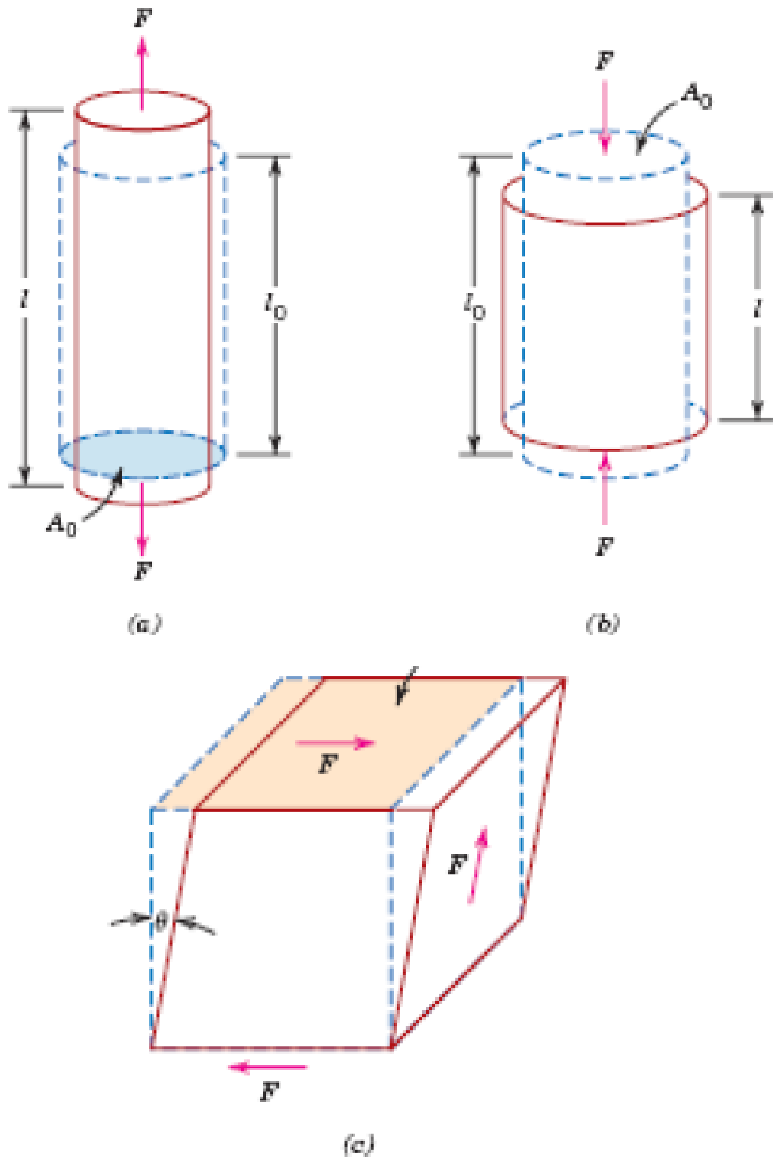
14. Mechanical properties of Materials

Different mechanical tests

- Tension
- Compression
- Shearing

Stress vs. Strain



Normal stress

$$\sigma = \frac{F}{A_0}$$

Normal strain

$$\varepsilon = \frac{l - l_0}{l_0} \times 100\%$$

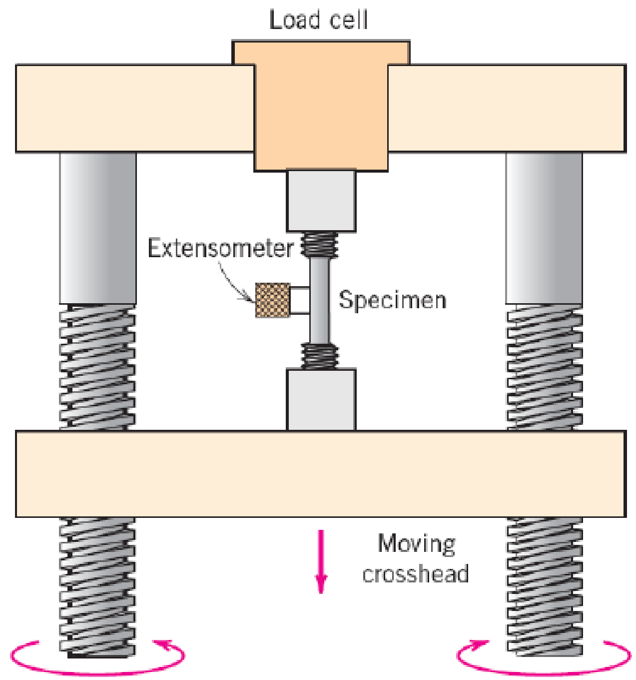
Shear stress

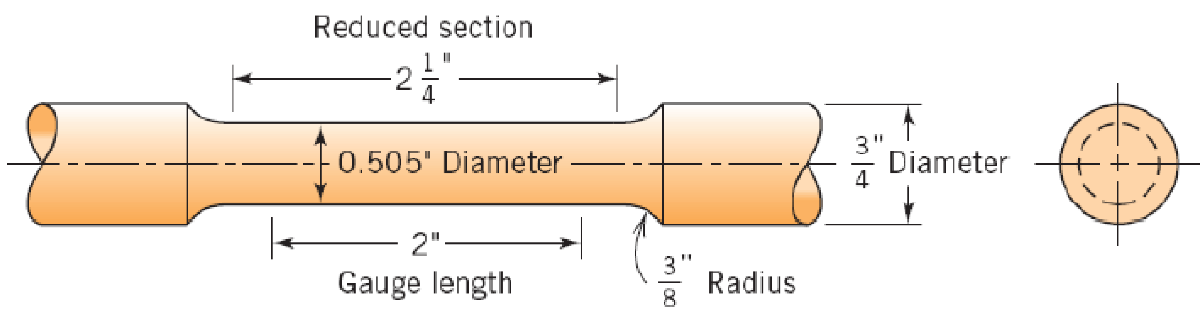
$$\tau = \frac{F}{A_0}$$

Shear strain

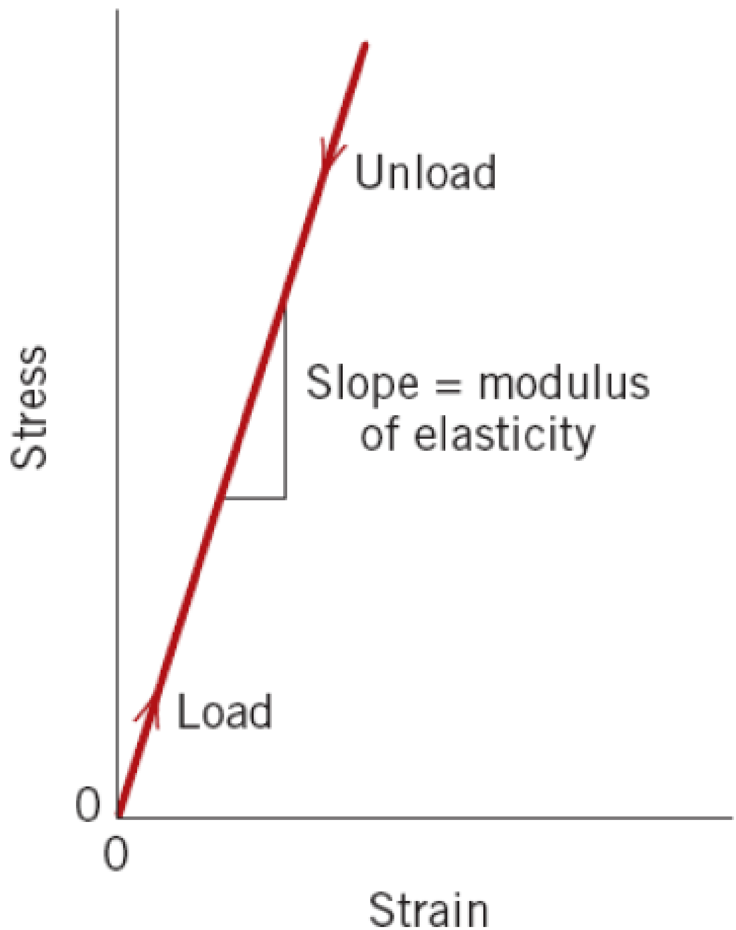
$$\varepsilon = \tan\theta$$

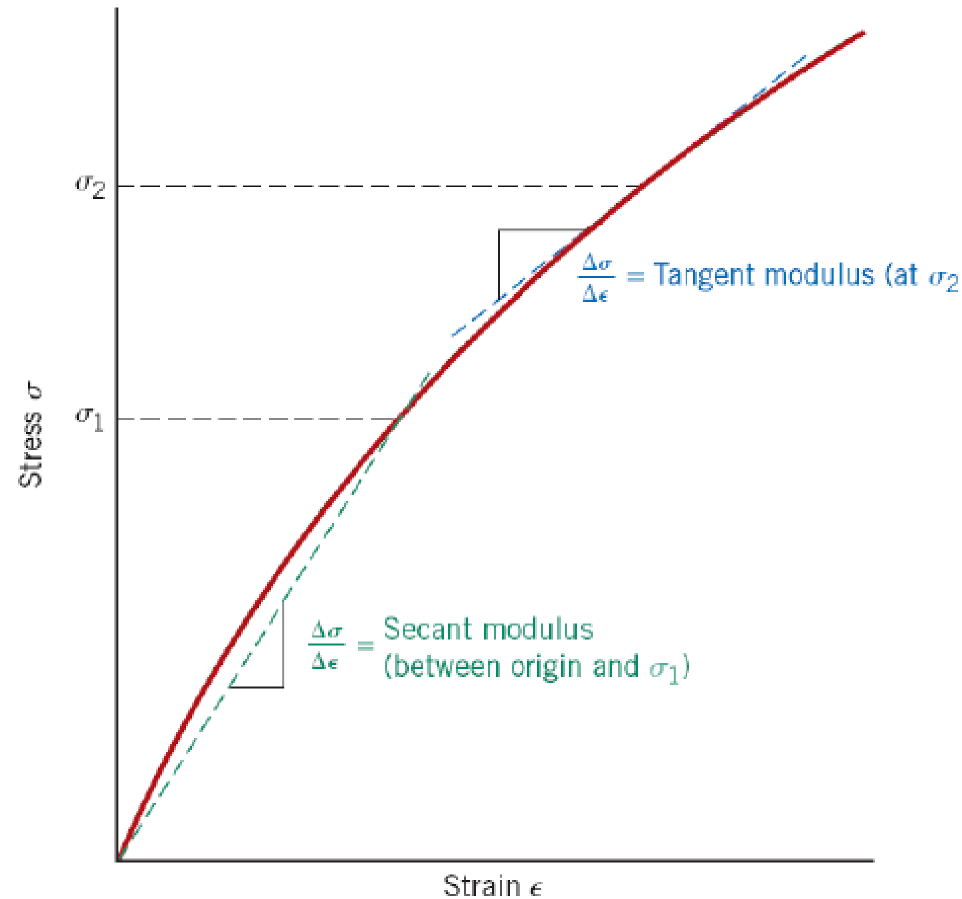
Stress-Strain diagram





Elasticity


Stress
Strain
0
0
Load
Unload
Slope = modulus of elasticity

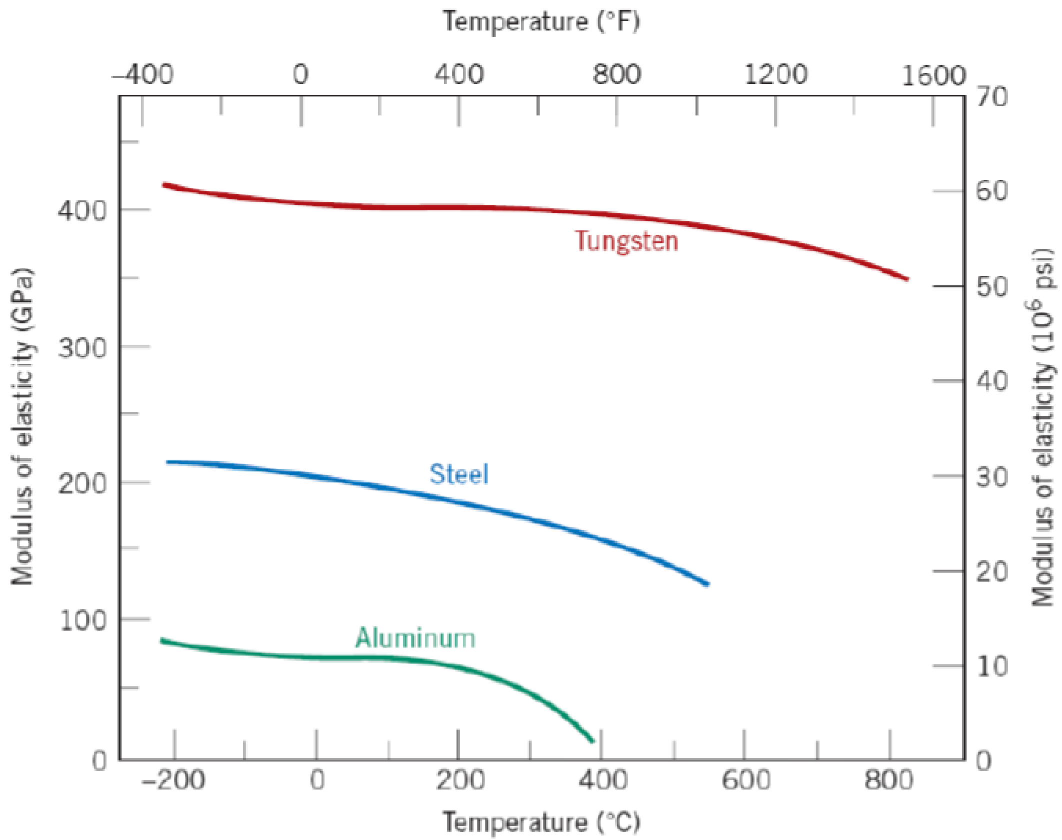

Stress σ
Strain ϵ
σ2
σ1
Δσ/Δϵ = Tangent modulus (at σ2)
Δσ/Δϵ = Secant modulus (between origin and σ1)

Material	*Modulus of Elasticity* GPa	*Modulus of Elasticity* 10^6 psi	*Shear Modulus* GPa	*Shear Modulus* 10^6 psi	*Poisson's Ratio*
		Metal Alloys			
Tungsten	407	59	160	23.2	0.28
Steel	207	30	83	12.0	0.30
Nickel	207	30	76	11.0	0.31
Titanium	107	15.5	45	6.5	0.34
Copper	110	16	46	6.7	0.34
Brass	97	14	37	5.4	0.34
Aluminum	69	10	25	3.6	0.33
Magnesium	45	6.5	17	2.5	0.35
		Ceramic Materials			
Aluminum oxide (Al_2O_3)	393	57	—	—	0.22
Silicon carbide (SiC)	345	50	—	—	0.17
Silicon nitride (Si_3N_4)	304	44	—	—	0.30
Spinel ($MgAl_2O_4$)	260	38	—	—	—
Magnesium oxide (MgO)	225	33	—	—	0.18
Zirconia (ZrO_2)[a]	205	30	—	—	0.31
Mullite ($3Al_2O_3$-$2SiO_2$)	145	21	—	—	0.24
Glass–ceramic (Pyroceram)	120	17	—	—	0.25
Fused silica (SiO_2)	73	11	—	—	0.17
Soda–lime glass	69	10	—	—	0.23
		Polymers[b]			
Phenol-formaldehyde	2.76–4.83	0.40–0.70	—	—	—
Poly(vinyl chloride) (PVC)	2.41–4.14	0.35–0.60	—	—	0.38
Poly(ethylene terephthalate) (PET)	2.76–4.14	0.40–0.60	—	—	0.33
Polystyrene (PS)	2.28–3.28	0.33–0.48	—	—	0.33
Poly(methyl methacrylate) (PMMA)	2.24–3.24	0.33–0.47	—	—	0.37–0.44
Polycarbonate (PC)	2.38	0.35	—	—	0.36
Nylon 6,6	1.59–3.79	0.23–0.55	—	—	0.39
Polypropylene (PP)	1.14–1.55	0.17–0.23	—	—	0.40
Polyethylene—high density (HDPE)	1.08	0.16	—	—	0.46
Polytetrafluoroethylene (PTFE)	0.40–0.55	0.058–0.080	—	—	0.46
Polyethylene—low density (LDPE)	0.17–0.28	0.025–0.041	—	—	0.33–0.40

[a]Partially stabilized with 3 mol% Y_2O_3.
[b]*Modern Plastics Encyclopedia '96*, McGraw-Hill, New York, 1995.

Temperature-dependent modulus

Figure 6.8 Plot of modulus of elasticity versus temperature for tungsten, steel, and aluminum. (Adapted from K. M. Ralls, T. H. Courtney, and J. Wulff, *Introduction to Materials Science and Engineering*. Copyright © 1976 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.)



Temperature ↑

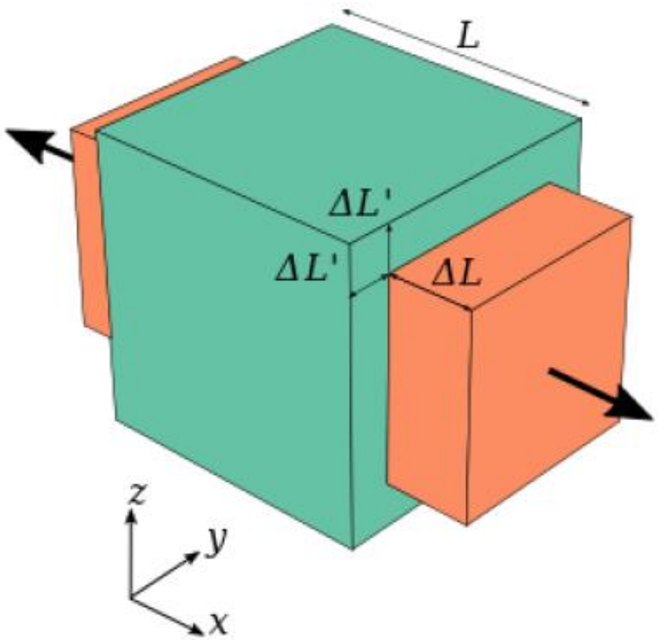
Atomic vibration ↑

Atomic distance ↓

Atomic force ↓

Modulus ↓

Poisson's Ratio



$$d\varepsilon_x = \frac{dx}{x} \qquad d\varepsilon_y = \frac{dy}{y} \qquad d\varepsilon_z = \frac{dz}{z}$$

$$-\nu \int_L^{L+\Delta L} \frac{dx}{x} = \int_L^{L+\Delta L'} \frac{dy}{y} = \int_L^{L+\Delta L'} \frac{dz}{z}$$

$$\left(1 + \frac{\Delta L}{L}\right)^{-\nu} = 1 + \frac{\Delta L'}{L}$$

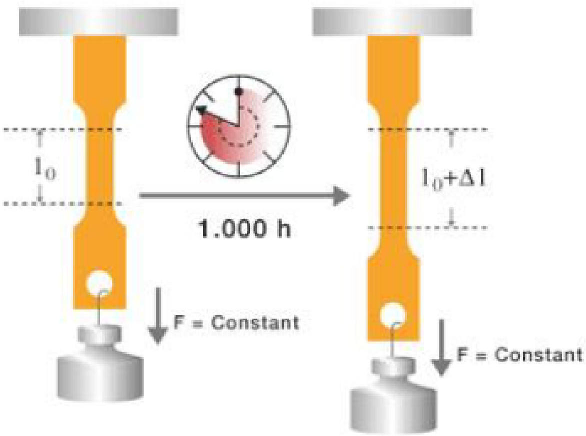
$$\nu \approx -\frac{\Delta L'}{\Delta L}$$

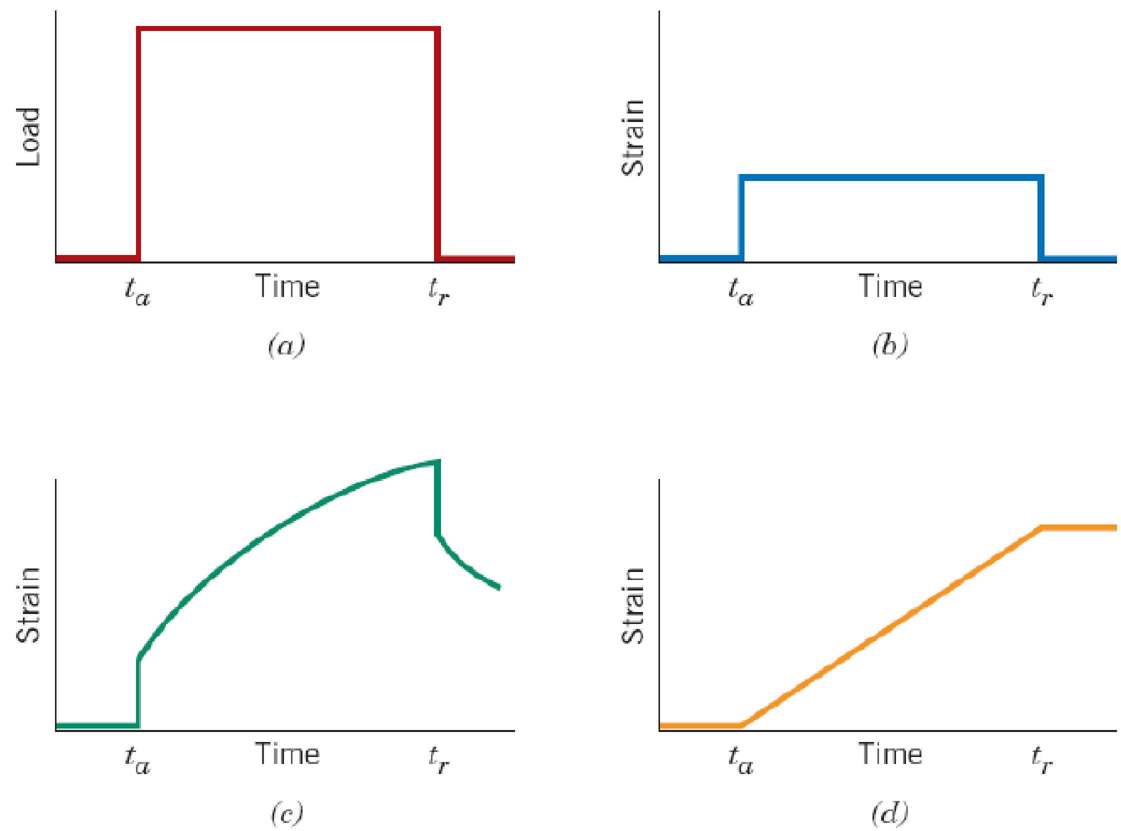
Inelasticity

Elastic deformation can be time-dependent:

- With constant stress, the strain may further increase;
- After stress is removed, strain goes back to zero after a while;
- For polymers, such time-dependent behavior is significant: **Viscoelasticity**

Viscoelasticity: Creep





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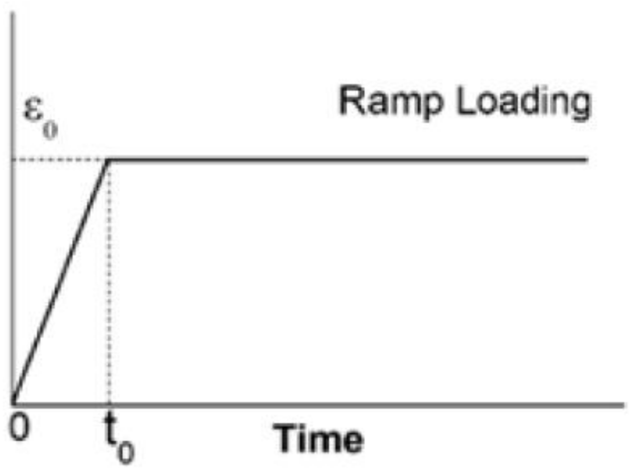
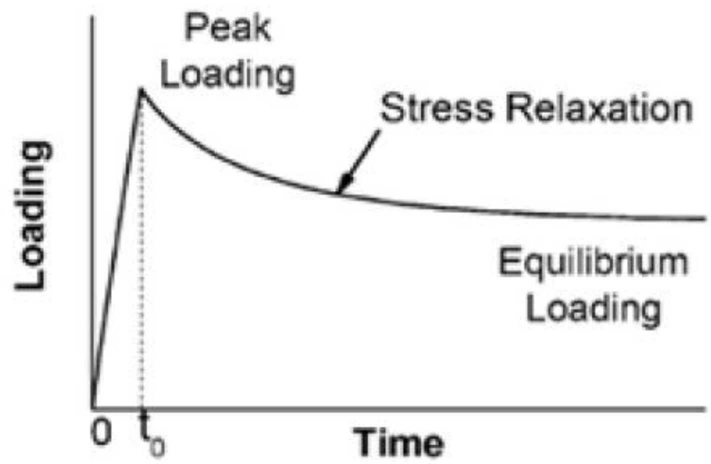
Viscoelasticity: Relaxation

Fix deformation

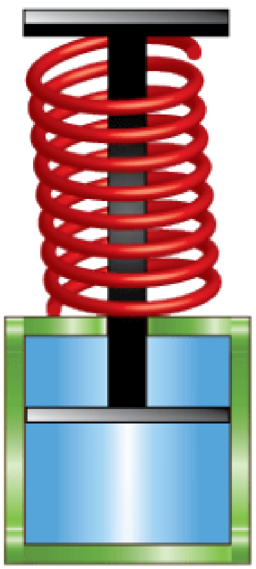




Spring dashpot for viscoelasticity

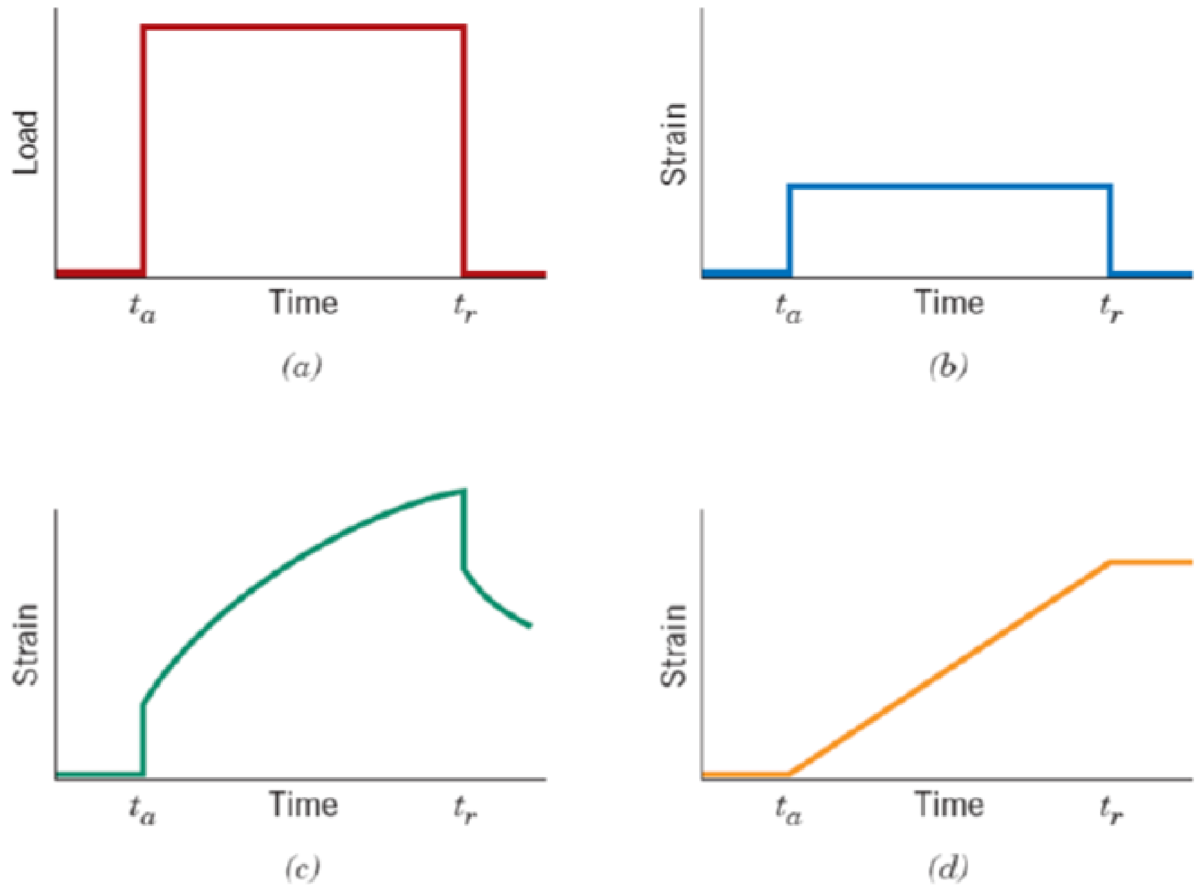
Conceptual model

Viscoelastic Model

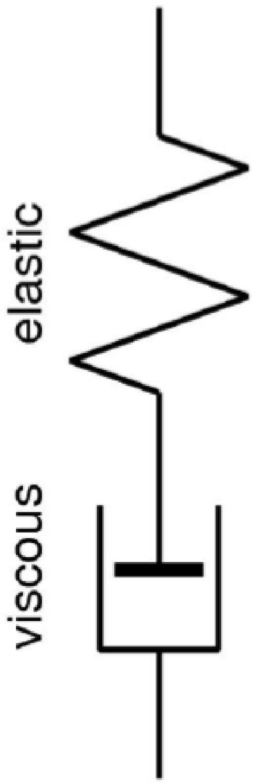


Spring models elasticity

Dashpot models viscosity



Viscoelasticity Model



Maxwell model

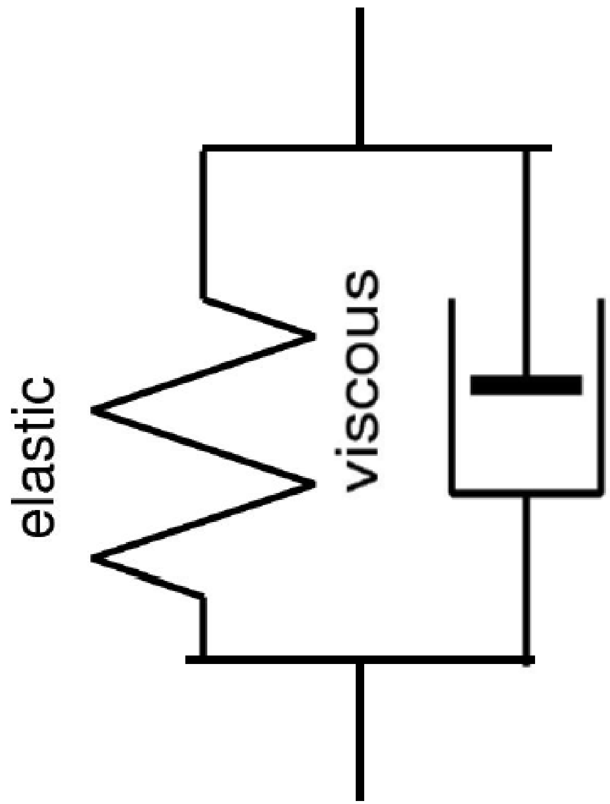
$$\sigma_{\mathrm{Total}} = \sigma_D = \sigma_S$$

$$\varepsilon_{\mathrm{Total}} = \varepsilon_D + \varepsilon_S$$

$$\frac{d\varepsilon_{\mathrm{Total}}}{dt} = \frac{d\varepsilon_D}{dt} + \frac{d\varepsilon_S}{dt} = \frac{\sigma}{\eta} + \frac{1}{E}\frac{d\sigma}{dt}$$

$$\frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta} = \dot{\varepsilon}$$

Viscoelasticity Model



$$\varepsilon_{\text{Total}} = \varepsilon_S = \varepsilon_D$$

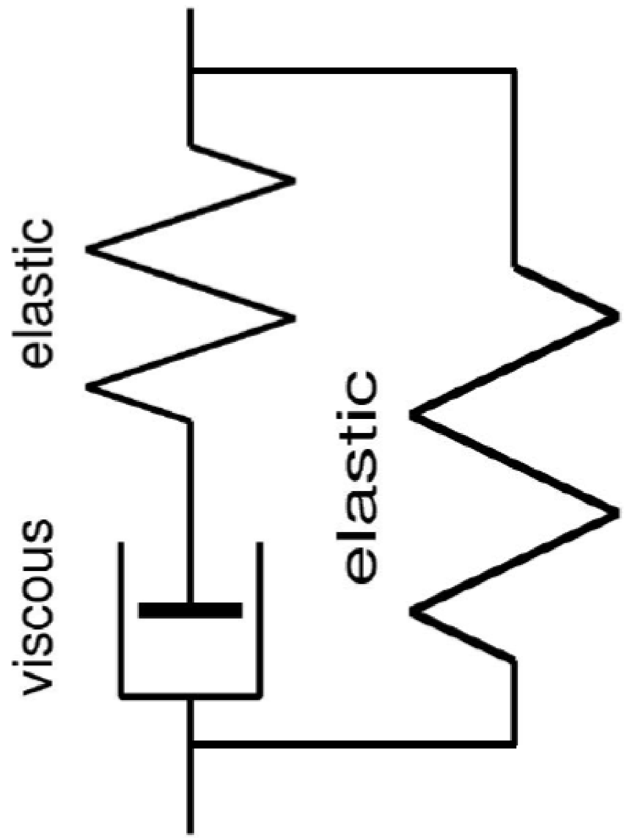
$$\sigma_{\text{Total}} = \sigma_S + \sigma_D$$

$$\sigma(t) = E\varepsilon(t) + \eta \frac{d\varepsilon(t)}{dt}$$

$$\sigma = E\varepsilon + \eta\dot{\varepsilon}$$

Kelvin-Voigt model

Viscoelasticity Model



Zener model

$$\sigma_{tot} = \sigma_m + \sigma_{S_1}$$

$$\varepsilon_{tot} = \varepsilon_m = \varepsilon_{S_1}$$

$$\sigma_m = \sigma_D = \sigma_{S_2}$$

$$\varepsilon_m = \varepsilon_D + \varepsilon_{S_2}$$

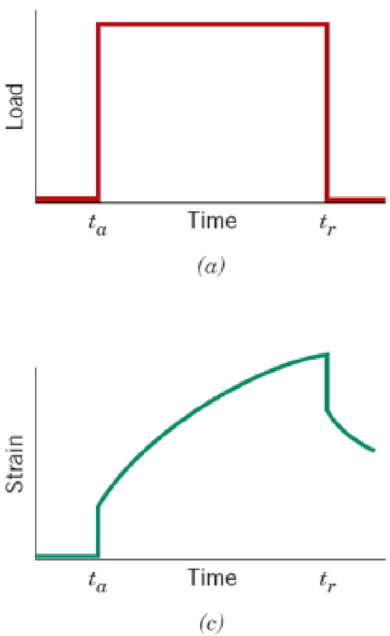
$$\frac{d\varepsilon(t)}{dt} = \frac{\frac{E_2}{\eta}\left(\frac{\eta}{E_2}\frac{d\sigma(t)}{dt} + \sigma(t) - E_1\varepsilon(t)\right)}{E_1 + E_2}$$

$$\sigma(t) + \frac{\eta}{E_2}\frac{d\sigma(t)}{dt} = E_1\varepsilon(t) + \frac{\eta(E_1 + E_2)}{E_2}\frac{d\varepsilon(t)}{dt}$$

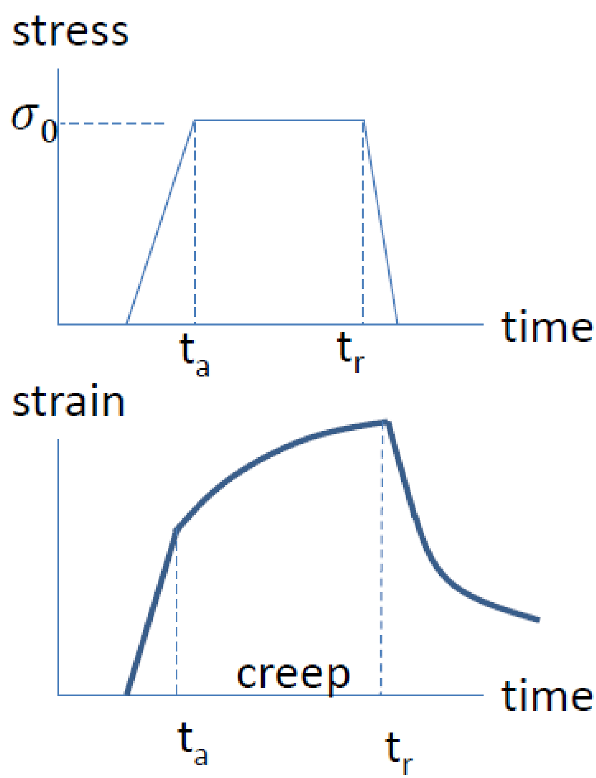
$$\sigma + \frac{\eta}{E_1 + E_2}\dot{\sigma} = \frac{E_1 E_2}{E_1 + E_2}\varepsilon + \frac{E_1\eta}{E_1 + E_2}\dot{\varepsilon}$$

Creep experiment

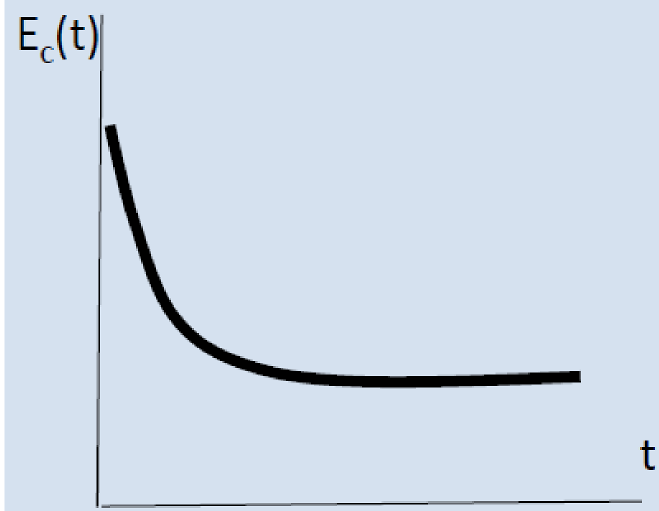
Ideal case



Real experiment



Creep modulus

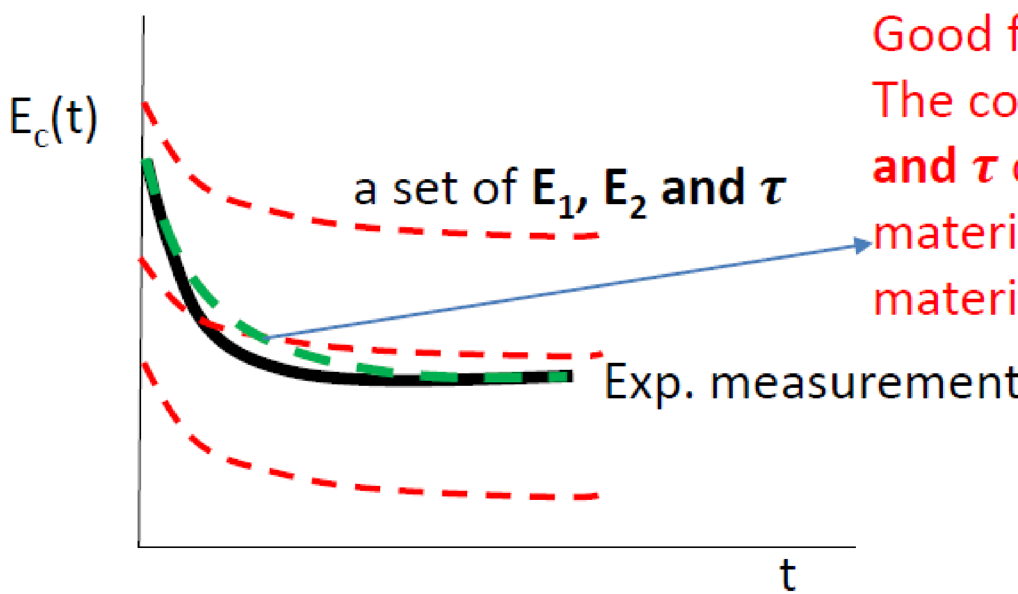


Data fittings

Zener model:

$$E_c(t) = \frac{1}{\frac{1}{E_1+E_2} + \left(\frac{1}{E_1} - \frac{1}{E_1+E_2}\right)\left(1-\exp\left(-\frac{t}{\tau}\right)\right)},$$

Selecting a group of material parameters: E_1, E_2 and τ, to make the prediction of the model close to experimental measurements

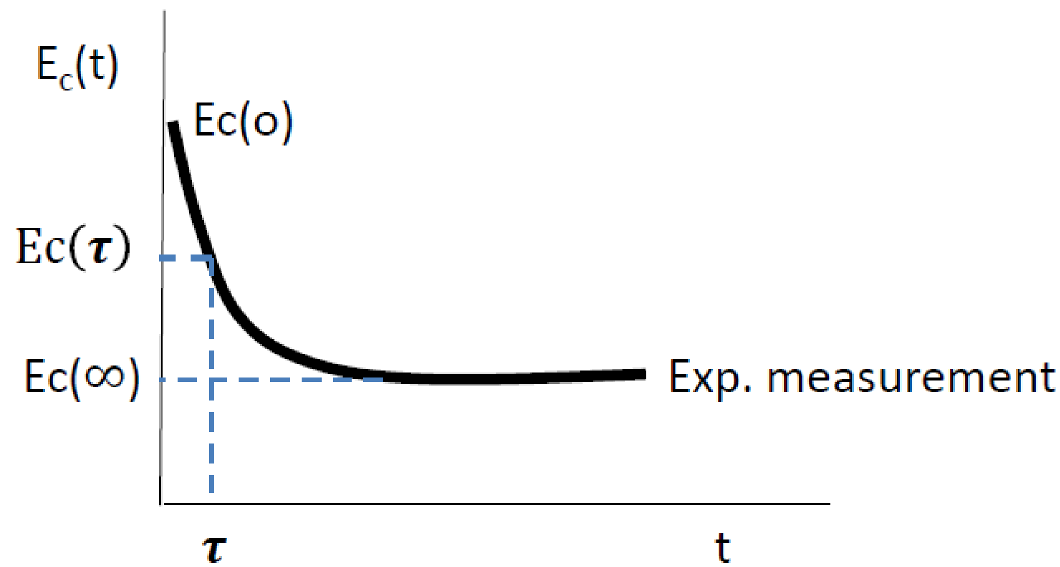


Good fitting!
The corresponding values **E_1, E_2 and τ** can be regarded as the material parameters for your material

Rough estimations of material parameters in the model

$$E_c(t) = \frac{1}{\frac{1}{E_1+E_2} + \left(\frac{1}{E_1} - \frac{1}{E_1+E_2}\right)\left(1-\exp\left(-\frac{t}{\tau}\right)\right)},$$

When t=0, Ec(o)= $\mathbf{E_1 + E_2}$;
when t=∞, Ec(∞)= $\mathbf{E_1}$;
When t= τ, Ec(τ) = $\mathbf{(E_1+E_2)/(1+(1-e^{-1})(E_2/E_1))}$;



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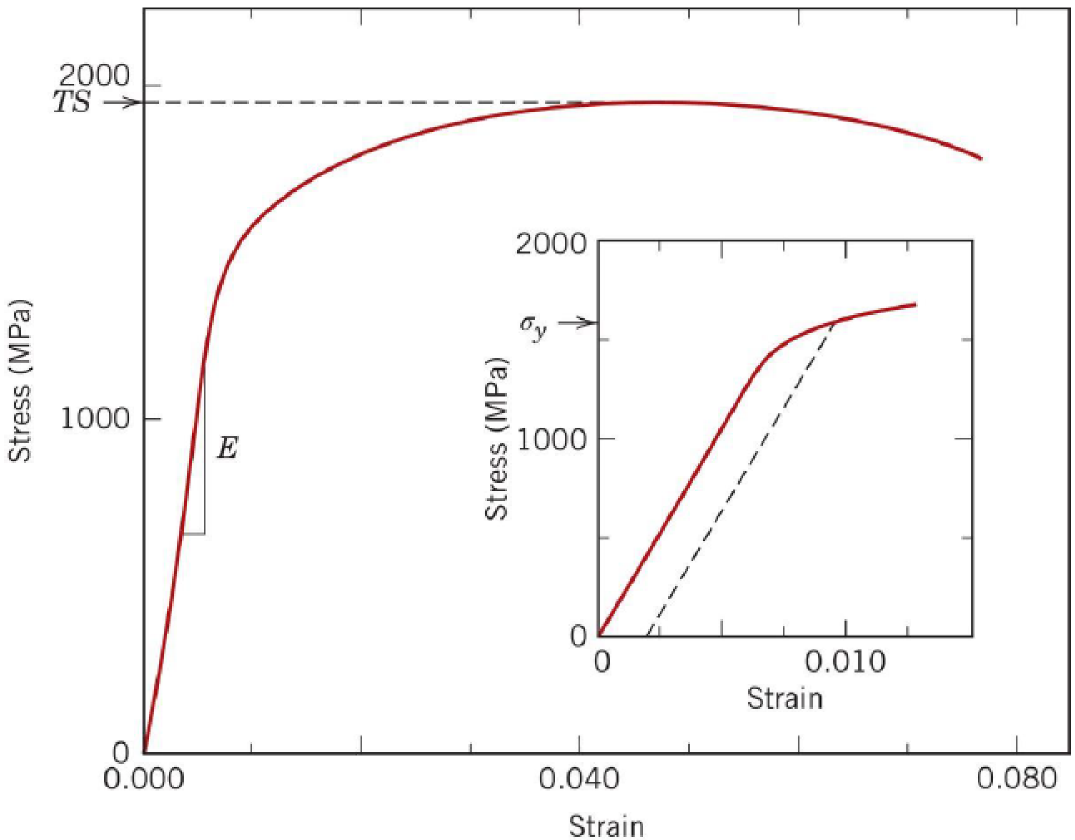
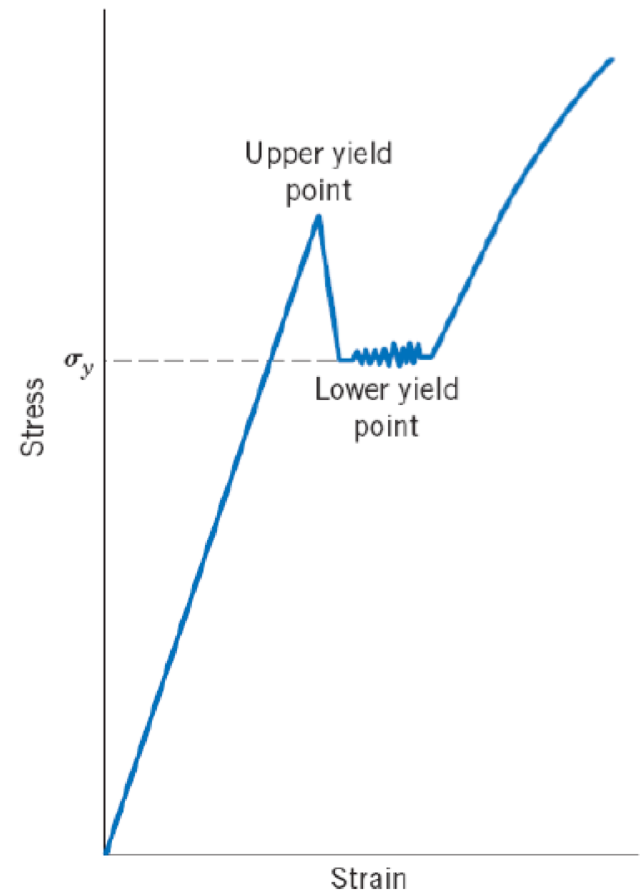
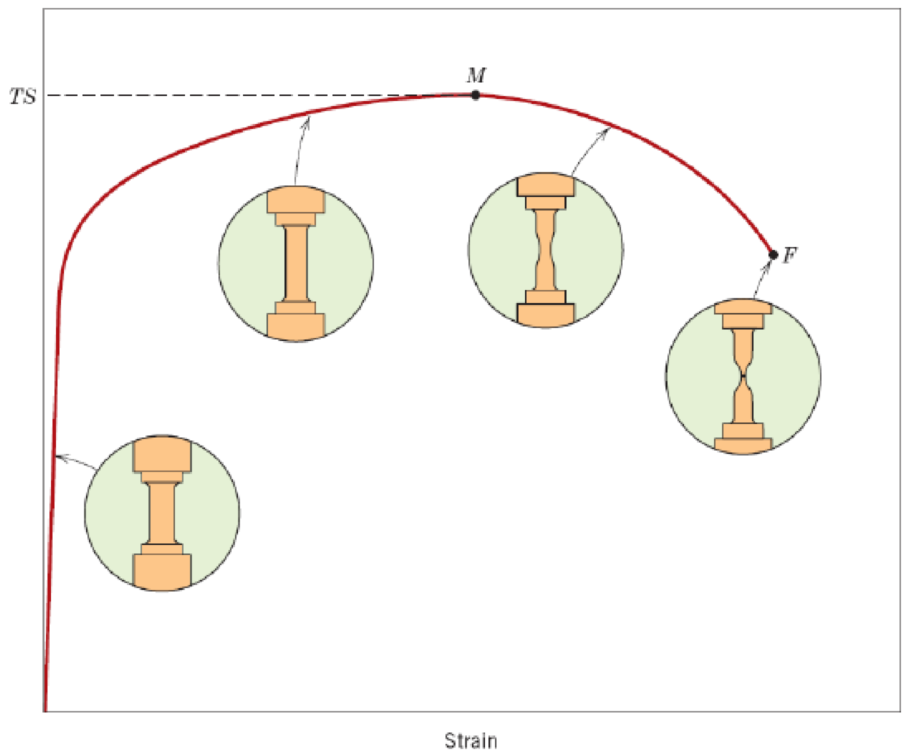
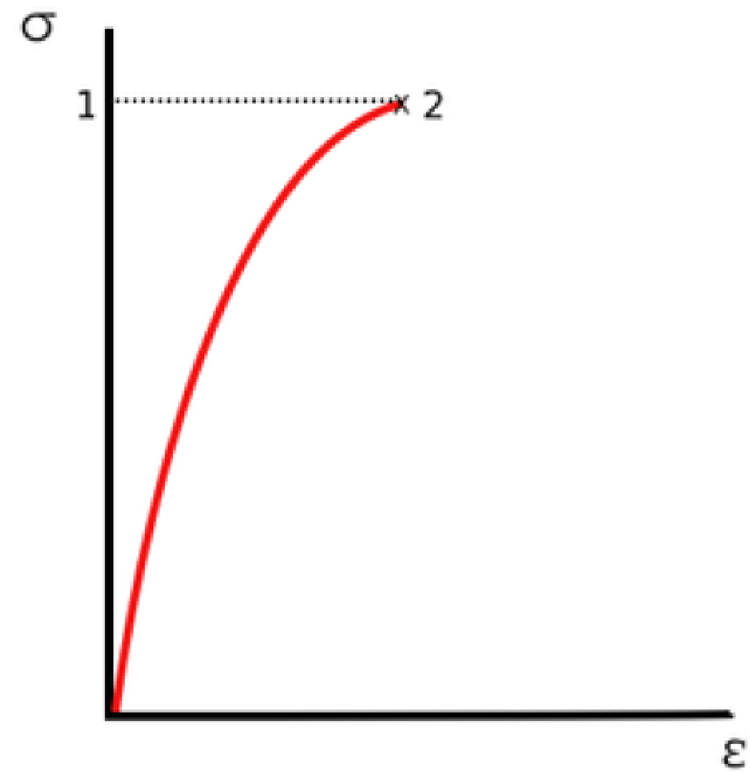
Plastic deformation



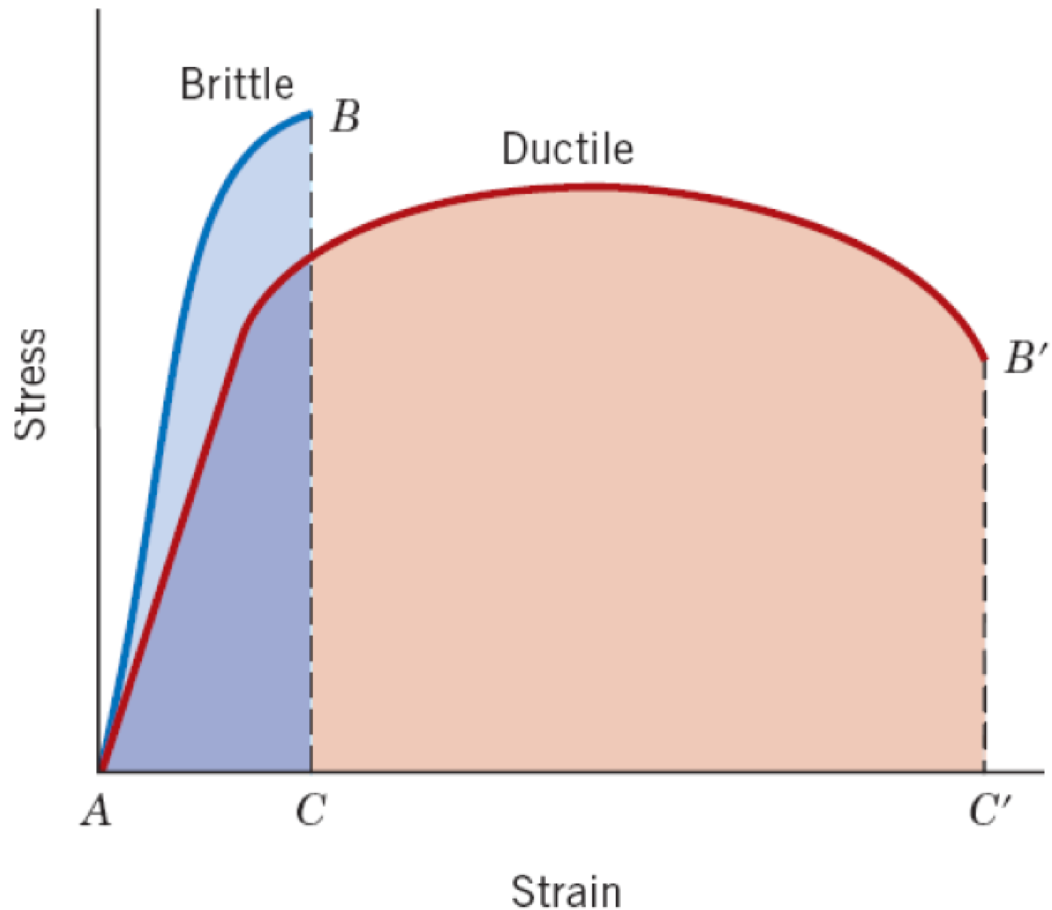


Strength





Ductility, Resilience and Toughness



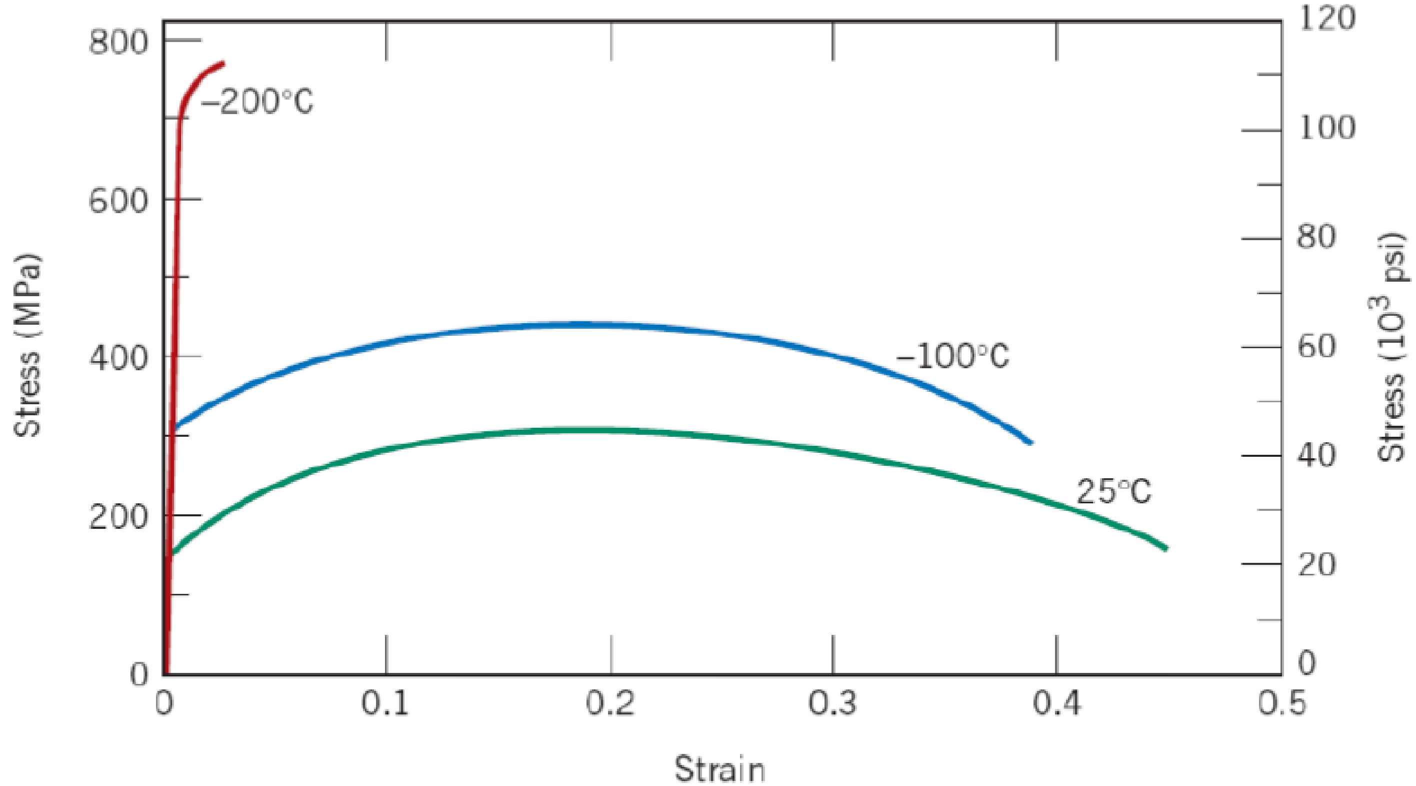
Ductility, Resilience and Toughness

Material	Yield Strength		Tensile Strength		Ductility, %EL [in 50 mm (2 in.)][a]
	MPa	ksi	MPa	ksi	
Metal Alloys[b]					
Molybdenum	565	82	655	95	35
Titanium	450	65	520	75	25
Steel (1020)	180	26	380	55	25
Nickel	138	20	480	70	40
Iron	130	19	262	38	45
Brass (70 Cu–30 Zn)	75	11	300	44	68
Copper	69	10	200	29	45
Aluminum	35	5	90	13	40
Ceramic Materials[c]					
Zirconia (ZrO_2)[d]	—	—	800–1500	115–215	—
Silicon nitride (Si_3N_4)	—	—	250–1000	35–145	—
Aluminum oxide (Al_2O_3)	—	—	275–700	40–100	—
Silicon carbide (SiC)	—	—	100–820	15–120	—
Glass–ceramic (Pyroceram)	—	—	247	36	—
Mullite ($3Al_2O_3$–$2SiO_2$)	—	—	185	27	—
Spinel ($MgAl_2O_4$)	—	—	110–245	16–36	—
Fused silica (SiO_2)	—	—	110	16	—
Magnesium oxide (MgO)[e]	—	—	105	15	—
Soda–lime glass	—	—	69	10	—
Polymers					
Nylon 6,6	44.8–82.8	6.5–12	75.9–94.5	11.0–13.7	15–300
Polycarbonate (PC)	62.1	9.0	62.8–72.4	9.1–10.5	110–150
Poly(ethylene terephthalate) (PET)	59.3	8.6	48.3–72.4	7.0–10.5	30–300
Poly(methyl methacrylate) (PMMA)	53.8–73.1	7.8–10.6	48.3–72.4	7.0–10.5	2.0–5.5
Poly(vinyl chloride) (PVC)	40.7–44.8	5.9–6.5	40.7–51.7	5.9–7.5	40–80
Phenol-formaldehyde	—	—	34.5–62.1	5.0–9.0	1.5–2.0
Polystyrene (PS)	25.0–69.0	3.63–10.0	35.9–51.7	5.2–7.5	1.2–2.5
Polypropylene (PP)	31.0–37.2	4.5–5.4	31.0–41.4	4.5–6.0	100–600
Polyethylene—high density (HDPE)	26.2–33.1	3.8–4.8	22.1–31.0	3.2–4.5	10–1200
Polytetrafluoroethylene (PTFE)	13.8–15.2	2.0–2.2	20.7–34.5	3.0–5.0	200–400
Polyethylene—low density (LDPE)	9.0–14.5	1.3–2.1	8.3–31.4	1.2–4.55	100–650

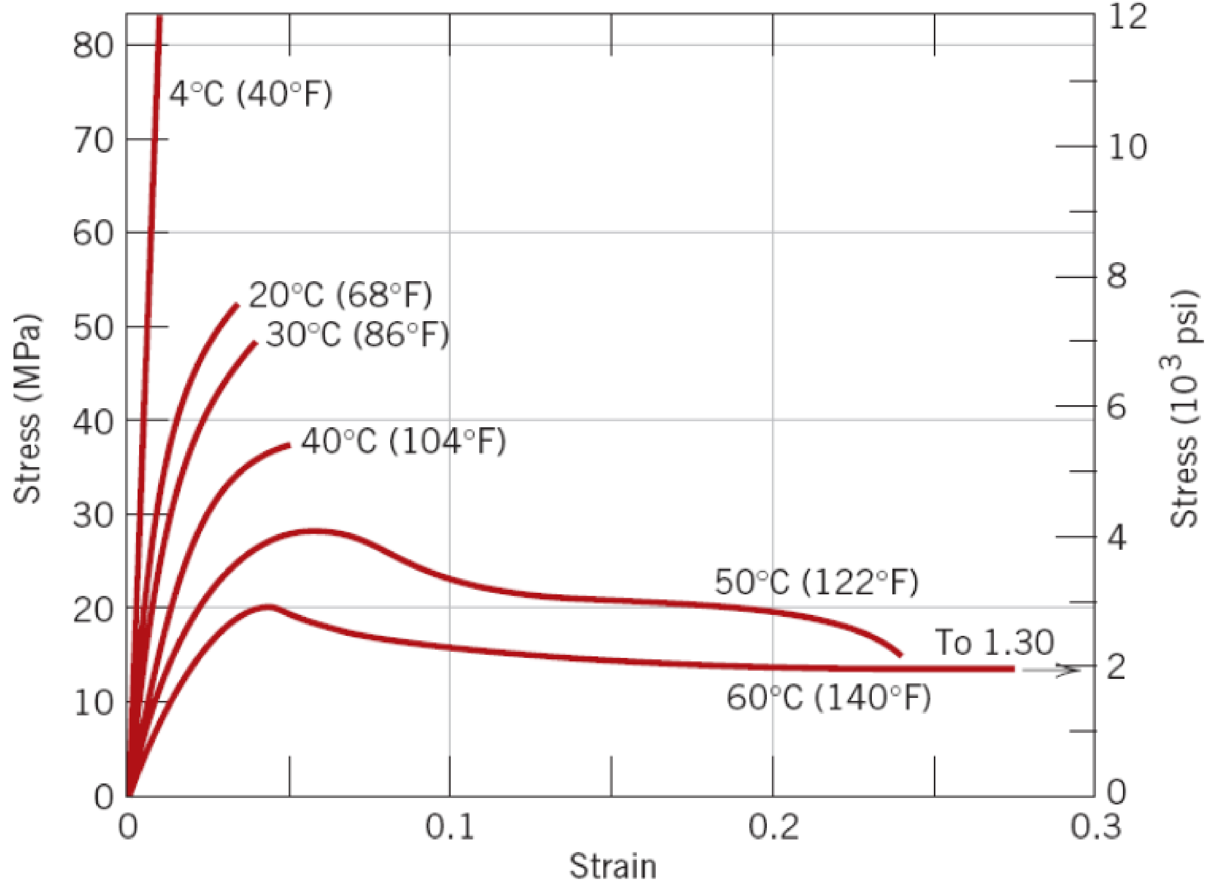
[a]For polymers, percent elongation at break.
[b]Property values are for metal alloys in an annealed state.
[c]The tensile strength of ceramic materials is taken as flexural strength (Section 7.10).
[d]Partially stabilized with 3 mol% Y_2O_3.
[e]Sintered and containing approximately 5% porosity.

Temperature-dependent mechanical properties

Figure 6.14 Engineering stress–strain behavior for iron at three temperatures.

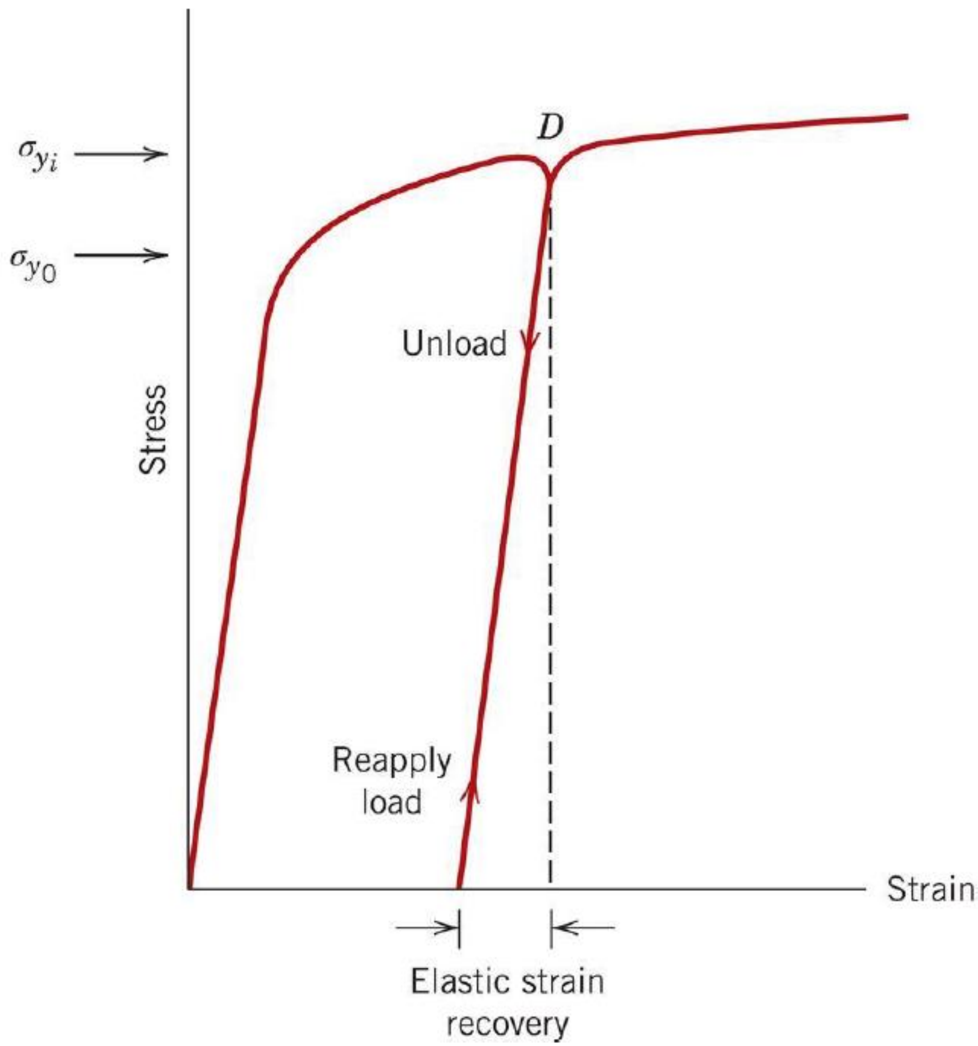


Temperature-dependent mechanical properties

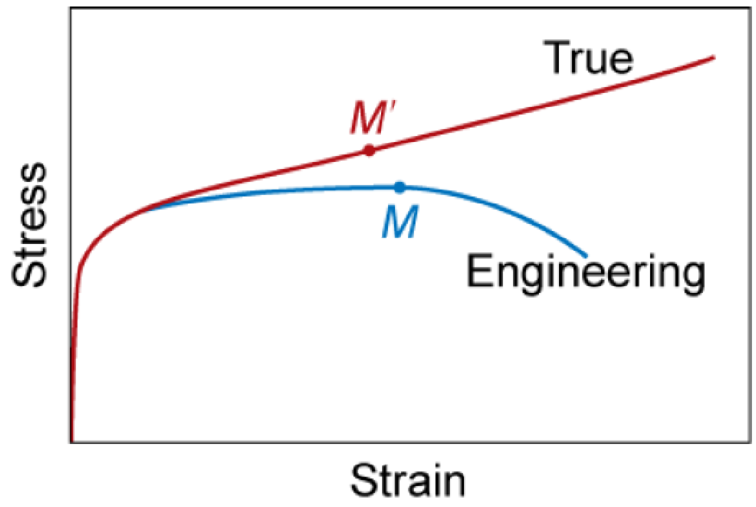


PMMA

Elastic recovery after plastic deformation



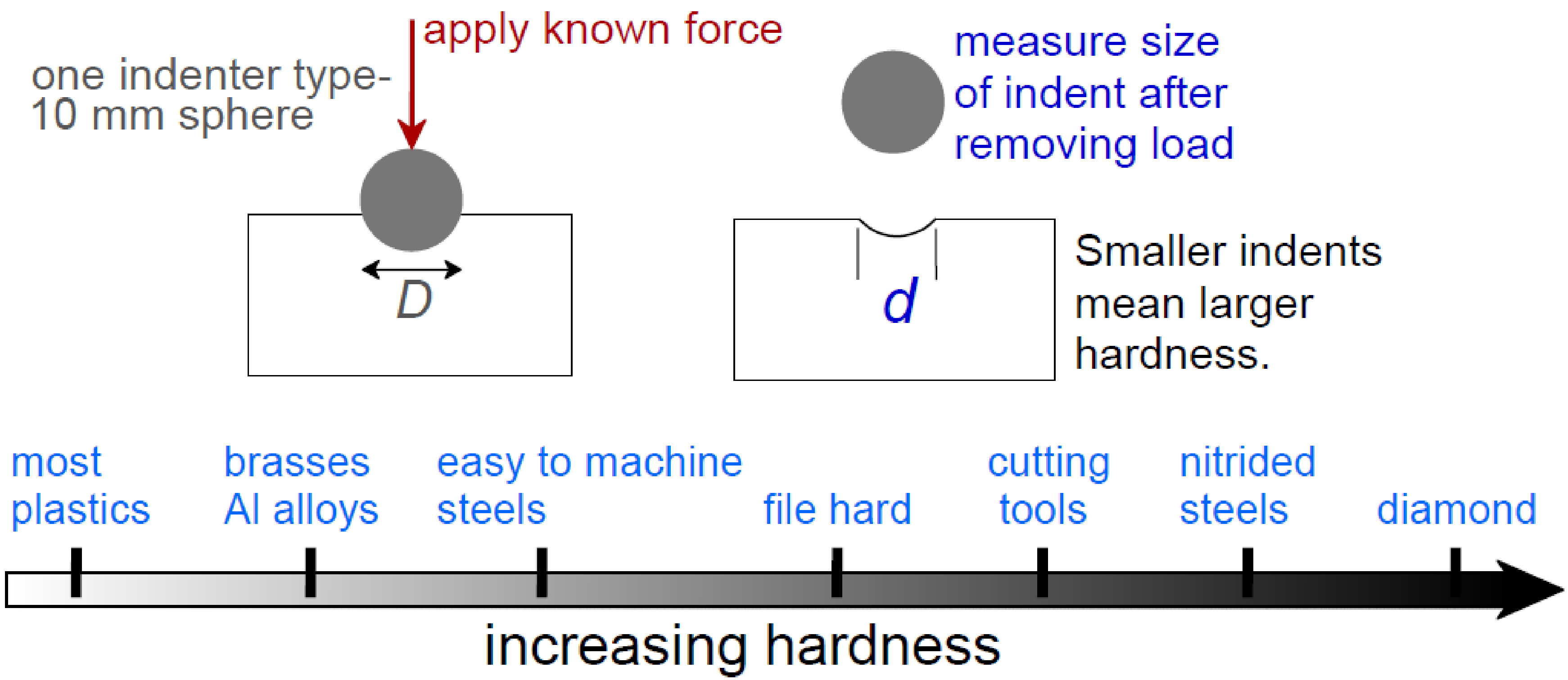
True stress and true strain



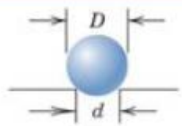
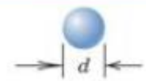
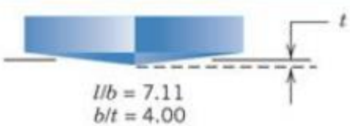
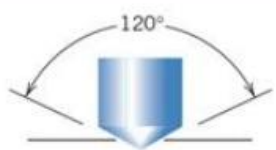
$$\sigma_T = K(\varepsilon_T)^n$$

		K	
Material	***n***	***MPa***	***psi***
Low-carbon steel (annealed)	0.21	600	87,000
4340 steel alloy (tempered at 315°C)	0.12	2650	385,000
304 stainless steel (annealed)	0.44	1400	205,000
Copper (annealed)	0.44	530	76,500
Naval brass (annealed)	0.21	585	85,000
2024 aluminum alloy (heat-treated—T3)	0.17	780	113,000
AZ-31B magnesium alloy (annealed)	0.16	450	66,000

Hardness


apply known force
one indenter type-
10 mm sphere
D
measure size
of indent after
removing load
d
Smaller indents
mean larger
hardness.
most
plastics
brasses
Al alloys
easy to machine
steels
file hard
cutting
tools
nitrided
steels
diamond
increasing hardness

Hardness

Test	Indenter	Shape of Indentation: Side View	Shape of Indentation: Top View	Load	Formula for Hardness Number[a]
Brinell	10-mm sphere of steel or tungsten carbide			P	$HB = \frac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$
Vickers microhardness	Diamond pyramid			P	$HV = 1.854P/d_1^2$
Knoop microhardness	Diamond pyramid			P	$HK = 14.2\ P/l^2$
Rockwell and superficial Rockwell	Diamond cone; $\frac{1}{16}$-, $\frac{1}{8}$-, $\frac{1}{4}$-, $\frac{1}{2}$- in. diameter tungsten carbide spheres	 		60 kg, 100 kg, 150 kg } Rockwell 15 kg, 30 kg, 45 kg } Superficial Rockwell	

[a]For the hardness formulas given, P (the applied load) is in kg and D, d, d_1, and l are all in mm.

Source: Adapted from H. W. Hayden, W. G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*. Copyright © 1965 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.

Variability of Material Properties

- Statistical treatments
- Typical value—take average value, $\overline{x}$ for some parameter x:

$$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

n = number of measurements

x_i = specific measured value

- Degree of scatter—use standard deviation, s

$$s = \left[\frac{\sum_{i=1}^{n} \left(x_i - \overline{x} \right)^2}{n-1} \right]^{\frac{1}{2}}$$

Design/Safety Factors

- Because of design uncertainties allowances must be made to protect against unanticipated failure
- For structural applications, to protect against possibility of failure—use working stress, σ_w, and a factor of safety, N

$$\sigma_w = \frac{\sigma_y}{N}$$

yield strength (σ_y)

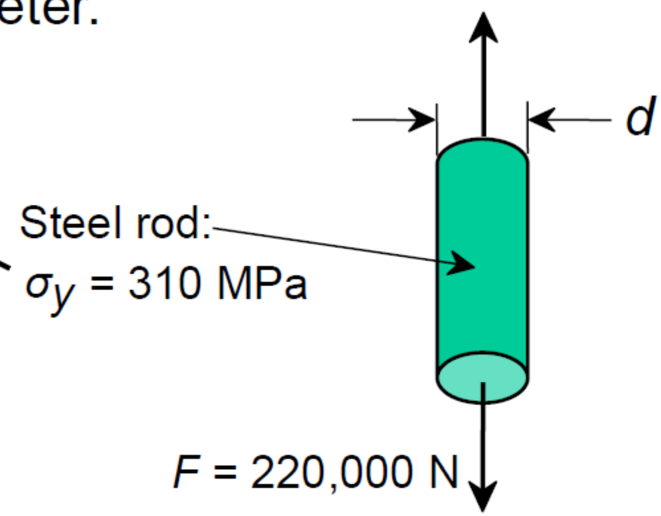
Depending on application, N is between 1.2 and 4

Design/Safety Factors

Example Problem: A cylindrical rod, to be constructed from a steel that has a yield strength of 310 MPa, is to withstand a load of 220,000 N without yielding. Assuming a value of 4 for *N*, specify a suitable bar diameter.

$$\sigma_w = \frac{\sigma_y}{N}$$

$$\frac{220{,}000\ \text{N}}{\pi\left(\frac{d}{2}\right)^2} = \frac{\sigma_y}{4}$$



Solving for the rod diameter *d* yields

$$d = 0.060\ \text{m} = 60\ \text{mm}$$

Fracture Phenomena

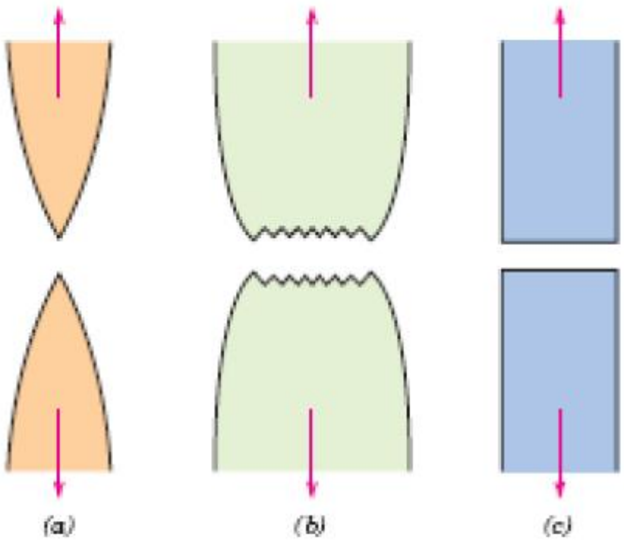


Figure 8.3 (*a*) Cup-and-cone fracture in aluminum. (*b*) Brittle fracture in a mild steel.

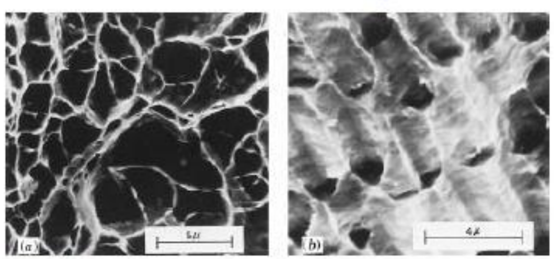
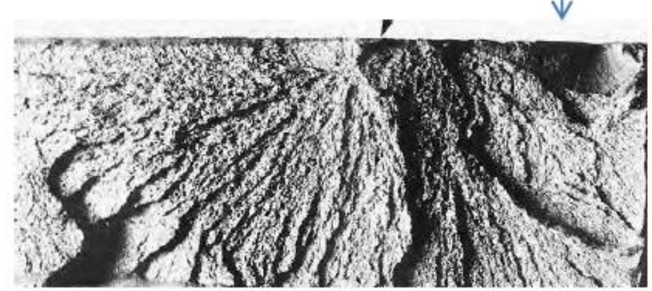


Figure 8.4 (*a*) Scanning electron fractograph showing spherical dimples characteristic of ductile fracture resulting from uniaxial tensile loads. 3300×. (*b*) Scanning electron fractograph showing parabolic-shaped dimples characteristic of ductile fracture resulting from shear loading. 5000×. (From R. W. Hertzberg, *Deformation and Fracture Mechanics of Engineering Materials*, 3rd edition. Copyright © 1989 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.)



(b)

Microscopic process of fracturing

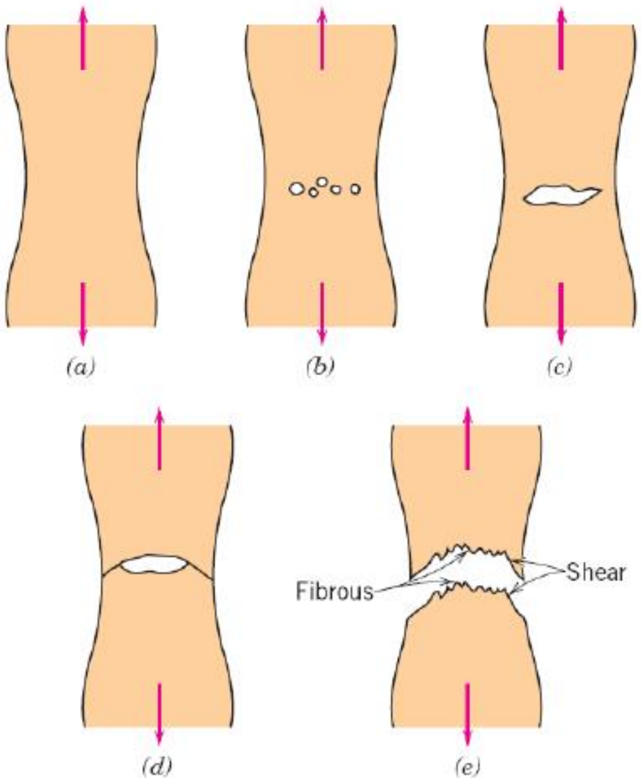
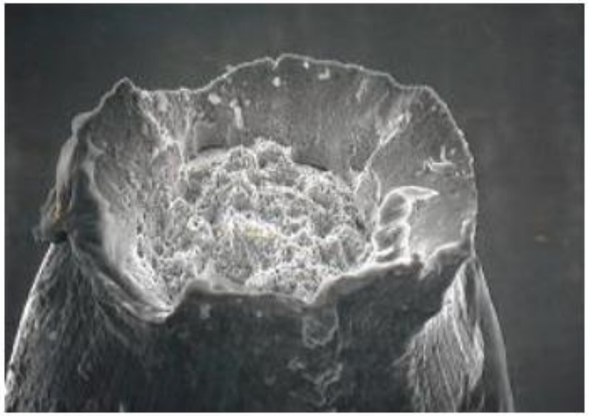


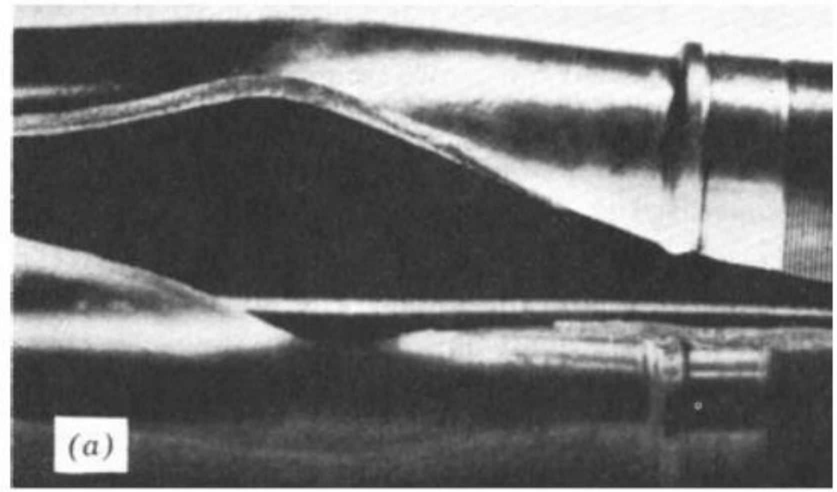
Figure 8.2 Stages in the cup-and-cone fracture. (*a*) Initial necking. (*b*) Small cavity formation. (*c*) Coalescence of cavities to form a crack. (*d*) Crack propagation. (*e*) Final shear fracture at a 45° angle relative to the tensile direction. (From K. M. Ralls, T. H. Courtney, and J. Wulff, *Introduction to Materials Science and Engineering*, p. 468. Copyright © 1976 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.)



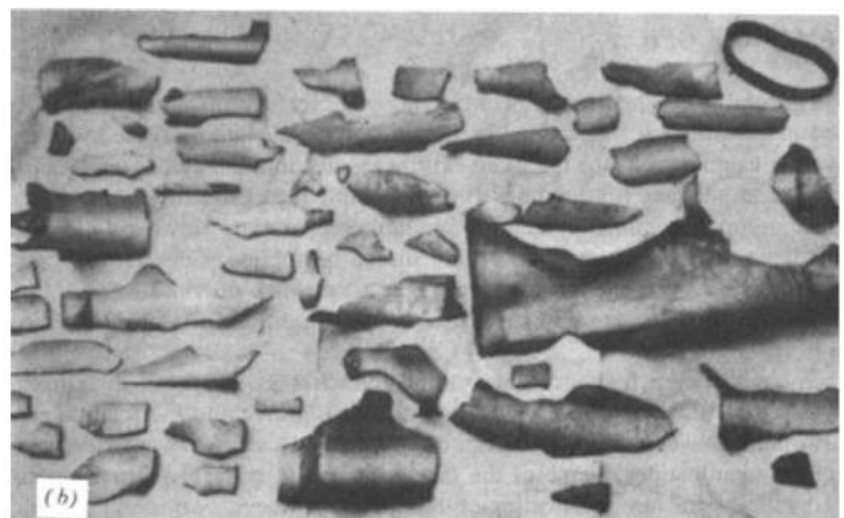
Fracture surface of tire cord wire loaded in tension. Courtesy of F. Roehrig, CC Technologies, Dublin, OH. Used with permission.

Examples of Ductile and Brittle Fracture of Pipes

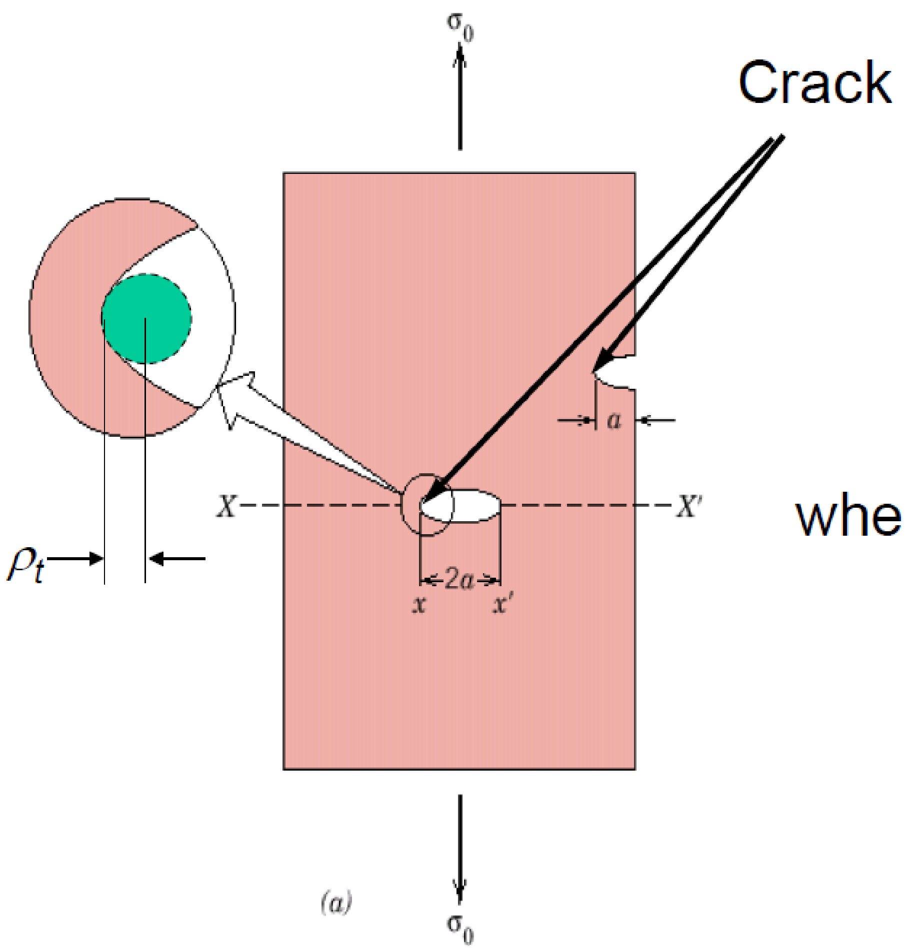
- **Ductile fracture:**
  -- one piece
  -- large deformation



- **Brittle fracture:**
  -- many pieces
  -- small deformations



Fracture Mechanics



$$\sigma_m = 2\sigma_o \left(\frac{a}{\rho_t}\right)^{1/2}$$

where

ρ_t = radius of curvature

σ_o = applied stress

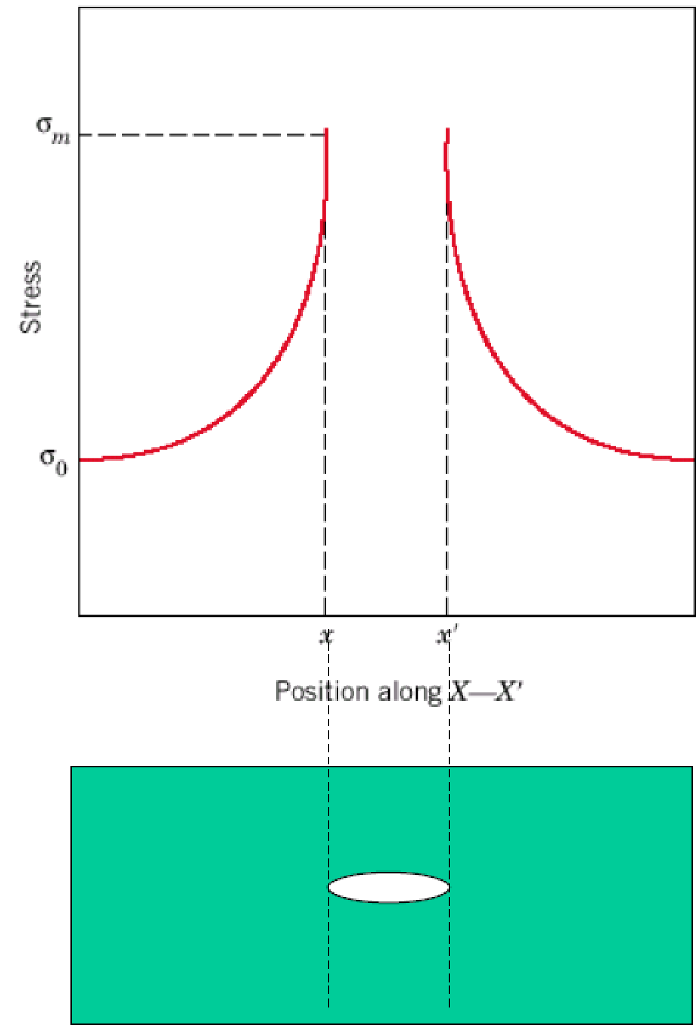
σ_m = stress at crack tip

Fracture Mechanics

Stress Concentration at Crack Tip

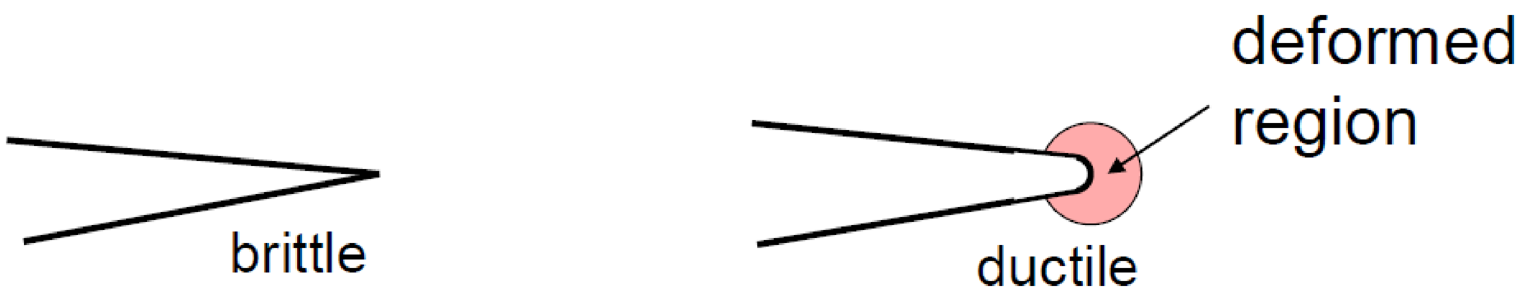
K_t = stress concentration factor

$$K_t = \frac{\sigma_m}{\sigma_o}$$



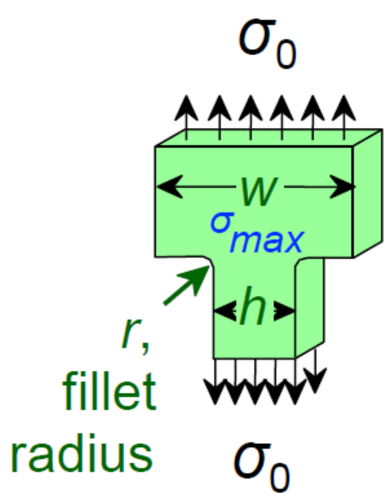
Crack Propagation

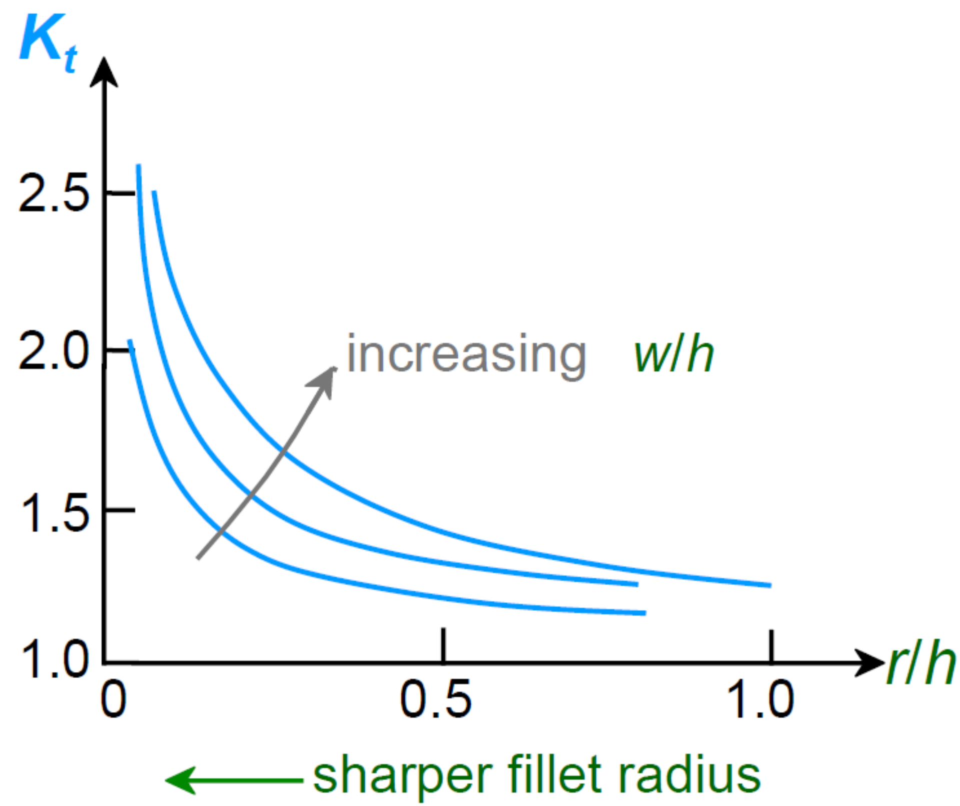
- Stress concentration higher for sharp cracks—propagate at lower stresses than cracks with blunt tips
- For ductile materials—plastic deformation at crack tip when stress reaches yield strength—tip blunted—lowers stress conc.



Crack Propagation

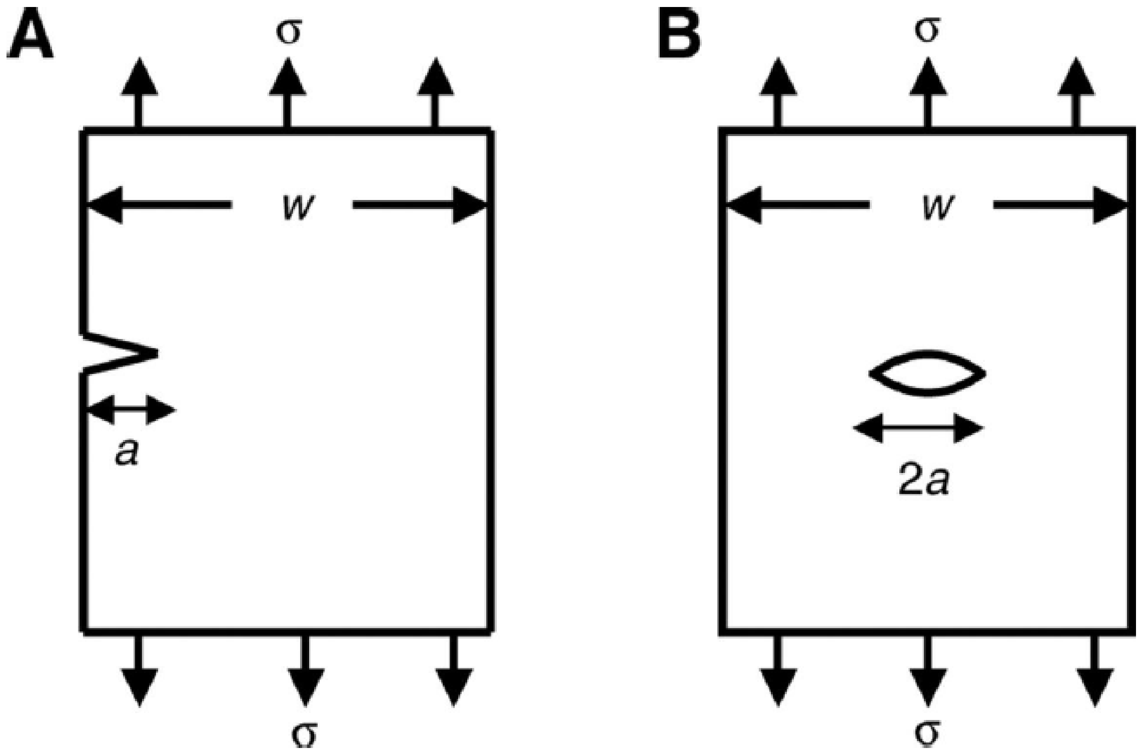
- **Avoid sharp corners!**



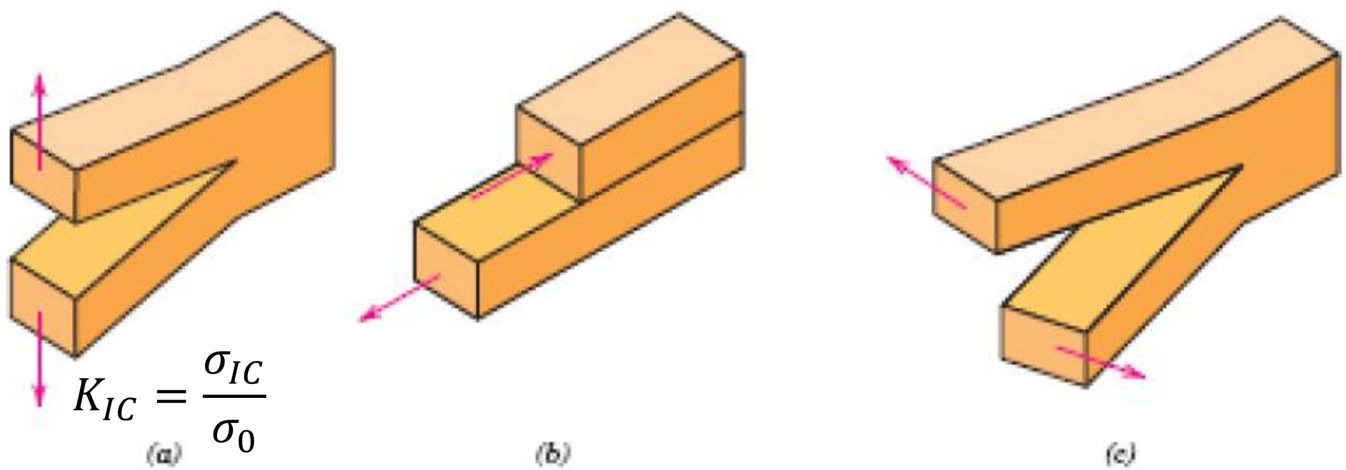


Adapted from Fig. 8.2W(c), *Callister 6e.* (Fig. 8.2W(c) is from G.H. Neugebauer, *Prod. Eng.* (NY), Vol. 14, pp. 82-87 1943.)

Fracture toughness



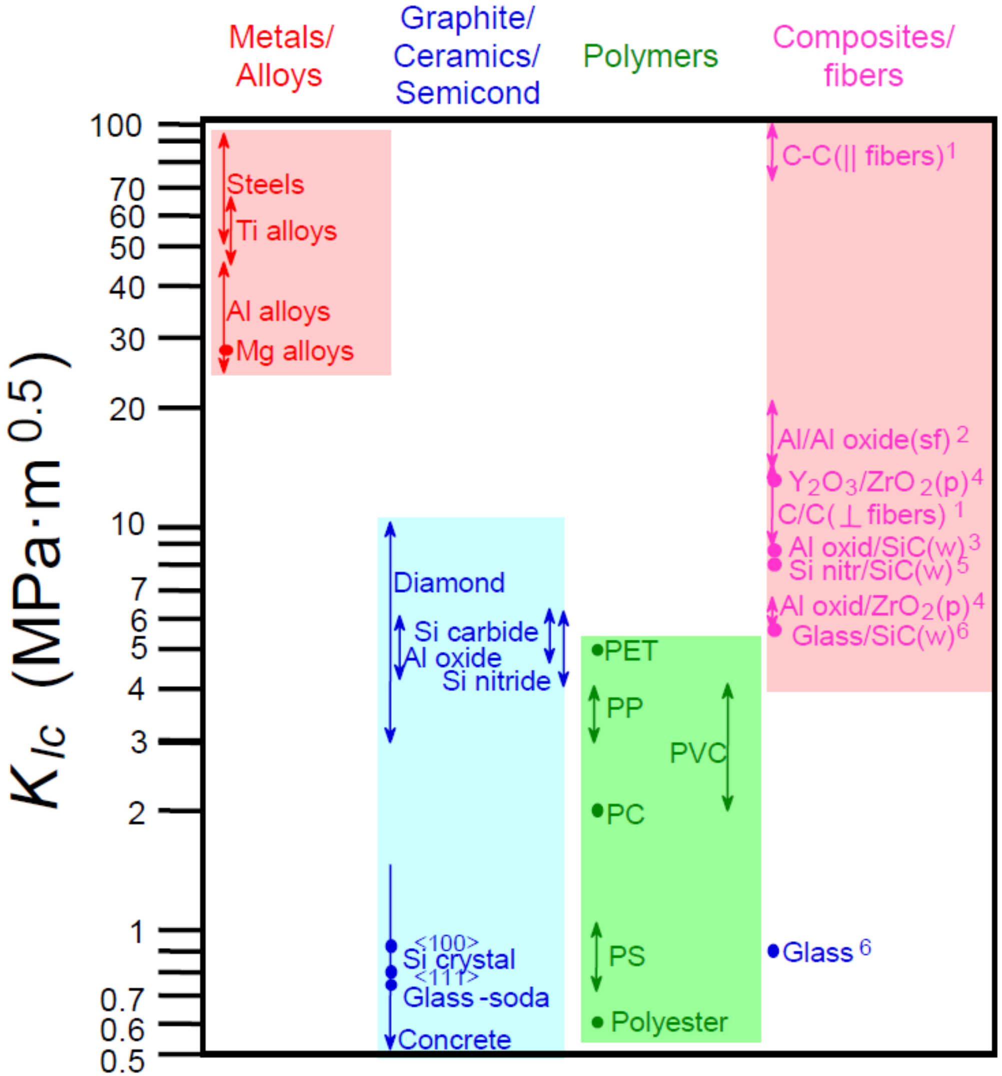
Fracture modes



$$K_{IC} = \frac{\sigma_{IC}}{\sigma_0}$$

Tensile mode | Sliding mode | Tearing mode

Fracture Toughness Ranges



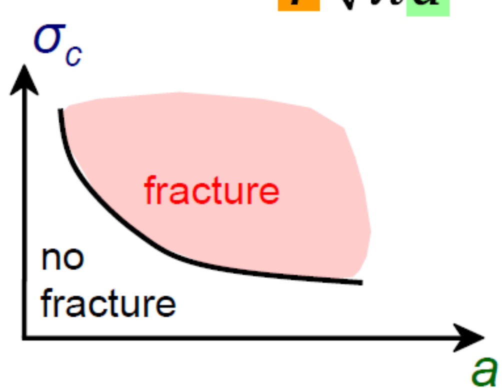
Design Against Fracture

- Crack growth condition:

$$K_{Ic} < Y\sigma_C\sqrt{\pi a}$$

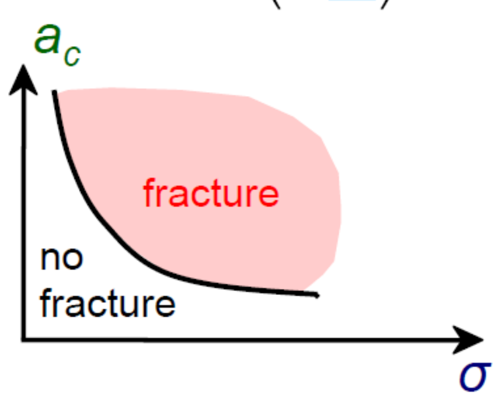
--Scenario 1: K_{Ic} and flaw size a specified - dictates max. design (critical) stress.

$$\sigma_c = \frac{K_{Ic}}{Y\sqrt{\pi a}}$$

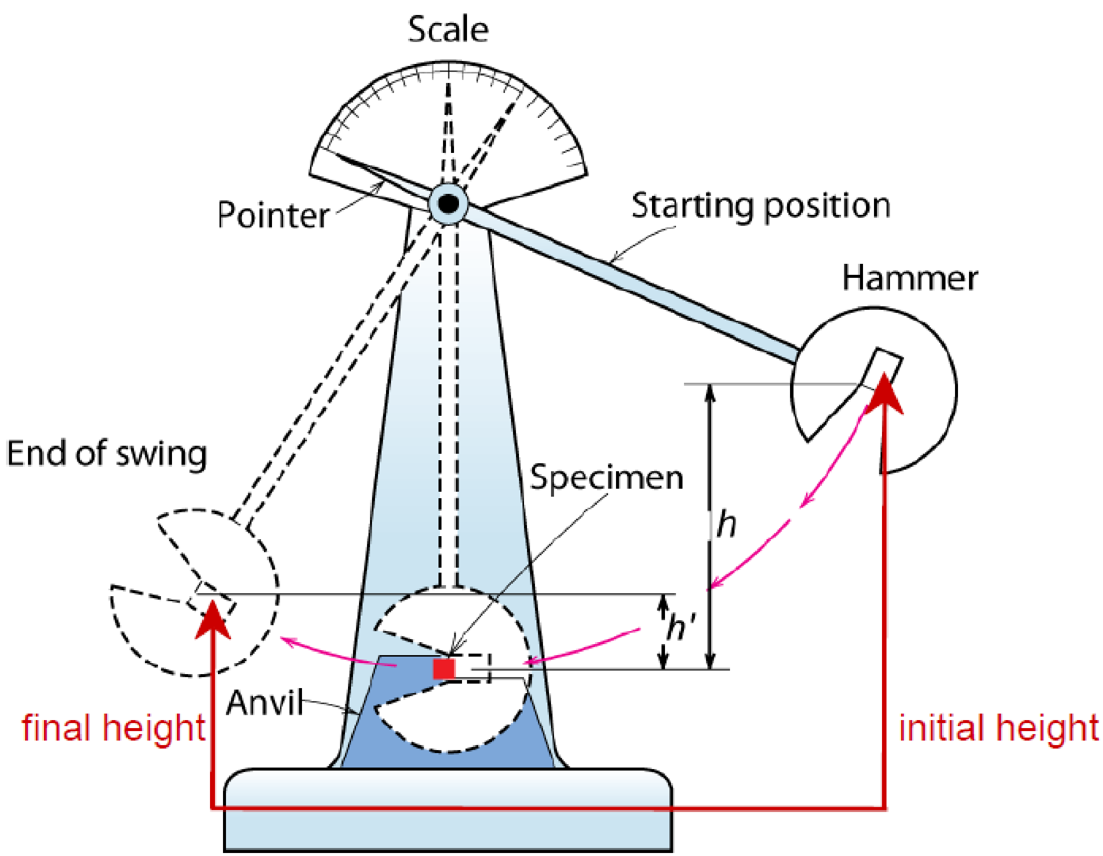


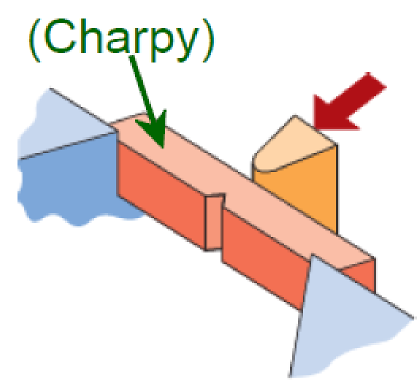
--Scenario 2: K_{Ic} and stress level specified - dictates max. allowable flaw size.

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{Y\sigma}\right)^2$$



Impact fracturing test





Ductile-to-Brittle Transition

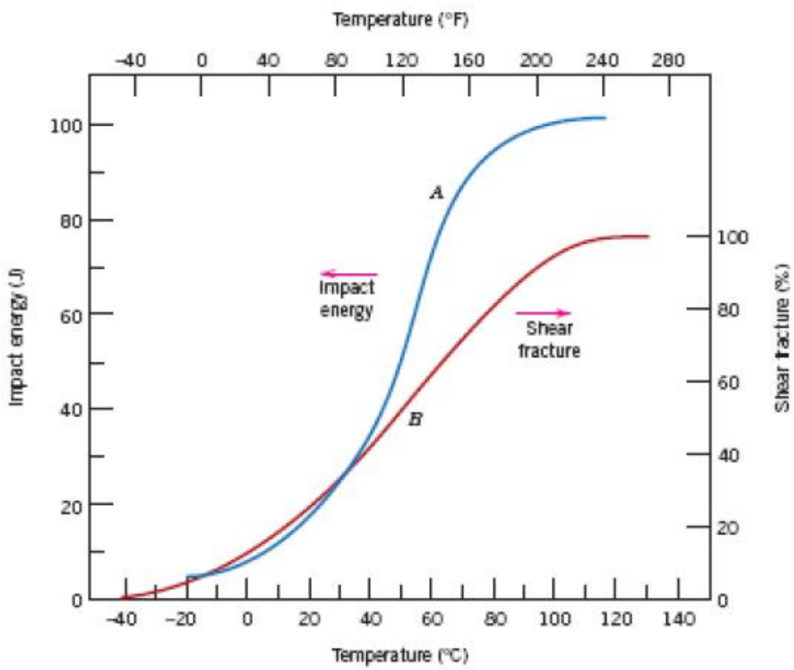
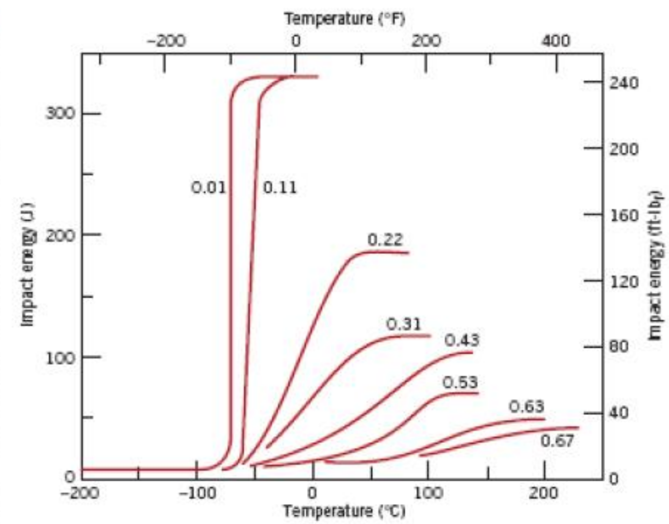


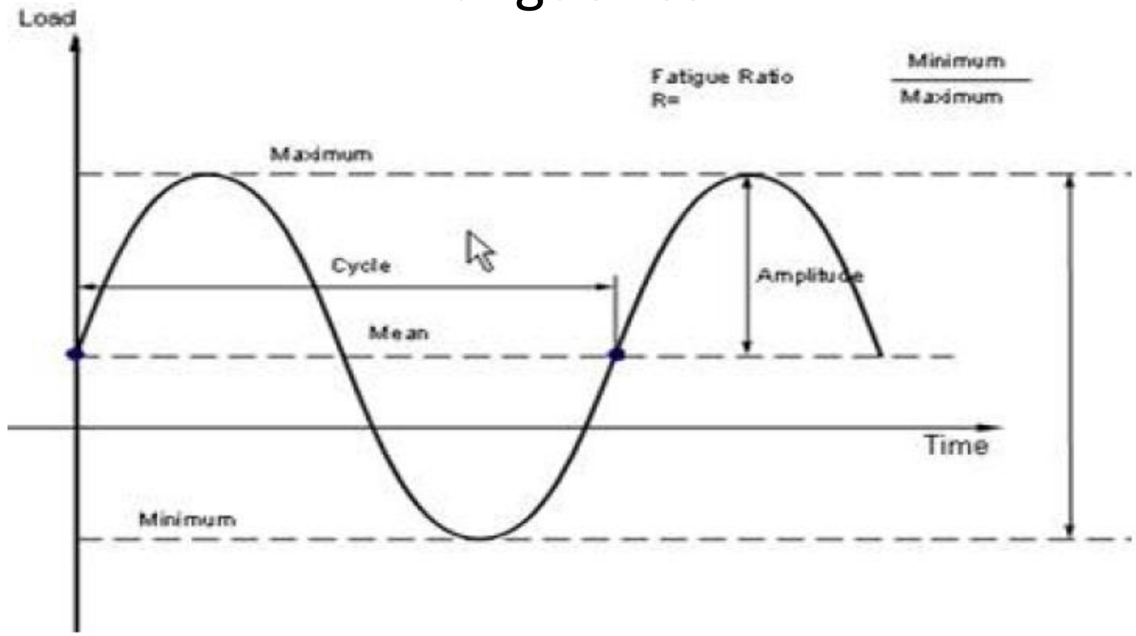
Figure 8.16 Influence of carbon content on the Charpy V-notch energy-versus-temperature behavior for steel. (Reprinted with permission from ASM International, Metals Park, OH 44073-9989, USA; J. A. Reinbolt and W. J. Harris, Jr., "Effect of Alloying Elements on Notch Toughness of Pearlitic Steels," *Transactions of ASM*, Vol. 43, 1951.)

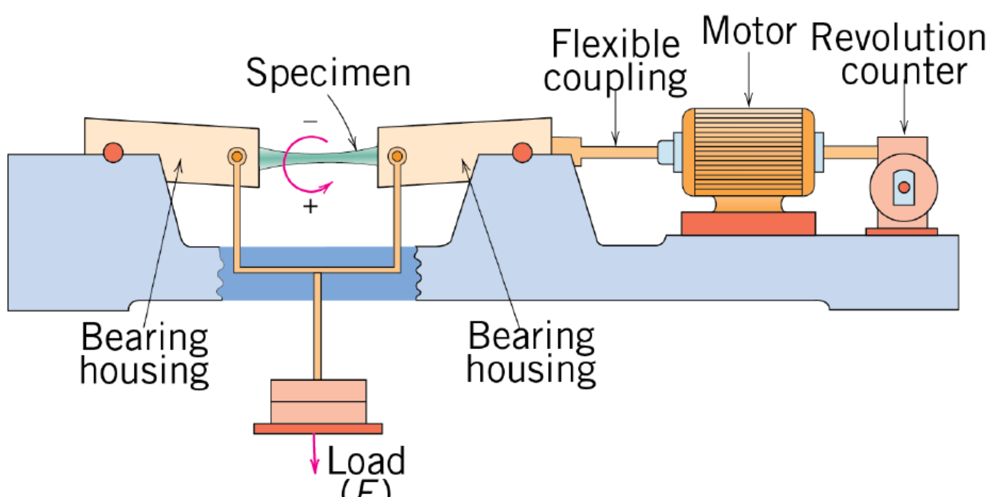


Brittle Fracture

Ductile Fracture

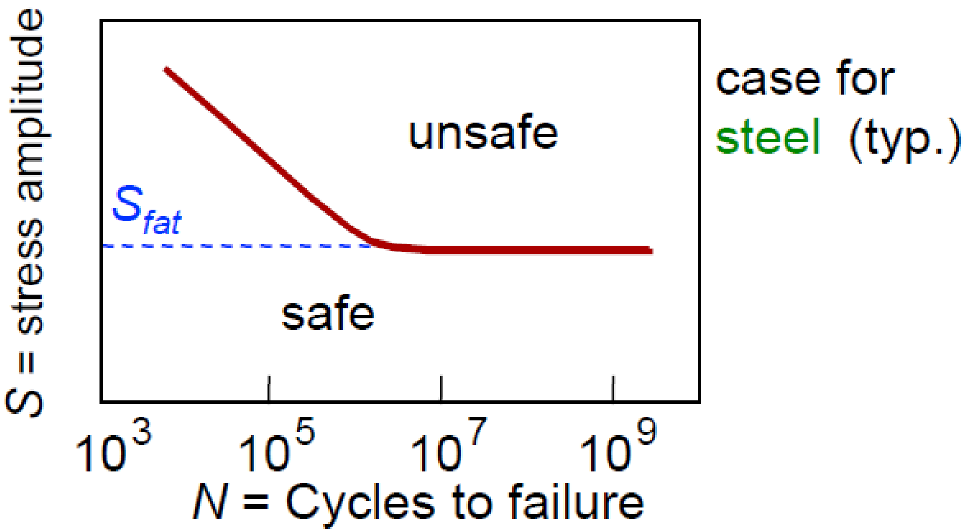
Fatigue Test

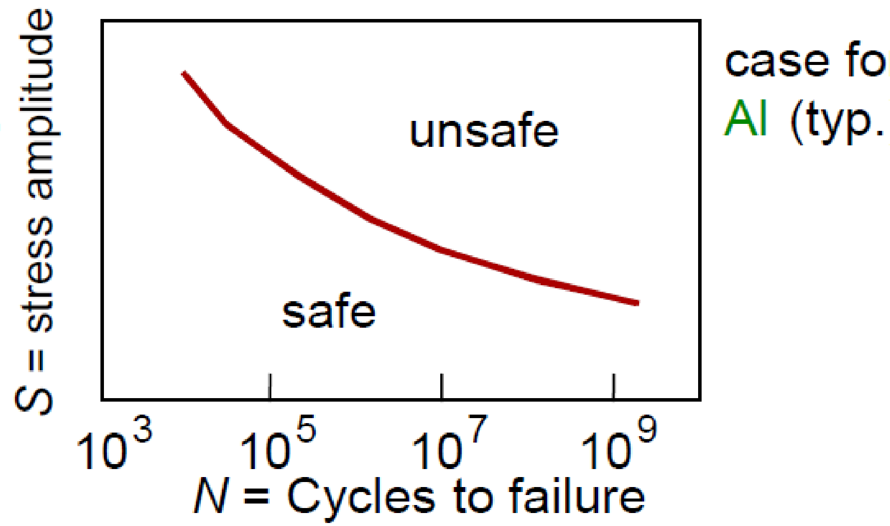




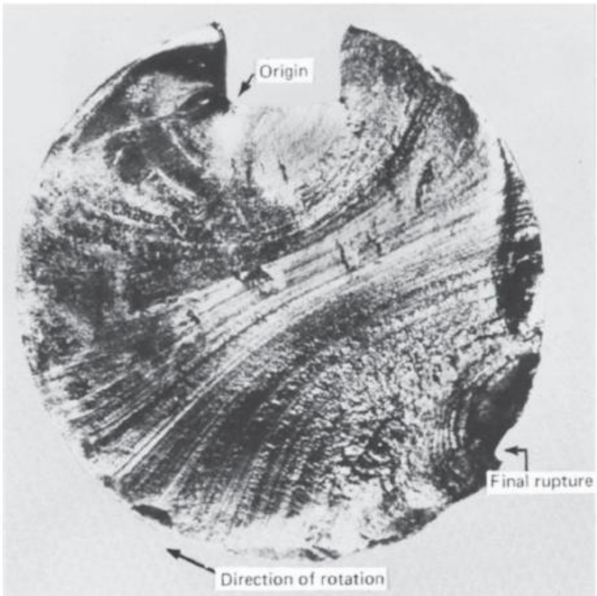
Fatigue Failure

Types of Fatigue Behavior

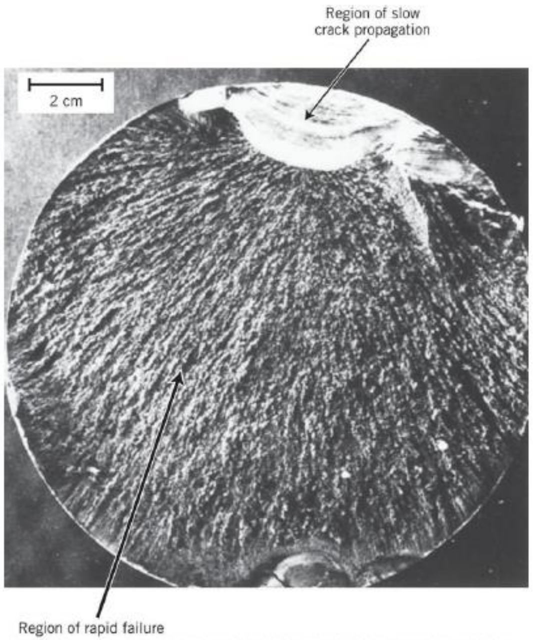




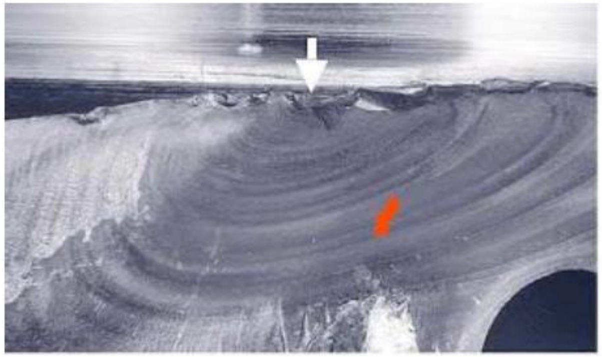
Types of Fatigue Behavior



Reproduced with permission from D. J. Wulpi, *Understanding How Components Fail*, American Society for Metals, Materials Park, OH, 1985.



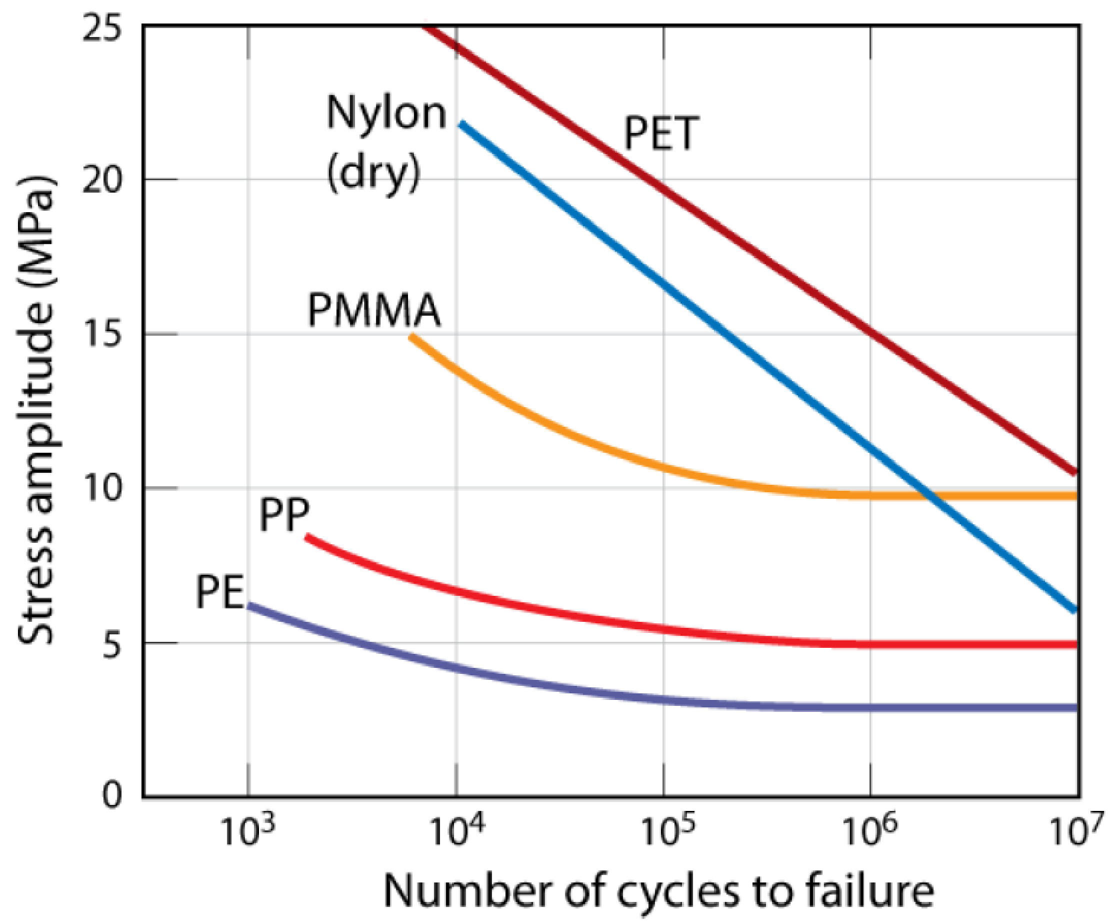
Reproduced by permission from *Metals Handbook: Fractography and Atlas of Fractographs*, Vol. 9, 8th edition, H. E. Boyer (Editor), American Society for Metals, 1974.



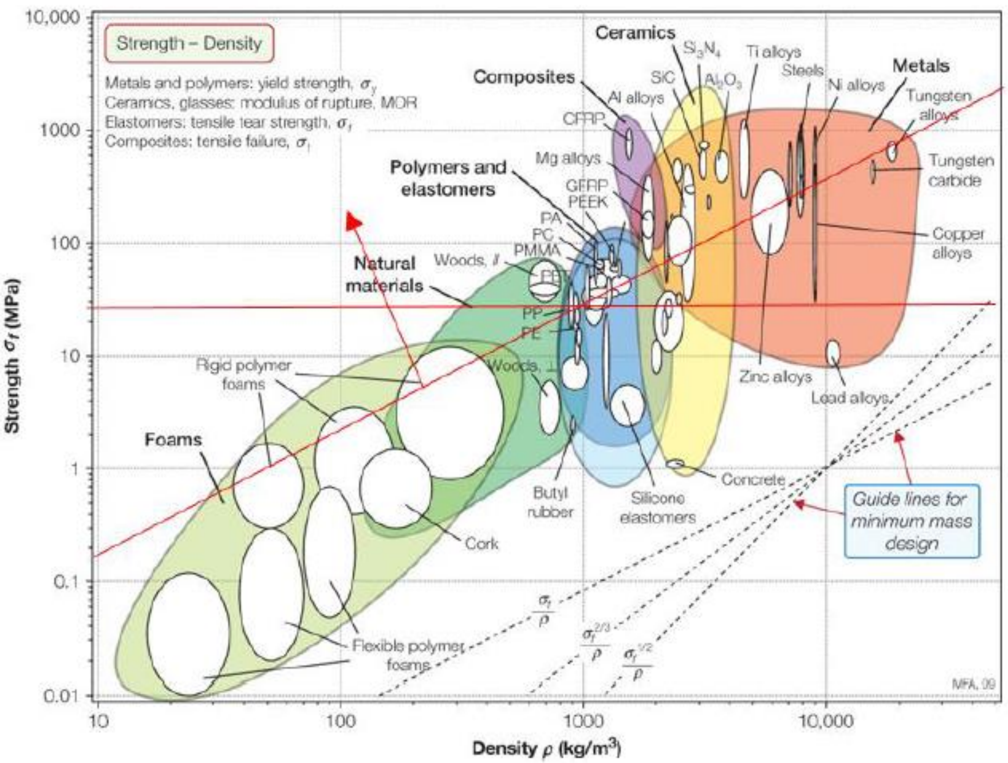
Fatigue Behavior of Polymers

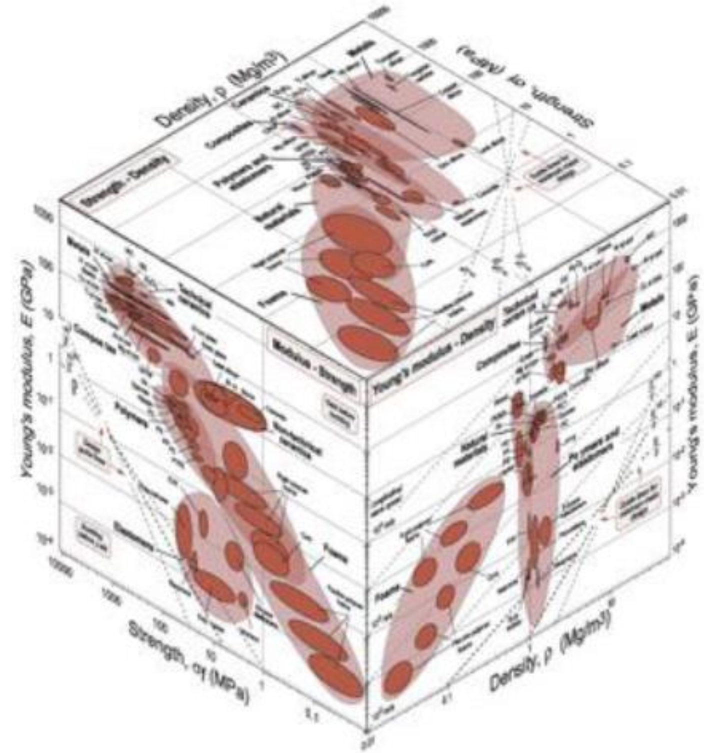
- Fatigue limit:
  - PMMA, PP, PE

- No fatigue limit:
  - PET, Nylon (dry)

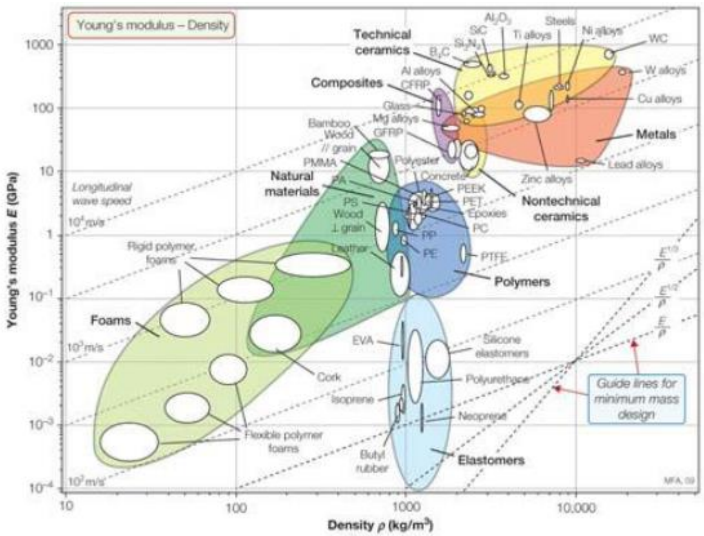


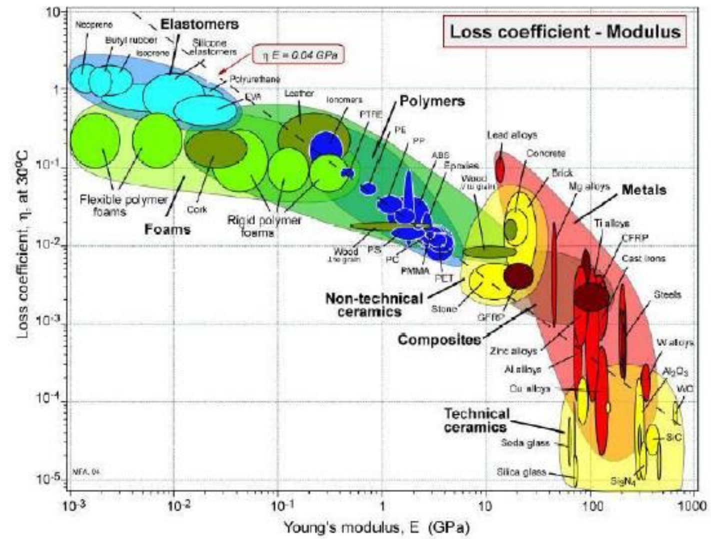
Ashby Chart

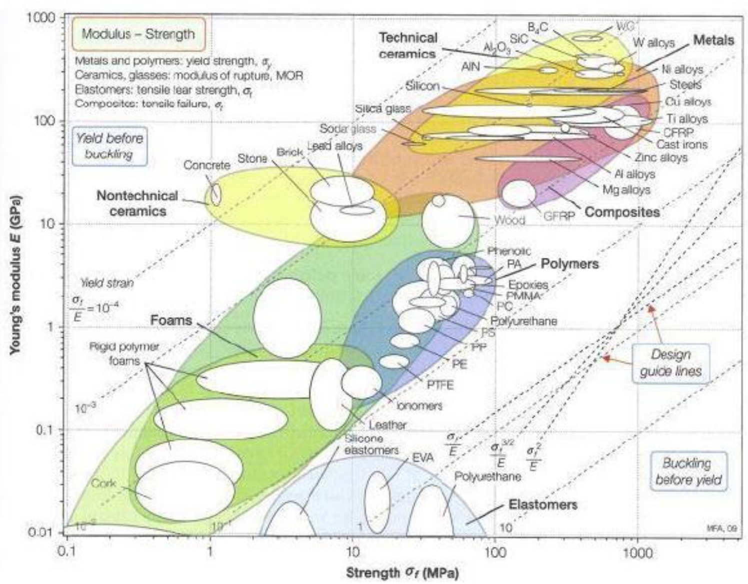

Strength – Density
Metals and polymers: yield strength, σ_y
Ceramics, glasses: modulus of rupture, MOR
Elastomers: tensile tear strength, σ_t
Composites: tensile failure, σ_t
Ceramics
Si_3N_4
Ti alloys
Steels
Metals
SiC
Al_2O_3
Ni alloys
Tungsten alloys
Composites
Al alloys
CFRP
Tungsten carbide
Polymers and elastomers
Mg alloys
GFRP
PEEK
PA
PC
PMMA
Copper alloys
Natural materials
Woods, //
PET
PP
PE
Woods, ⊥
Rigid polymer foams
Foams
Zinc alloys
Lead alloys
Butyl rubber
Silicone elastomers
Concrete
Cork
Guide lines for minimum mass design
Flexible polymer foams
$\frac{\sigma_f}{\rho}$
$\frac{\sigma_f^{2/3}}{\rho}$
$\frac{\sigma_f^{1/2}}{\rho}$
MFA, 09
Strength σ_f (MPa)
Density ρ (kg/m^3)
10,000
1000
100
10
1
0.1
0.01
10
100
1000
10,000

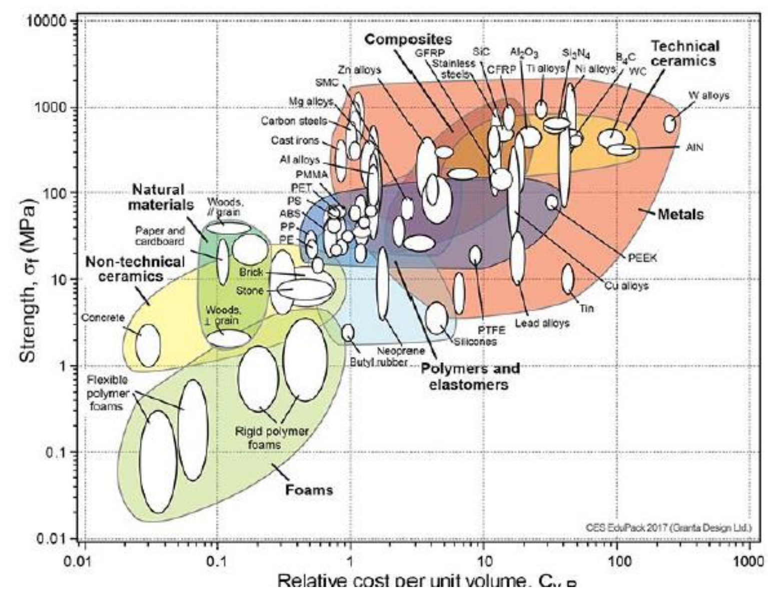


Ashby Chart


Young's modulus – Density
Technical ceramics
Composites
Metals
Natural materials
Nontechnical ceramics
Polymers
Foams
Elastomers
Guide lines for minimum mass design
Longitudinal wave speed
Young's modulus E (GPa)
Density ρ (kg/m³)


Loss coefficient - Modulus
Elastomers
Polymers
Metals
Flexible polymer foams
Foams
Rigid polymer foams
Non-technical ceramics
Composites
Technical ceramics
Loss coefficient, η, at 30°C
Young's modulus, E (GPa)


Modulus – Strength
Metals and polymers: yield strength, σy
Ceramics, glasses: modulus of rupture, MOR
Elastomers: tensile tear strength, σt
Composites: tensile failure, σt
Yield before buckling
Technical ceramics
Metals
Nontechnical ceramics
Composites
Polymers
Foams
Elastomers
Design guide lines
Buckling before yield
Young's modulus E (GPa)
Strength σf (MPa)


Composites
Technical ceramics
Natural materials
Non-technical ceramics
Metals
Polymers and elastomers
Foams
Strength, σf (MPa)
Relative cost per unit volume, Cv,R
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