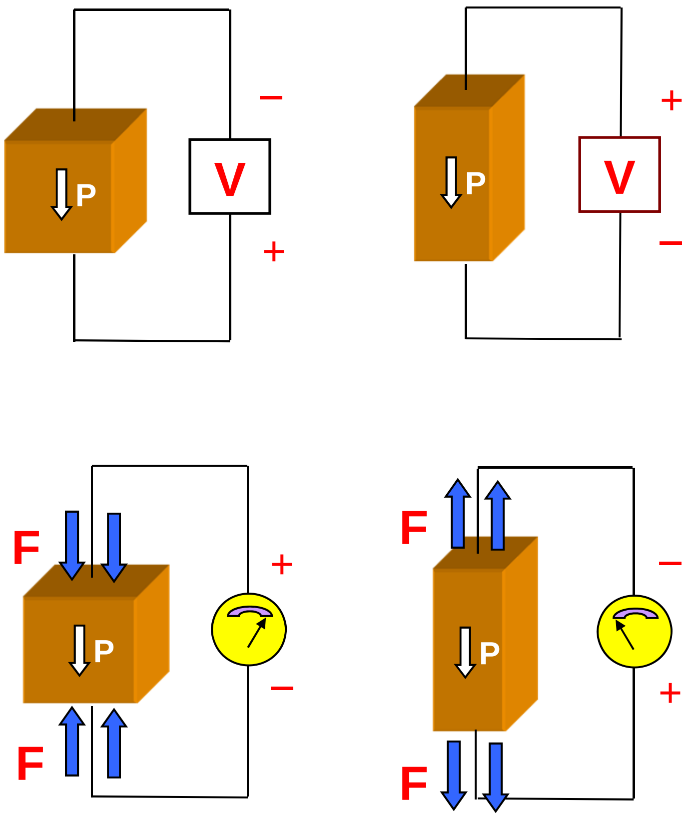
The temperature at which the crystal structure changes from a non-symmetrical (piezoelectric) to a symmetrical (non-piezoelectric) form, expressed in degrees Celsius.

and force (for a given voltage level) are determined by a line drawn between these points on a force versus deflection diagram as shown below. In practice, a bending motor must move a specified amount and exert a specified force, which determines its operating point on the force versus deflection diagram. Work is maximized when the deflection performed permits one half the blocked force to be developed. This occurs when the deflection equals one half the free deflection.

For cantilevered bending motors, X(f) and F(b) are approximated by observing the tip deflection after energizing, and by holding a force gauge against the tip during energization.



# Piezoelectric Materials




Mechanical Response to Electrical Input
and Electrical Response to Mechanical Input

**TABLE-1. PIEZOELECTRIC AND MATERIAL PROPERTIES FOR PSI-5A-S4-ENH PIEZOCERAMIC**

| | | | | |
|---|---|---|---|---|
| **PIEZOELECTRIC** | | | | |
| Composition | | | Lead Zirconate Titanate, Navy Type-II | |
| Material Designation | | | PSI-5A-S4-ENH | |
| Relative Dielectric Constant (@1KHz) | | $K^T_3$ | 1800 | |
| | | $K^T_1$ | 1800 | |
| Piezoelectric Strain Coefficient | | $d_{33}$ | $390 \times 10^{-12}$ | Meters / Volt |
| | | $d_{31}$ | $-190 \times 10^{-12}$ | Meters / Volt |
| | | $d_{15}$ | $\sim 550 \times 10^{-12}$ | Meters / Volt |
| Piezoelectric Voltage Coefficient | | $g_{33}$ | $24.0 \times 10^{-3}$ | Volt Meters / Newton |
| | | $g_{31}$ | $-11.8 \times 10^{-3}$ | Volt Meters / Newton |
| | | $g_{15}$ | $\sim 26.0 \times 10^{-3}$ | Volt Meters / Newton |
| Coupling Coefficient | | $k_{33}$ | 0.72 | |
| | | $k_{31}$ | 0.32 | |
| | | $k_{15}$ | 0.59 | |
| Polarization Field | | Ep | $2 \times 10^6$ | Volts / Meter |
| Coercive Field | (DC) | Ec | $5 \times 10^5$ | Volts / Meter |
| | (@ 60 Hz) | | $6 \times 10^5$ | Volts / Meter |

## MECHANICAL

| | | | | |
|---|---|---|---|---|
| Density | | $\rho$ | 7750 | Kg / Meter$^3$ |
| Elastic Modulus | | $Y^E_{33}$ | $4.9 \times 10^{10}$ | Newtons / Meter$^2$ |
| | | $Y^E_{11}$ | $6.2 \times 10^{10}$ | Newtons / Meter$^2$ |
| Poisson' Ratio | | $\upsilon$ | 0.31 | |
| Compressive Strength | | | $5.2 \times 10^8$ | Newtons / Meter$^2$ |
| Tensile Strength | (Static) | | $5.2 \times 10^8$ | Newtons / Meter$^2$ |
| | (Dynamic) | | $2.1 \times 10^7$ | Newtons / Meter$^2$ |
| Mechanical Q | | | 80 | |

## THERMAL

| | | | |
|---|---|---|---|
| Curie Temperature | | 350 | °C |
| Pyroelectric Coefficient | | $\sim 420 \times 10^{-6}$ | Coulombs / Meter$^2$ °C |
| Thermal Expansion Coefficient | | $\sim 4 \times 10^{-6}$ | Meters / Meter °C |
| Specific Heat | Cp | 440 | Joules / Kg °C |

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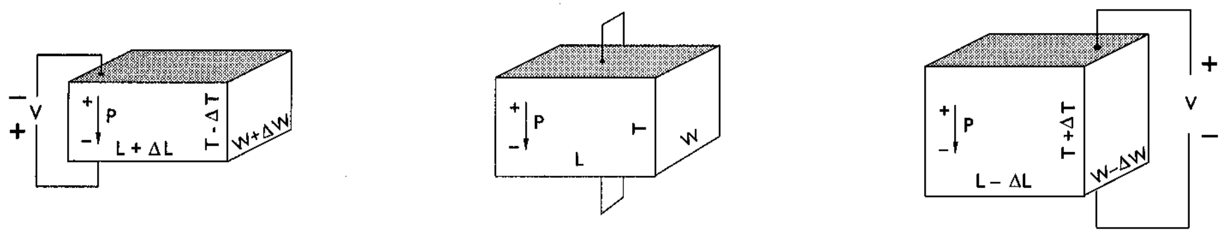
# MOTOR TRANSDUCER RELATIONSHIPS

## EXPANSION OR CONTRACTION MOTOR



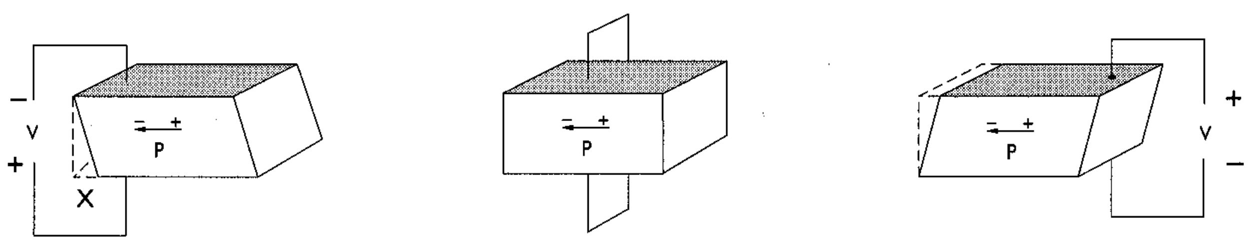

**Direct**

$$\Delta T = Vd_{33}$$

**Transverse**

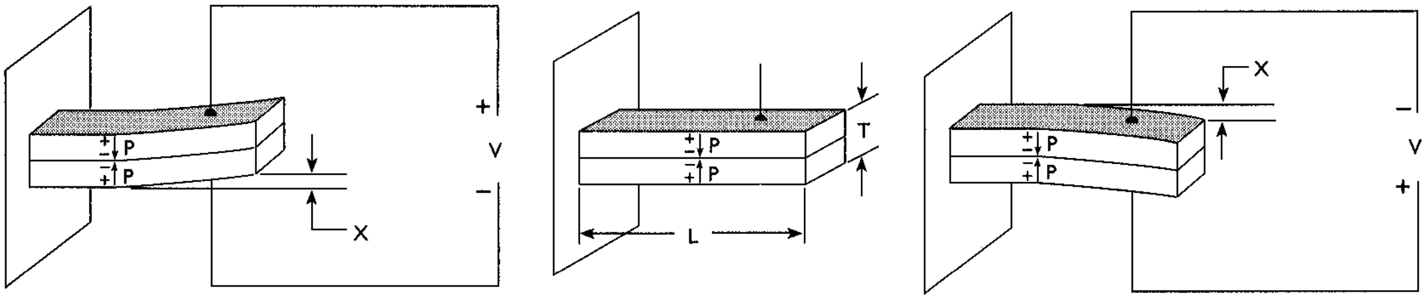
$$\frac{\Delta L}{L} = \frac{\Delta W}{W} = \frac{V}{T} d_{31}$$

## SHEAR MOTOR



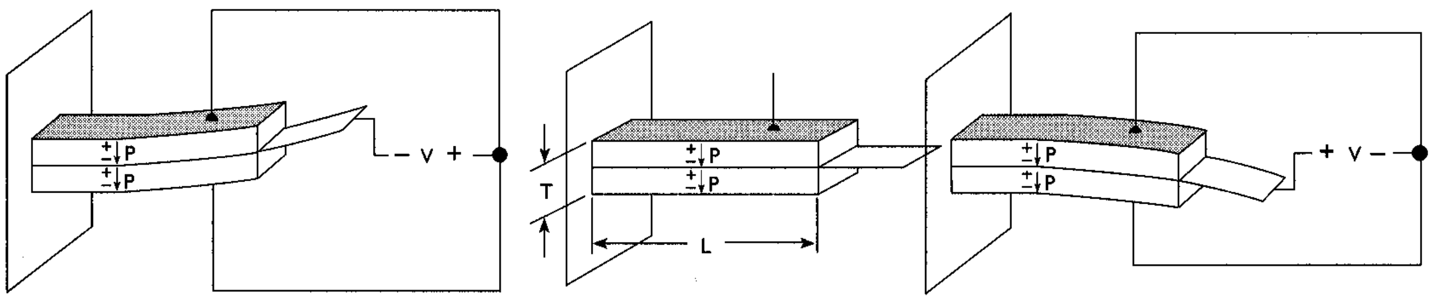

$$X = Vd_{15}$$

# BENDING MOTOR




**Series Connection**

$$X = \frac{2L^2Vd_{31}}{T^2}$$




**Parallel Connection**

$$X = \frac{4L^2Vd_{31}}{T^2}$$

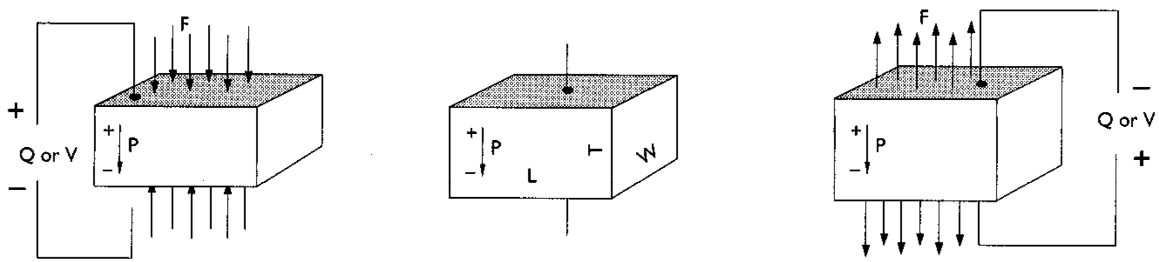
The piezoceramic bender is a versatile low power electro-mechanical transducer.

As a motor, the application of an electric field across the two outer electrodes of the bender causes one layer to expand while the other contracts. The net result is a bending displacement much greater than the length deformation of either of the two layers. In this mode, the application of an electric field would be analogous to a temperature change on a bimetallic thermostat.

NOTE: Quantities must be in compatible units. Equations give magnitudes only. Signs are shown on drawing.

# GENERATOR TRANSDUCER RELATIONSHIPS

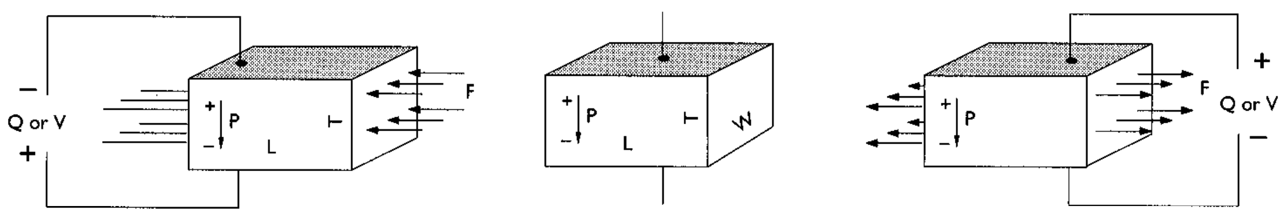
## PARALLEL COMPRESSION OR TENSION GENERATOR




$$Q = Fd_{33}$$

$$\frac{V}{T} = \frac{Fg_{33}}{LW}$$

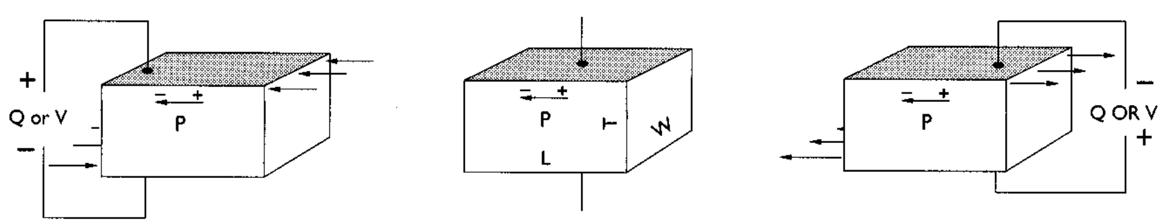
## TRANSVERSE COMPRESSION OR TENSION GENERATOR




$$\frac{Q}{LW} = \frac{F}{TW}\, d_{31}$$

$$\frac{V}{T} = \frac{F}{TW}\, g_{31}$$

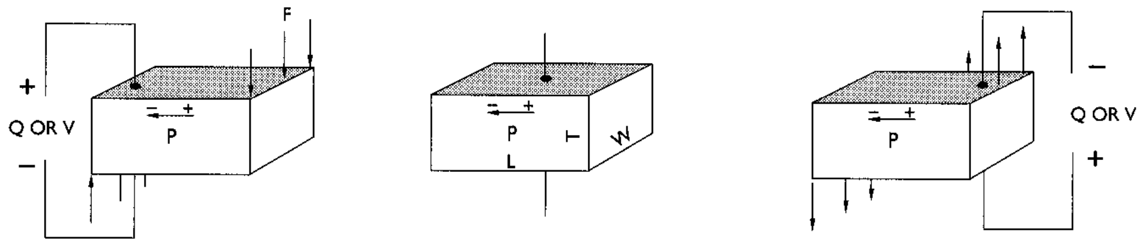
## PARALLEL SHEAR GENERATOR




$$Q = Fd_{15}$$

$$\frac{V}{T} = \frac{F}{TW}\, g_{15}$$

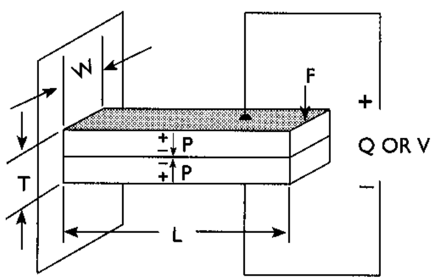
## TRANSVERSE SHEAR GENERATOR




$$\frac{Q}{LW} = \frac{F}{TW} d_{15}$$

$$\frac{V}{T} = \frac{F}{TW} g_{15}$$

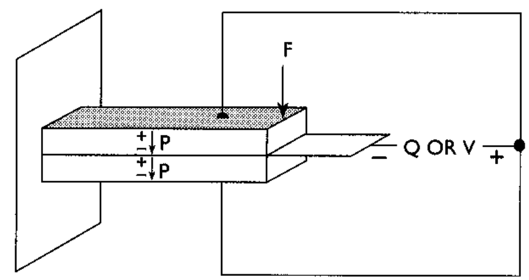
## BENDING GENERATOR




### Series Connection

$$Q = \frac{3FL^2}{2T^2} d_{31}$$

$$V = \frac{3FL}{2WT} g_{31}$$




### Parallel Connection

$$Q = \frac{3FL^2}{2T^2} d_{31}$$

$$V = \frac{3FL}{4WT} g_{31}$$

When a bilaminar element is forced to bend, one layer will be in tension while the other is in compression. Since the two layers are polarized in opposite directions, the opposite stresses in each layer will produce electrical outputs of like polarity. The electrical output of the bender will then simply be the summation of the outputs of each layer.

Benders may also be used as strain gauges for easy and rapid determination of the characteristics of dynamic strains in structures. They consist of polarized piezoelectric plates which can be cemented to a structure. They exhibit extremely high sensitivities, in the order of 50 times that of wire strain gauges. Elements are so small that on most structures they will not materially affect the vibrational characteristics of the structure.