6. Determination of residual stress

Contents

6.1 Generation and classification of residual stress within objects

- Residual stress is an internal stress.
- Internal stress refers to the stress that remains inside the component due to uneven deformation and volume changes. It maintains its balance when the various factors that generate external stress no longer exist.
- The various factors that generate stress no longer exist, which means that the external load is removed, processing is completed, the temperature is uniform, and the phase change process is terminated, etc.
- The currently recognized internal stress classification method was proposed by E. Marklauch of Germany in 1979. Internal stress is divided into three categories according to its equilibrium range, namely, Type I internal stress, Type II internal stress and Type III. internal stress.

6.1 Basic principles of X-ray residual stress measurement

- Classification of internal stress

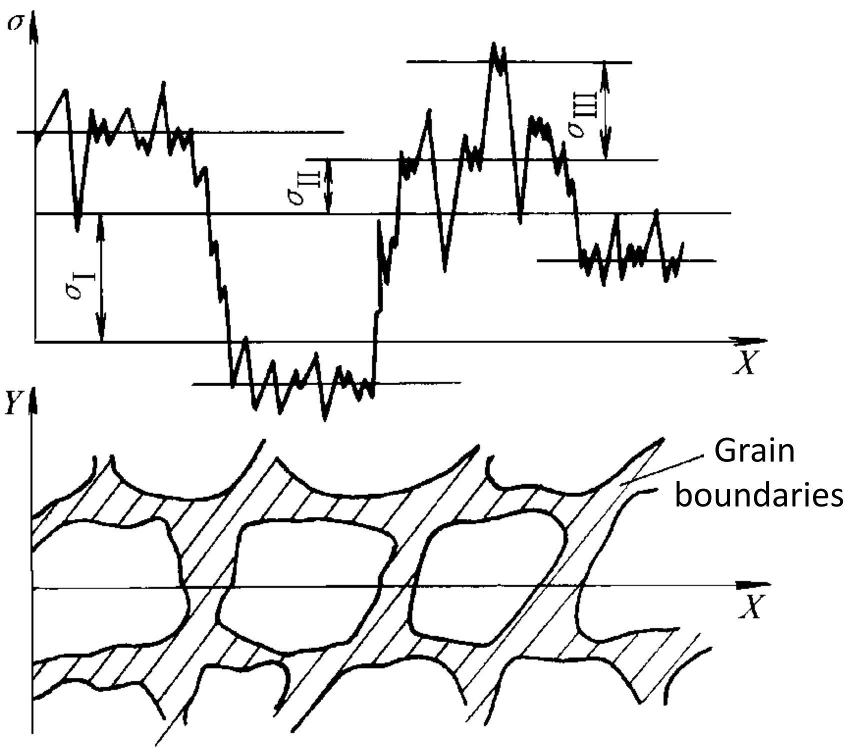
Type I internal stress (σ_{I}) refers to the internal stress that exists and is balanced within the macroscopic volume of the object. When it is released, the macroscopic volume or shape of the object will change.

Type II internal stress (σ_{II}) refers to the internal stress that exists and is balanced within the scope of several grains. Dimensional changes also occur when this balance is disrupted.

Type III internal stress (σ_{III}) refers to the internal stress that exists and is balanced within the range of several atoms. Such as various crystal defects (vacancies, interstitial atoms, dislocations, etc.), no size changes will occur when this balance is destroyed.

6.1 Generation and classification of residual stress within objects

- Distribution of internal stress



- Type I internal stress is the average value of the internal stress existing in each grain within the range of many grains, which is the result of uncoordinated macroscopic deformation of a larger volume.

- Type II internal stress is the average stress within the grain size range, which is the result of uncoordinated deformation between individual grains or grain regions.

- Type III internal stress is the fluctuation of the local internal stress within the grain relative to the value of Type II internal stress, which is related to the strain field formed by crystal defects.

6.1 Generation and classification of residual stress within objects

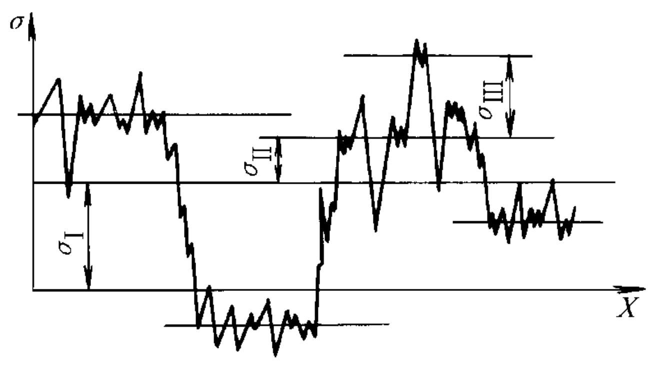
- Diffraction effect of internal stress

Type I internal stress is also called macroscopic stress or residual stress. Its diffraction effect causes the diffraction line to shift.

Type II internal stress is also called microscopic stress. Its diffraction effect mainly causes changes in the shape of diffraction lines.

Type III internal stress is also called lattice distortion stress or ultra-microscopic stress. Its diffraction effect reduces the diffraction intensity.

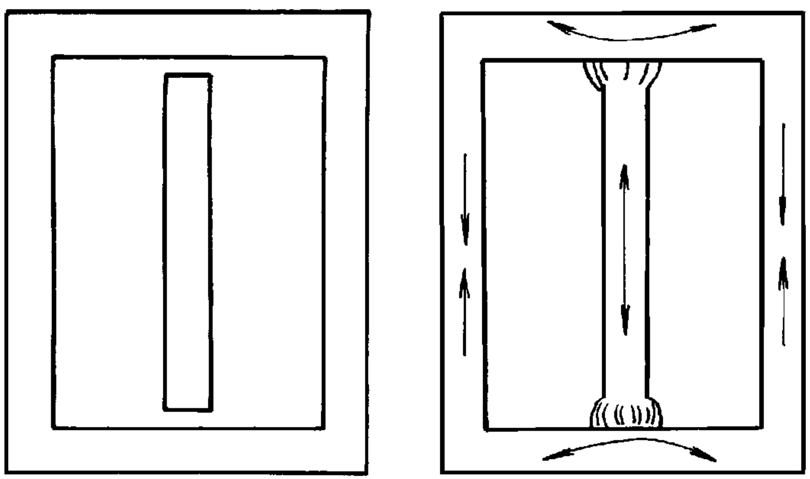
Type II internal stress is a very important intermediate connection, through which Type I internal stress and Type III internal stress can be connected to form a complete internal stress system.



6.1 Generation and classification of residual stress within objects

- Generation of internal stress

1. Macroscopic stress



a) Before welding b) After welding

Figure is an example of generating macroscopic stress. The frame and the middle beam have no stress before welding.

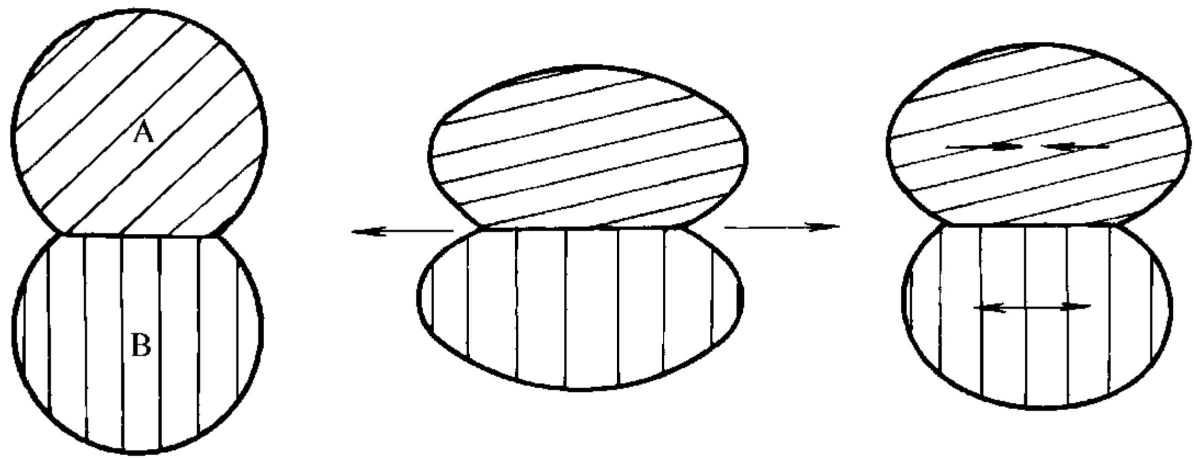
After the two ends of the beam are welded to the frame, the middle beam is subject to tensile stress, the frames on both sides are subject to compressive stress, and the upper and lower beams are subject to bending stress. It can be seen that residual stress is a balanced and evenly distributed stress within the macroscopic area inside the material.

6.1 Generation and classification of residual stress within objects

- Generation of internal stress

2. Microscopic stress

The figure schematically illustrates the generation of type II internal stress. Under the uniaxial tensile load, since the A grain is in the easy-slip orientation, when the load exceeds the critical shear stress, plastic deformation will occur; while the B grain only undergoes elastic deformation. After the load is removed, the deformation of grain B will be restored, while grain A will only partially recover, causing grain B to be under tensile stress and grain A to be under compressive stress, forming a mutually balanced stress between grains.



6.1 Generation and classification of residual stress within objects

- Detection of internal stress

Residual stress is a kind of elastic stress, which is closely related to the fatigue performance, stress corrosion resistance and dimensional stability of components. Residual stress detection is of great significance for process control, failure analysis, etc. The main methods are:

- The stress relaxation method is to relax the stress by drilling, grooving or thin layers, and use resistance strain gauges to measure the deformation to calculate the residual stress. It is a destructive test.

- Non-destructive methods use stress-sensitive methods, such as ultrasound, magnetism, neutron diffraction, X-ray diffraction, etc.

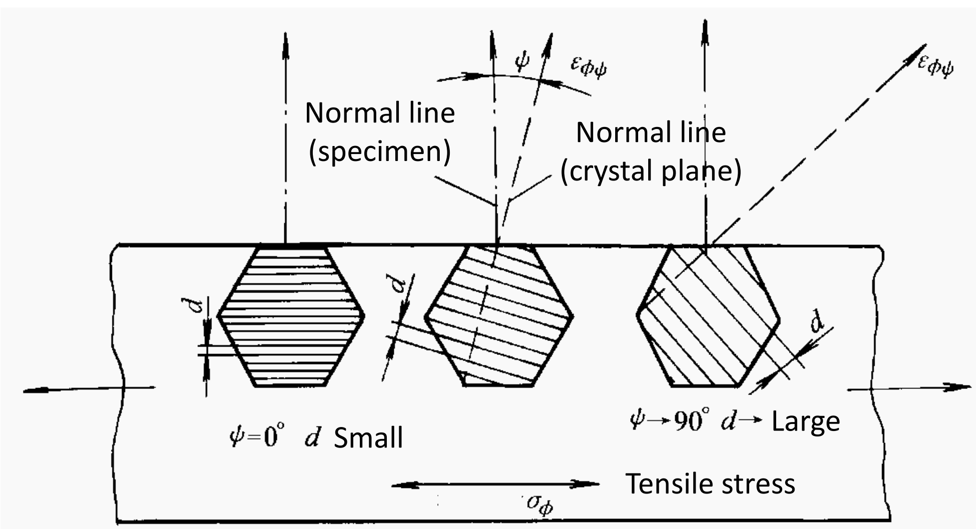
- X-ray diffraction method is a non-destructive method. It has the advantages of fast, accurate and reliable, small measurement area, etc., and can distinguish and measure three different categories of internal stress.

6.2 Basic principles of X-ray residual stress measurement

- Principle

Use X-ray diffraction method to measure residual stress.

First, measure the strain, and then determine the stress with the help of elastic characteristic parameters of the material.

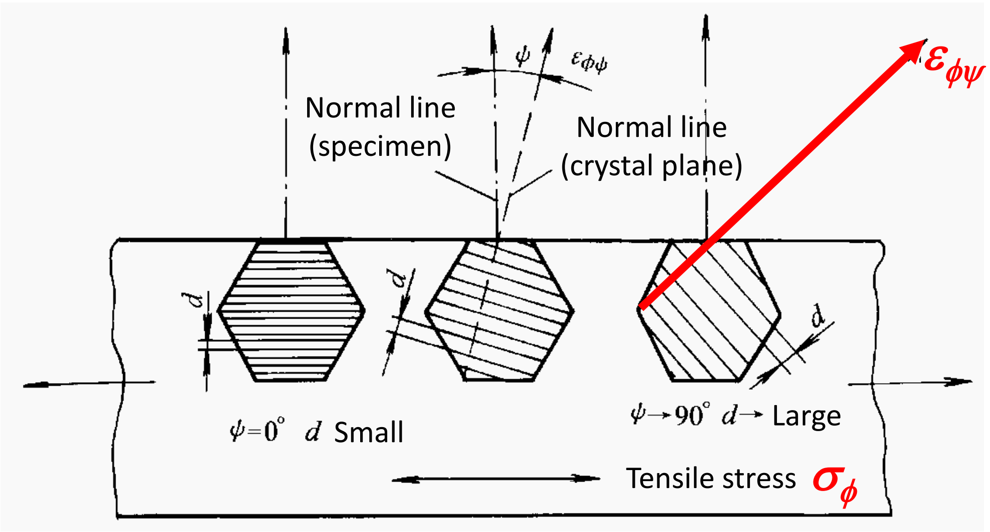


For an ideal polycrystal, in a stress-free state, the orientation and stress magnitude of homogeneous crystal planes in different orientations change regularly, as shown in the figure.

As the angle ψ between the normal line of the crystal plane and the normal line of the sample surface increases, the interplanar spacing ***d*** increases.

6.2 Basic principles of X-ray residual stress measurement

- Principle



Along the $\varepsilon_{\phi\psi}$ direction, the change of a certain crystal plane spacing $d_{\phi\psi}$ relative to the state without stress $d_{\phi\psi}$- d_0)/d_0= $\Delta d/d_0$ reflects the elastic strain $\varepsilon_{\phi\psi} = \Delta d/d_0$ at normal direction of the crystal plane caused by stress.

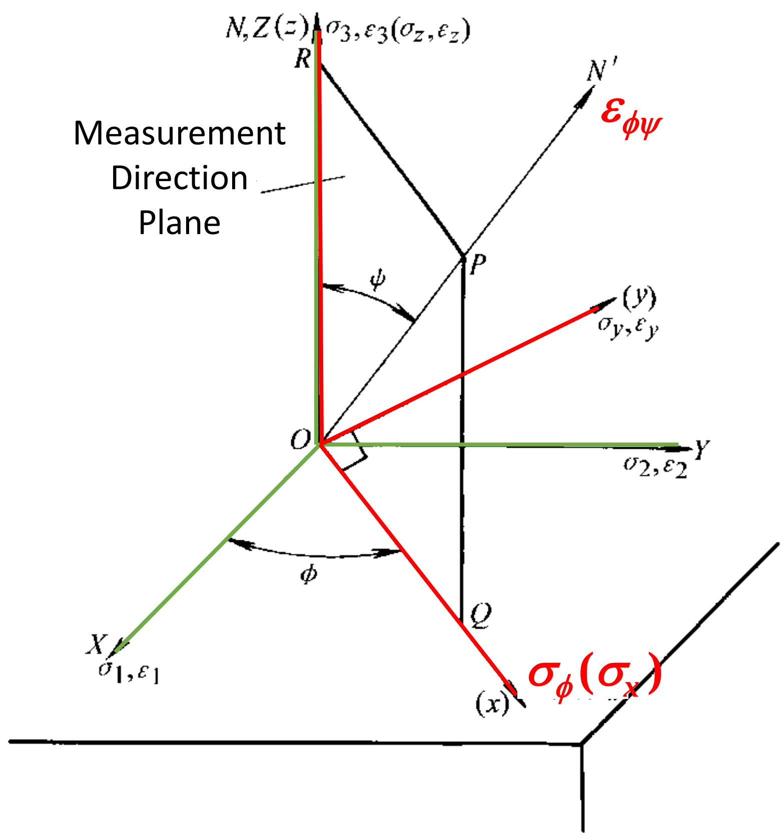
There is a certain functional relationship between the rate of change of interplanar spacing with orientation and the applied stress.

Therefore, establishing the relationship between the residual stress to be measured σ_ϕ and the strain $\varepsilon_{\phi\psi}$ in a certain orientation in space is the key to solving the problem of stress measurement.

6.2 Basic principles of X-ray residual stress measurement

- Principle

$$\sigma_\phi(\sigma_x) \longleftrightarrow \varepsilon_{\phi\psi}$$



O-XYZ is the principal stress coordinate system. (σ_1, σ_2, σ_3) are three principal stress. (ε_1, ε_2, ε_3) are three principal strain.

O-xyz is the direction of a stress to be measured $\sigma_\phi(\sigma_x)$ and the direction of σ_y and σ_z.

σ_3 and σ_z are parallel to the specimen normal direction "***ON***".

ϕ is the angle between σ_ϕ and σ_1.

The plane determined by ***ON*** and σ_ϕ is called the measurement direction plane.

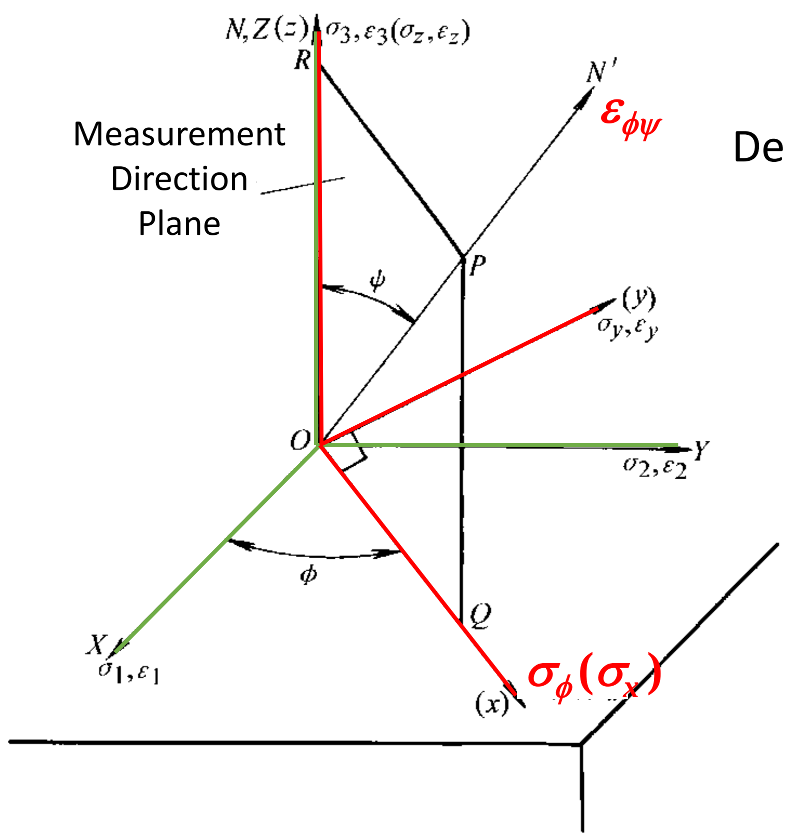
$\varepsilon_{\phi\psi}$ is the strain in a certain direction on this plane, and the angle between it and ***ON*** is called the azimuth angle ψ.

6.2 Basic principles of X-ray residual stress measurement

$\sigma_\phi(\sigma_x) \longleftrightarrow \varepsilon_{\phi\psi}$

- Stress determination equation

According to the principle of elasticity, for a continuous, homogeneous, isotropic object, under the plane stress state, $\sigma_z = 0$, $\varepsilon_z = \varepsilon_3$, according to the coordinate system shown in Figure, any The strain in the ***ON*** direction is:



$$\varepsilon_{\phi\psi} = \frac{1+\nu}{E}\sigma_\phi \sin^2\psi + \varepsilon_3$$

Derivative $\varepsilon_{\phi\psi}$ with respect to $\sin^2\psi$

$$\frac{\partial \varepsilon_{\phi\psi}}{\partial \sin^2\psi} = \frac{1+\nu}{E}\sigma_\phi$$

$$\sigma_\phi = \frac{E}{1+\nu}\frac{\partial \varepsilon_{\phi\psi}}{\partial \sin^2\psi}$$

E is elastic modulus, ν is Poisson's ratio.

6.2 Basic principles of X-ray residual stress measurement

- Principle of uniaxial stress measurement

- In an ideal polycrystalline material, the grains are moderately uniform in size and arbitrarily oriented. When there is no stress, the distance d_0 between the same (***HKL***) crystal planes of each grain remains unchanged.

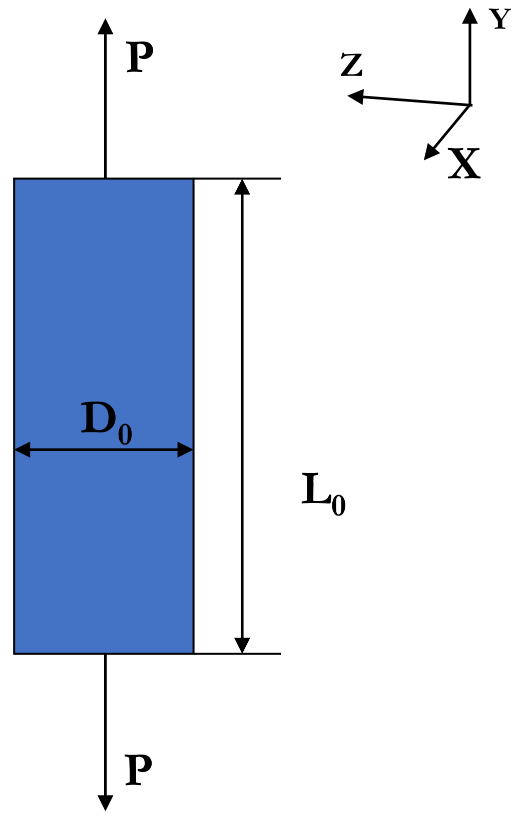
- When subjected to stress, the spacing between each crystal plane changes due to the angle between the plane and the stress axis and the magnitude of the stress.

6.2 Basic principles of X-ray residual stress measurement

- Principle of uniaxial stress measurement

The stress on the specimen along the Y direction:

$$\sigma_Y = \frac{P}{S} \qquad \varepsilon_Y = \frac{L - L_0}{L_0}$$



In the elastic range, the relationship between stress and strain:

$$\sigma_Y = E \cdot \varepsilon_Y = E \cdot \frac{L - L_0}{L_0}$$

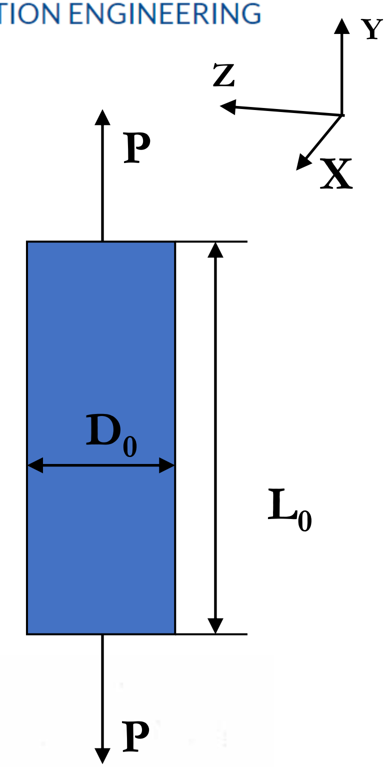
The strain in the Y direction can be expressed as the strain in the Z direction or the X direction:

$$-\nu \cdot \varepsilon_Y = \varepsilon_X = \varepsilon_Z = \frac{D - D_o}{D_0}$$

E is elastic modulus, ν is Poisson's ratio.

6.2 Basic principles of X-ray residual stress measurement

- Principle of uniaxial stress measurement



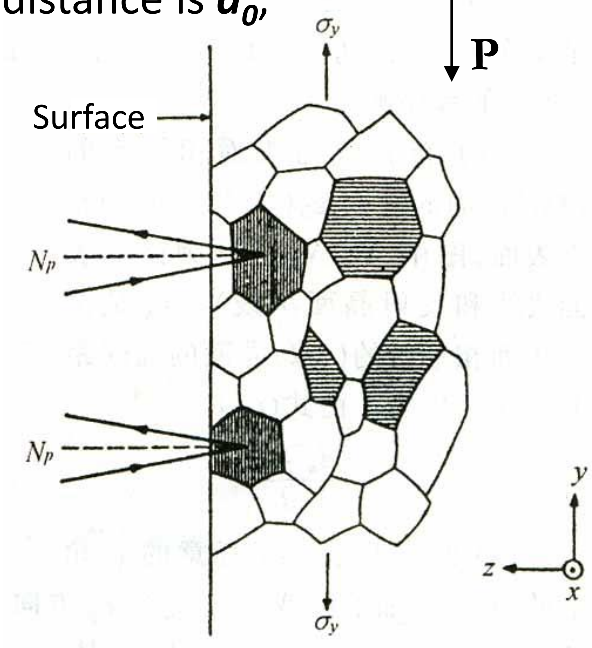
$$-\nu \cdot \varepsilon_Y = \varepsilon_X = \varepsilon_Z = \frac{D-D_0}{D_0}$$

$$\sigma_Y = E \cdot \varepsilon_Y = -E \cdot \frac{\varepsilon_Z}{\nu} = -\frac{E}{\nu} \cdot \frac{D-D_0}{D_0}$$

Assume that a diffraction surface (***HKL***) in some grains in the sample is basically parallel to the Y direction, and the inter-plane distance is d_0, then there is:

$$\varepsilon_Z = \frac{d-d_0}{d_0}$$

$$\sigma_Y = -\frac{E}{\nu} \cdot \frac{d-d_0}{d_0}$$

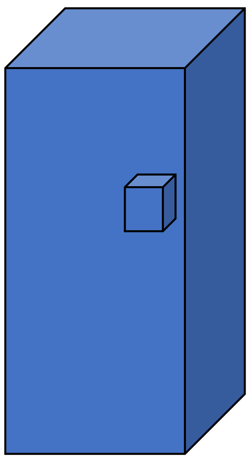


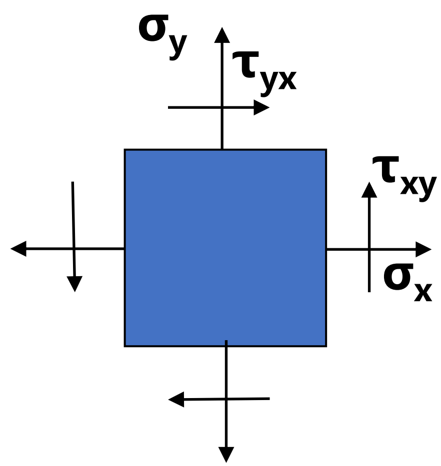
d: distance under stress

d_0: distance without stress

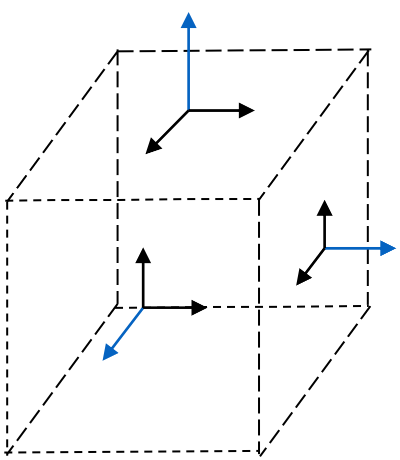
6.2 Basic principles of X-ray residual stress measurement

- Principle of uniaxial stress measurement





- Materials are usually in a triaxial stress state. X-rays only irradiate the surface layer with a depth of about 20 μm, so the X-ray method measures the two-dimensional plane stress on the surface.



Normal stress： σ_x、σ_y、σ_z

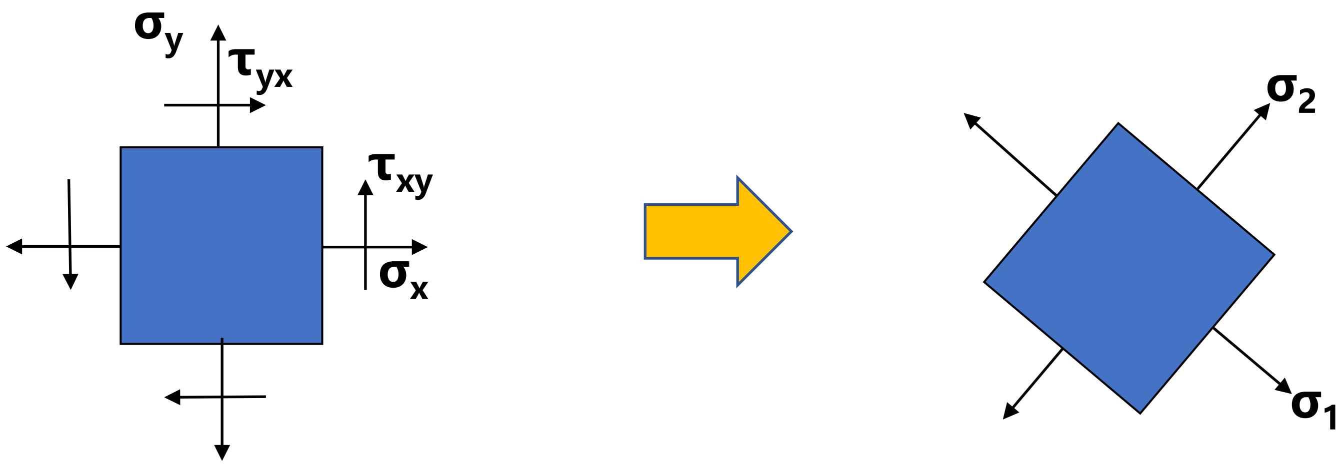
Shear stress： τ_{xy}、τ_{xz}、τ_{yx}、τ_{yz}、τ_{zx}、τ_{zy}

- According to elasticity, a unit body can be selected in a force-bearing object, and the stress can be decomposed into normal stress and shear stress in all directions of the unit body.

6.2 Basic principles of X-ray residual stress measurement

- Principle of uniaxial stress measurement

By appropriately adjusting the direction of the unit body, a suitable orientation can always be found so that the shear stress on each plane of the unit body is zero, and there are only three mutually perpendicular principal stresses σ_1, σ_2, and σ_3.



6.2 Basic principles of X-ray residual stress measurement

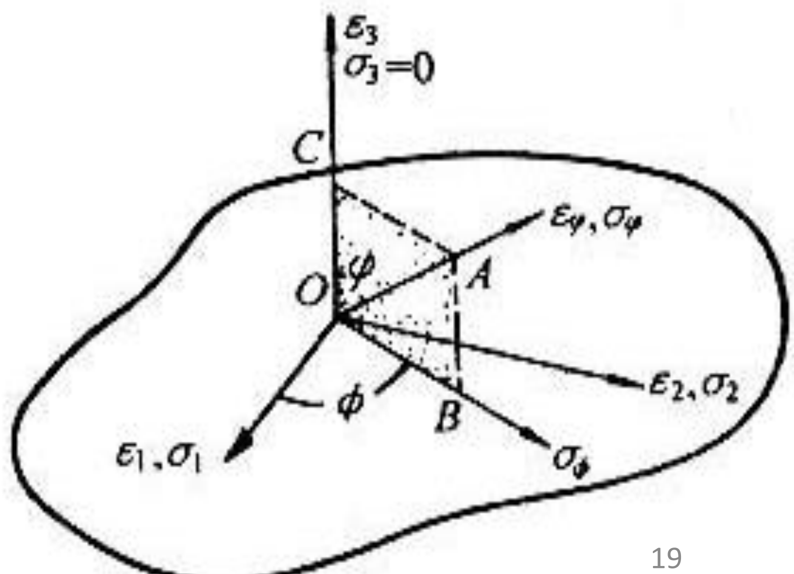
- Principle of residual stress measurement

For plane stress, there are only two principal stresses σ_1 and σ_2 parallel to the specimen surface, and the surface principal stress $\sigma_3 = 0$ perpendicular to the surface. However, the principal strain on the surface $\varepsilon_3 \neq 0$, for isotropic materials, is:

$$\varepsilon_3 = -\nu(\varepsilon_1 + \varepsilon_2) = -\nu(\sigma_1 + \sigma_2)/E$$

ε_3 can be measured by the change in the ***d*** value of the crystal plane spacing parallel to the sample surface, that is:

$$\varepsilon_3 = (d_n - d_0)/d_0 = -\nu(\sigma_1 + \sigma_2)/E$$



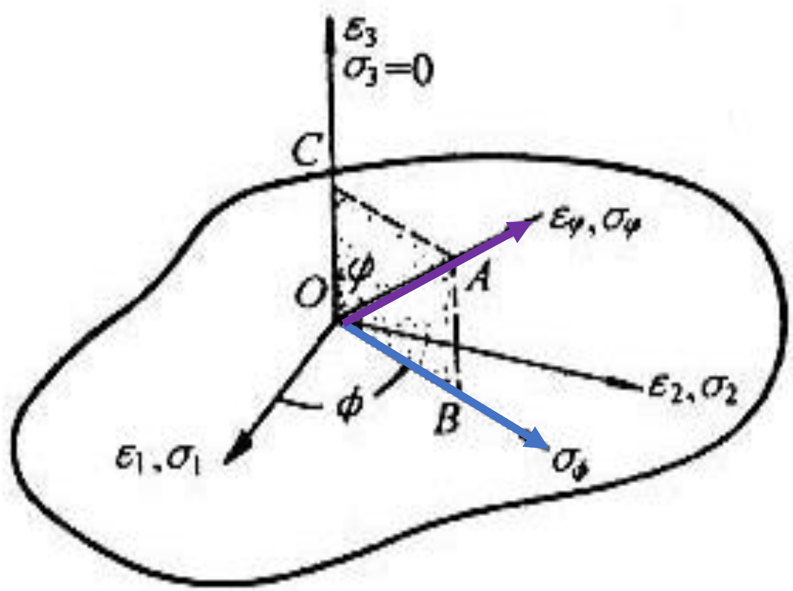
6.2 Basic principles of X-ray residual stress measurement

- Principle of residual stress measurement

The sum of the two principal stresses in the plane can be measured ($\sigma_1 + \sigma_2$).

However, in engineering practice, stress in a certain direction on the plane is required, such as the stress σ_ϕ in the OB direction with an angle ϕ with σ_1 in the figure.

It is necessary to first consider the relationship between the stress σ_ψ, strain ε_ψ and the principal stress and principal strain in any direction ψ in space.



6.2 Basic principles of X-ray residual stress measurement

- Principle of residual stress measurement

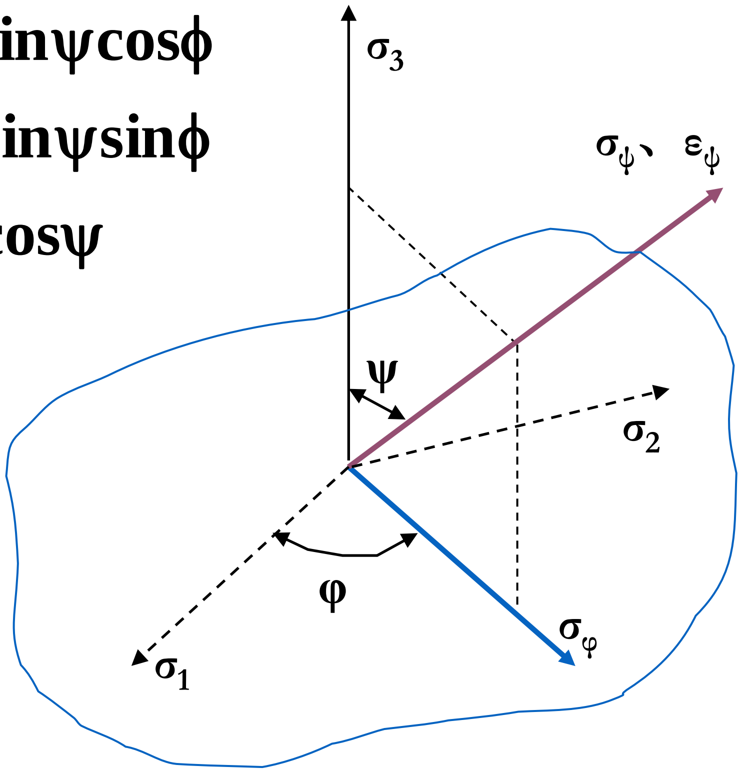
The relationship between strain and principal strain in any direction ψ in space is:

$$\varepsilon_\psi = \alpha_1^2\varepsilon_1 + \alpha_2^2\varepsilon_2 + \alpha_3^2\varepsilon_3$$

$$\alpha_1 = \sin\psi\cos\phi$$

$$\alpha_2 = \sin\psi\sin\phi$$

$$\alpha_3 = \cos\psi$$



The relationship between stress and principal stress in any direction ψ in space is:

$$\sigma_\psi = \alpha_1^2\sigma_1 + \alpha_2^2\sigma_2 + \alpha_3^2\sigma_3$$

6.2 Basic principles of X-ray residual stress measurement

- Principle of residual stress measurement

$$\varepsilon_\psi = \alpha_1^2\varepsilon_1 + \alpha_2^2\varepsilon_2 + \alpha_3^2\varepsilon_3$$

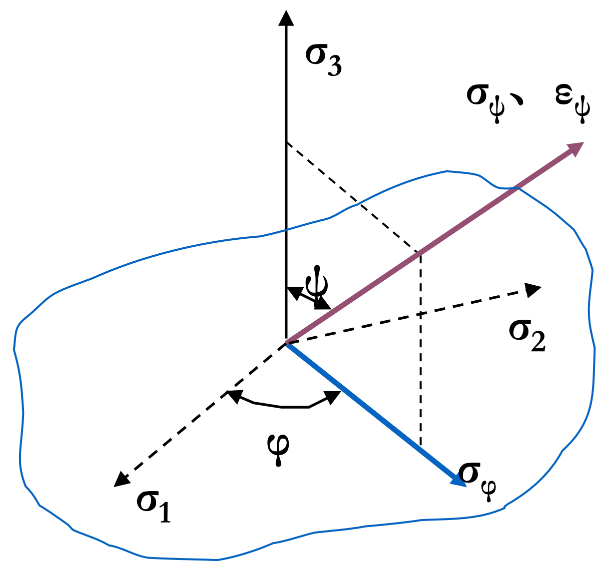
$$\sigma_\psi = \alpha_1^2\sigma_1 + \alpha_2^2\sigma_2 + \alpha_3^2\sigma_3$$

Within the elastic range, the relationship between principal stress and principal strain satisfies the generalized Hooke's theorem:

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)]$$

$$\varepsilon_2 = \frac{1}{E}[\sigma_2 - \nu(\sigma_3 + \sigma_1)]$$

$$\varepsilon_3 = \frac{1}{E}[\sigma_3 - \nu(\sigma_1 + \sigma_2)]$$



When $\sigma_3 = 0$, then:

$$\varepsilon_\psi = \frac{1+\nu}{E}\sin^2\psi(\sigma_1\cos^2\phi + \sigma_2\sin^2\phi) + \varepsilon_3$$

6.2 Basic principles of X-ray residual stress measurement

- Principle of residual stress measurement

$$\varepsilon_\psi - \varepsilon_3 = \frac{1+\nu}{E}\sin^2\psi(\sigma_1\cos^2\phi + \sigma_2\sin^2\phi)$$

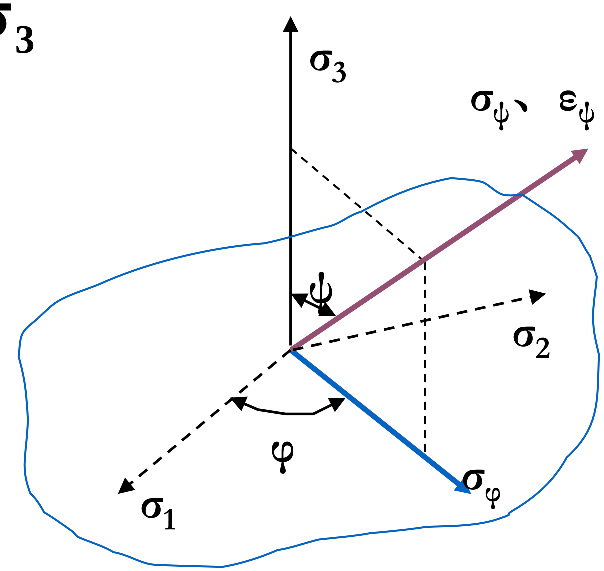
Substitute α_1, α_2, and α_3 into the following formula to get:

$$\alpha_1 = \sin\psi\cos\phi$$
$$\alpha_2 = \sin\psi\sin\phi \longrightarrow \sigma_\psi = \alpha_1^2\sigma_1 + \alpha_2^2\sigma_2 + \alpha_3^2\sigma_3$$
$$\alpha_3 = \cos\psi$$

$$\sigma_\psi = \sin^2\psi(\sigma_1\cos^2\phi + \sigma_2\sin^2\phi)$$

When $\psi = 90°$, $\sigma_\psi = \sigma_\phi$

$$\sigma_{\psi=90°} = \sigma_\phi = \sigma_1\cos^2\phi + \sigma_2\sin^2\phi$$



6.2 Basic principles of X-ray residual stress measurement

- Principle of residual stress measurement

$$\varepsilon_\psi - \varepsilon_3 = \frac{1+\nu}{E}\sin^2\psi(\sigma_1\cos^2\phi + \sigma_2\sin^2\phi) \leftarrow \sigma_\phi = \sigma_1\cos^2\phi + \sigma_2\sin^2\phi$$

$$\varepsilon_\psi - \varepsilon_3 = \frac{1+\nu}{E}\sin^2\psi\,\sigma_\phi$$

$$\varepsilon_\psi = \frac{d_\psi - d_0}{d_0} \qquad \varepsilon_3 = \frac{d_3 - d_0}{d_0}$$

$$\downarrow$$

$$\varepsilon_\psi - \varepsilon_3 = \frac{d_\psi - d_3}{d_0}$$

$$\sigma_\phi = \frac{E}{1+\nu}\frac{\varepsilon_\psi - \varepsilon_3}{\sin^2\psi}$$

$$\downarrow$$

$$\sigma_\phi = \frac{E}{1+\nu}\frac{1}{\sin^2\psi}\frac{d_\psi - d_3}{d_0}$$

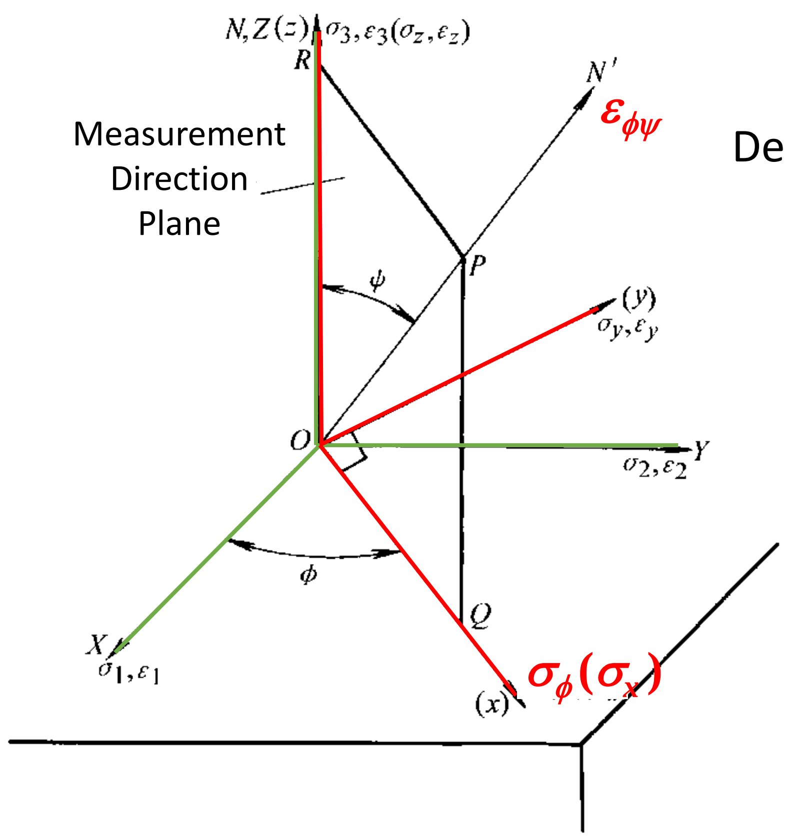
d_0 ： Diffraction surface spacing without stress;
d_ψ ： Diffraction surface spacing perpendicular to the selected direction;
d_3 ： The distance between the diffraction surfaces parallel to the specimen surface.

6.2 Basic principles of X-ray residual stress measurement

$\sigma_\phi(\sigma_x) \longleftrightarrow \varepsilon_{\phi\psi}$

- Stress determination equation

According to the principle of elasticity, for a continuous, homogeneous, isotropic object, under the plane stress state, $\sigma_z = 0$, $\varepsilon_z = \varepsilon_3$, according to the coordinate system shown in Figure, any The strain in the ***ON*** direction is:



$$\varepsilon_{\phi\psi} = \frac{1+\nu}{E}\sigma_\phi \sin^2\psi + \varepsilon_3$$

Derivative $\varepsilon_{\phi\psi}$ with respect to $\sin^2\psi$

$$\frac{\partial \varepsilon_{\phi\psi}}{\partial \sin^2\psi} = \frac{1+\nu}{E}\sigma_\phi$$

$$\sigma_\phi = \frac{E}{1+\nu}\frac{\partial \varepsilon_{\phi\psi}}{\partial \sin^2\psi}$$

E is elastic modulus, ν is Poisson’s ratio.

6.2 Basic principles of X-ray residual stress measurement

- Stress constant K

Differential form of Bragg's equation: $\Delta d/d = -\Delta\theta \cot\theta_0$

When λ is a constant, $\theta \approx \theta_0$ is the diffraction angle without stress: $\Delta\theta = (2\theta_{\phi\psi} - 2\theta_0)/2$

Then, we have $\varepsilon_{\phi\psi} = -(2\theta_{\phi\psi} - 2\theta_0)\cot\theta_0/2$

$$\sigma_\phi = \frac{E}{1+\nu}\frac{\partial \varepsilon_{\phi\psi}}{\partial \sin^2\psi}$$

$$\sigma_\phi = -\frac{E}{2(1+\nu)}\cot\theta_0 \frac{\partial 2\theta_{\phi\psi}}{\partial \sin^2\psi}$$

Unit changes from "radians" to "degrees".

$$\sigma_\phi = -\frac{E}{2(1+\nu)}\cot\theta_0 \frac{\pi}{180^\circ}\frac{\Delta 2\theta_{\phi\psi}}{\Delta \sin^2\psi}$$

6.2 Basic principles of X-ray residual stress measurement

- Stress constant K

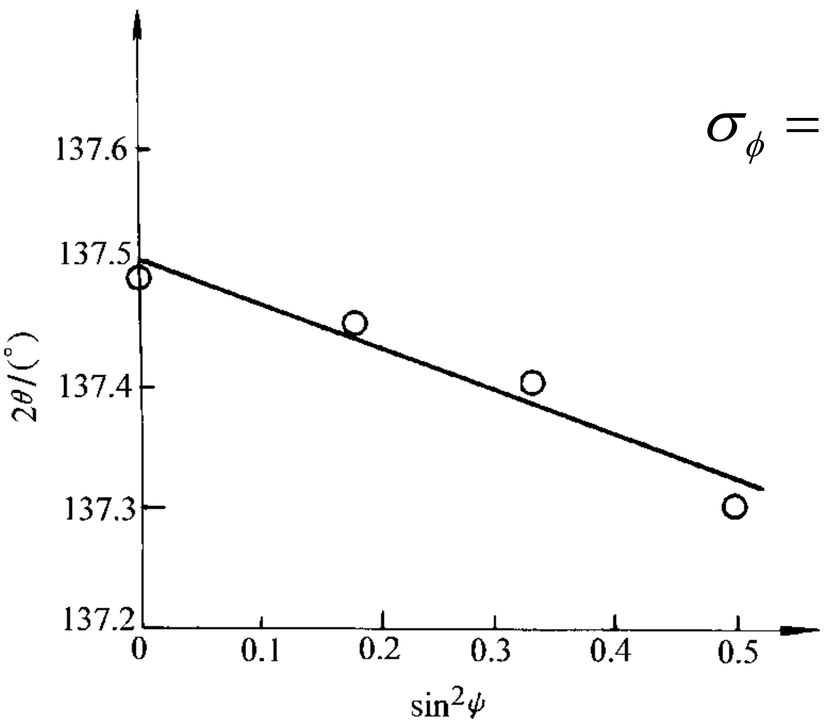
$$\sigma_\phi = -\frac{E}{2(1+\nu)}\cot\theta_0 \frac{\pi}{180^\circ}\frac{\Delta 2\theta_{\phi\psi}}{\Delta \sin^2\psi}$$

The above formula shows that in the plane stress state, $2\theta_{\phi\psi}$ has a linear relationship with $\sin^2\psi$, as shown in the figure.

$$K = -\frac{E}{2(1+\nu)}\cot\theta_0 \frac{\pi}{180^\circ}$$

$$\sigma_\phi = KM$$

$$M = \frac{\Delta 2\theta_{\phi\psi}}{\Delta \sin^2\psi}$$



K is called the stress constant, which is determined by the elastic properties of the material and the diffraction angle of the diffraction crystal plane (determined by the crystal plane spacing ***d*** and the wavelength ***λ***).

6.2 Basic principles of X-ray residual stress measurement

- Discussion

- X-rays are incident from different ψ angles, and their respective 2θψ angles are measured respectively. The changes in the 2θψ angle reflect the changes in the interplanar spacing at different orientations with the sample surface.

- Stress is proportional to the slope of the 2θψ ~ sin2ψ relationship curve. You only need to obtain the 2θψ ~ sin2ψ relationship curve, and then find its slope (***M***) to calculate the stress σϕ.

- If the material of the sample to be tested is uniform, continuous and isotropic, the relationship curve between 2θψ ~ sin2ψ will generally be a straight line.

6.2 Basic principles of X-ray residual stress measurement

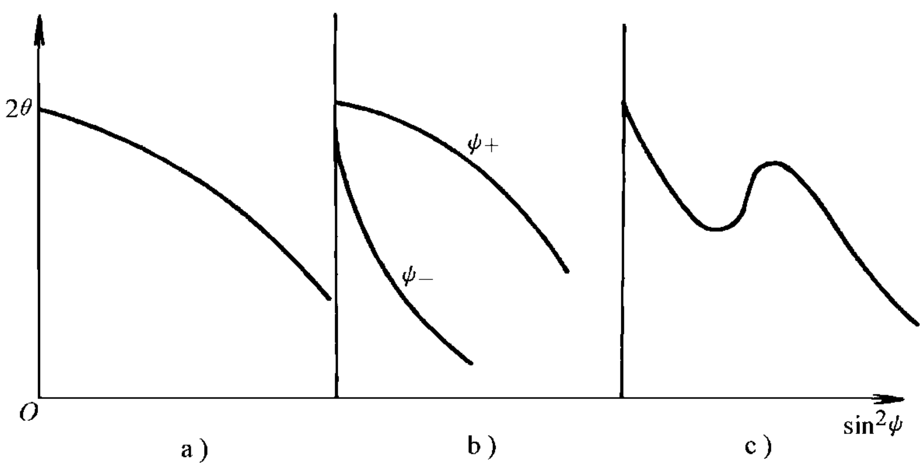
- Stress constant K

$$M = \frac{\Delta 2\theta_{\phi\psi}}{\Delta \sin^2\psi}$$

M is the slope of the $2\theta_{\phi\psi}$ - $\sin^2\psi$ line.

Since ***K*** is a negative value, if ***M*** > 0, the stress is negative, that is, compressive stress; when ***M*** < 0, the stress is positive, that is, tensile stress.

If the $2\theta_{\phi\psi}$ - $\sin^2\psi$ relationship loses linearity, it means that the material deviates from the plane stress state. The effects of the three non-plane stress states are shown in the figure.



Stress gradient | 3D Stress | Texture

When there is a stress gradient, three-dimensional stress state or texture in the sample testing range, special methods need to be used to calculate the residual stress.

6.2 Basic principles of X-ray residual stress measurement

- Stress constant K

$$K = -\frac{E}{2(1+\nu)}\cot\theta_0 \frac{\pi}{180^\circ}$$

Materials	Lattice type	Parameters /Å	Source	{ *hkl* }	2θ/(°)	*K*/[MPa/(°)]
α-Fe	BCC	2.8664	CrK_α CoK_α	211 310	156.8 161.4	-318.1 -230.4
γ-Fe	FCC	3.656	CrK_β MnK_α	311 311	149.6 154.8	-355.35 -292.73
Al	FCC	4.049	CrK_α CoK_α	222 420	156.7 162.1	-92.12 -70.36
Cu	FCC	3.6153	CrK_β CoK_α	311 400	146.5 163.5	-245.0 -118.0
Ti	HCP	*a* 2.9504 *c* 4.6831	CoK_α CoK_α	114 211	154.2 142.2	-171.6 -256.7
Ni	FCC	3.5238	CrK_β CuK_α	311 420	157.7 155.6	-273.22 -289.39

6.3 Macroscopic stress measurement method

If you want to measure the residual stress $\boldsymbol{\sigma_\phi} = \boldsymbol{KM}$ in a certain direction on the surface of the sample, you need to follow the following steps:

- Measure at least two diffraction angles $\mathbf{2}\boldsymbol{\theta_{\phi\psi}}$ with different azimuths $\boldsymbol{\psi}$ in the measuring direction plane;
- Find the slope ***M*** of the $\mathbf{2}\boldsymbol{\theta_{\phi\psi}}$ - $\mathbf{sin^2}\boldsymbol{\psi}$ line;
- Get the stress constant K according to the test conditions;
- Substitute M and K into equation ($\boldsymbol{\sigma_\phi} = \boldsymbol{KM}$) to calculate the residual stress.

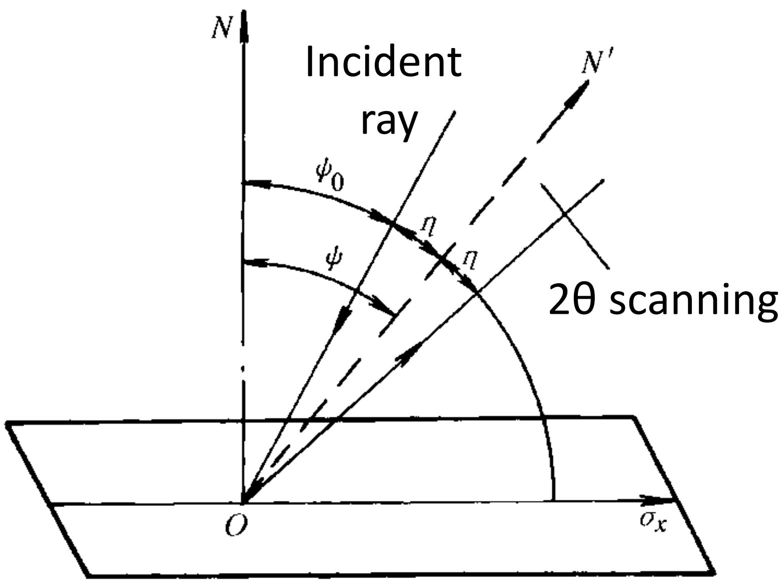
To determine and change the orientation of the diffraction crystal plane $\boldsymbol{\psi}$, it is necessary to use a certain diffraction geometry method. At present, residual stress is mostly measured on a diffractometer or stress meter. There are two commonly used diffraction geometric methods, the Iso-inclination method and the Sid-inclination method.

6.3 Macroscopic stress measurement method

- Iso-inclination method

The diffraction geometric feature of the co-tilt method is that the measurement direction plane and the scanning plane coincide, as shown in the figure.

The measurement direction plane is the plane where ***ON*** and σ_x are located; the scanning plane is the plane where the incident ray, the normal line of the diffraction crystal plane (***ON***', $\varepsilon_{\phi\psi}$ direction) and the diffraction line are located.



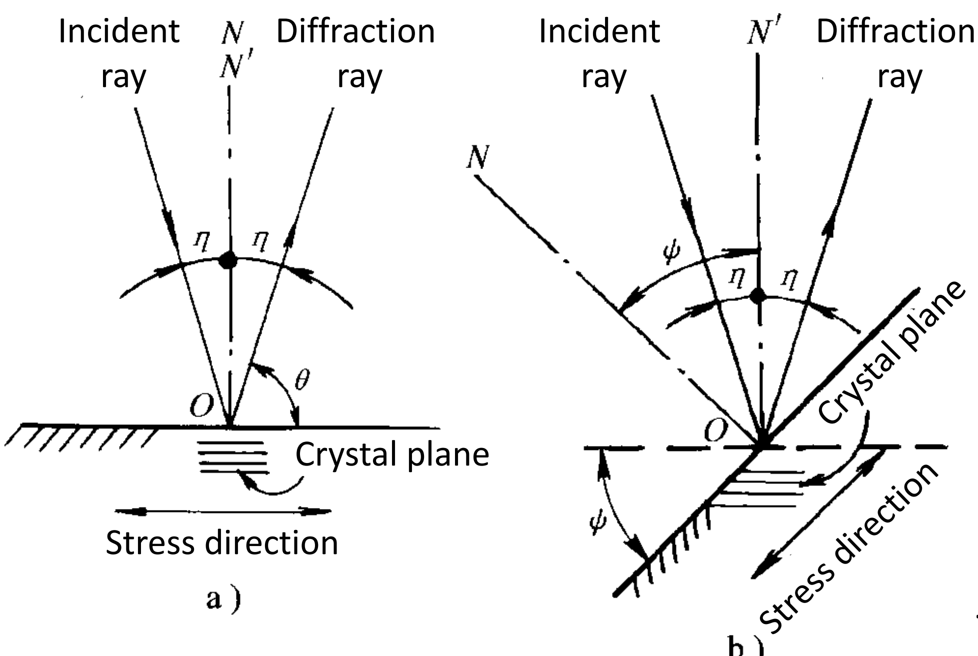
6.3 Macroscopic stress measurement method

- Iso-inclination method

1. fixed ψ method

$2\theta_{\phi\psi}$ - $\sin^2\psi$

When ***ON*** and ***ON'*** coincide, that is, $\psi = 0°$, the counting tube and the sample rotate at an angular speed of 2:1. At this time, the diffraction crystal plane is parallel to the sample surface, see Figure a).



a) $\psi = 0°$ b) $\psi = 45°$

The sample rotates around the axis of the diffractometer through an angle of ψ, and the angle between ***ON*** and ***ON'*** is ψ, see Figure b).

The method of directly determining and changing the orientation ψ of the diffraction surface by setting the diffraction geometric conditions is called the fixed ψ method.

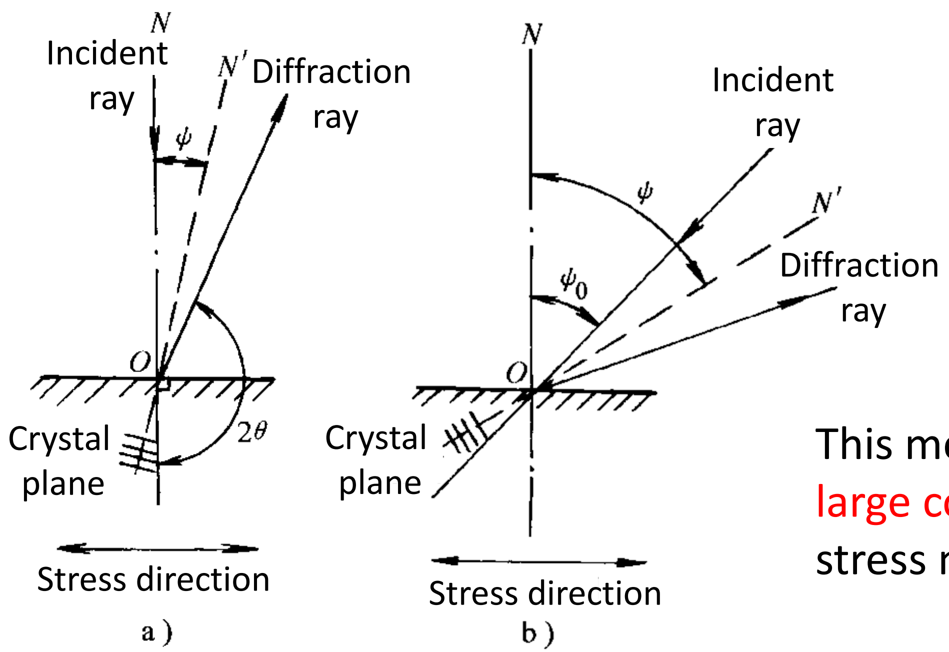
This method is suitable for measuring the macroscopic residual stress of smaller-sized samples on a diffractometer.

6.3 Macroscopic stress measurement method

- Iso-inclination method

2. fixed ψ_0 method

ψ_0 is the angle between the incident ray and the normal line ***ON*** of the specimen surface. With the ψ_0 method, the test sample is fixed and does not move, and different ψ orientations are obtained by changing the incident direction of X-rays, as shown in the figure.



a) $\psi_0 = 0°$ b) $\psi_0 = 45°$

According to the diffraction geometric conditions shown in the figure, calculate ψ from ψ_0 and θ:

$$\psi = \psi_0 + (90° - \theta)$$

This method is suitable for mechanical parts or large components, and is mostly used on special stress measuring instruments.

6.3 Macroscopic stress measurement method

- Iso-inclination method

3. Selection of crystal plane azimuth angle ψ

There are two ways to select the crystal plane azimuth angle using the Iso-inclination method (fixed ψ or ψ_0).

a. 0°- 45° method (two-point method) ψ or ψ_0 . Select 0° and 45° for measurement, and calculate the slope M of the $2\theta_{\phi\psi}$ - $\sin^2\psi$ line from the two data.

This method is suitable for situations where it is known that $2\theta_{\phi\psi}$ - $\sin^2\psi$ has a good linear relationship or the measurement accuracy is not high.

For fixed ψ 0°- 45° method: $\Delta\sin^2\psi = \sin^2 45° - \sin^2 0° = 0.5$

$$M = \frac{\Delta 2\theta_{\phi\psi}}{\Delta \sin^2 \psi}$$

Then the stress calculation formula is simplified to: $\sigma_\phi = 2K\Delta 2\theta_{\phi\psi}$

$$\sigma_\phi = KM$$

6.3 Macroscopic stress measurement method

- Iso-inclination method

3. Selection of crystal plane azimuth angle ψ

b. $\sin^2\psi$ method: There must be accidental errors in measurement ($2\theta_{\phi\psi}$), so the two-point method will affect the measurement accuracy. For this purpose, take several (n≥4) azimuth measurements, and then use the graph method or the least squares method to find the optimal slope M of the $2\theta_{\phi\psi}$ - $\sin^2\psi$ straight line. Then we can get the equation of the straight line.

$$2\theta_{\phi\psi i} = 2\theta_{\psi=0} + M\sin^2\psi_i$$

The slope M satisfies the minimum deviation. According to the principle of least squares, its M value is:

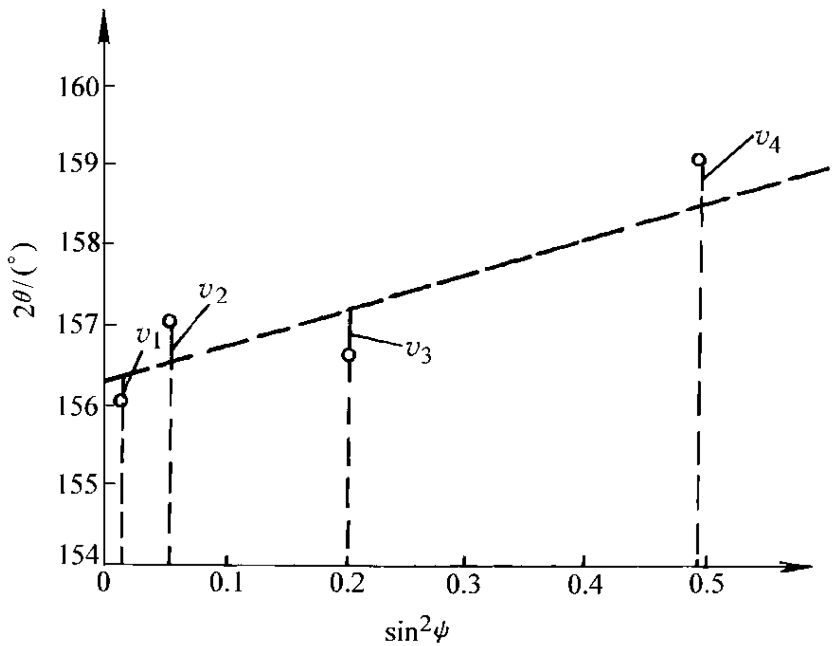
$$M = \frac{n\sum(2\theta'_{\phi\psi i}\sin^2\psi_i) - \sum\sin^2\psi_i\sum 2\theta'_{\phi\psi i}}{n\sum\sin^4\psi_i - (\sum\sin^2\psi_i)^2}$$

6.3 Macroscopic stress measurement method

- Iso-inclination method

3. Selection of crystal plane azimuth angle ψ

The four azimuth angles ψ_i and ψ_{0i} in the $\sin^2\psi$ method are selected as follows. The fixed ψ method ψ_i usually takes 0°, 25°, 35°, and 45°; the fixed ψ_0 method can estimate the appropriate value based on the value of θ_0 and ψ_{0i} .



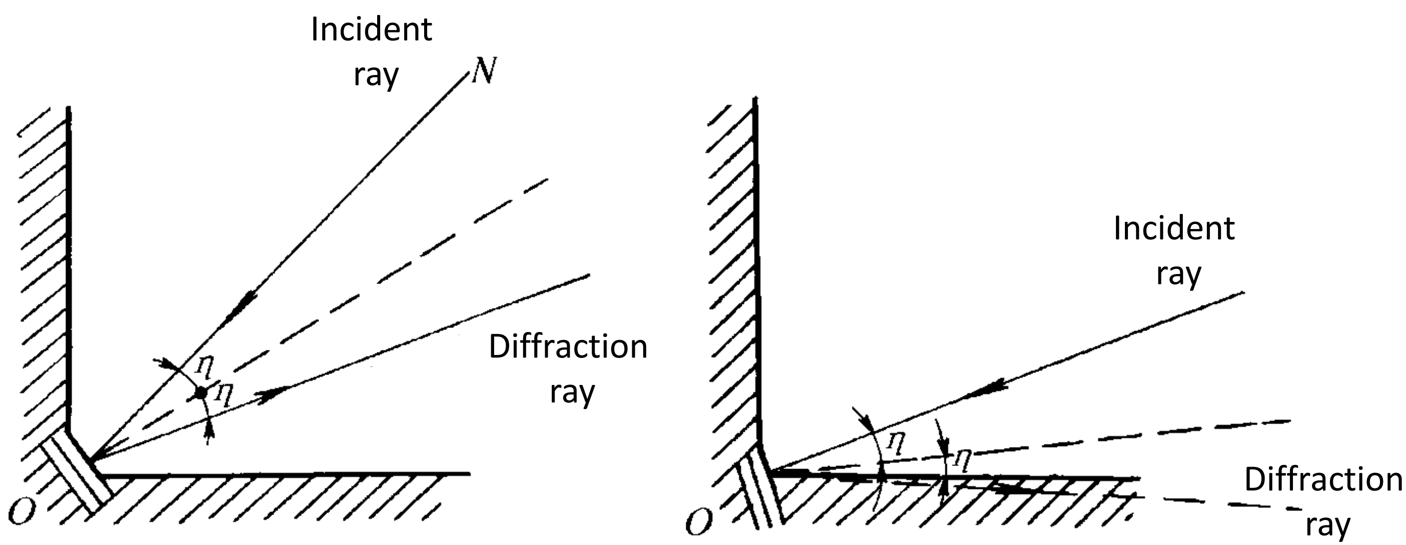
Using computers to process data can take more measurement points to improve the accuracy of M.

Determine the optimal slope of the line $2\theta_{\phi\psi}$ - $\sin^2\psi$

6.3 Macroscopic stress measurement method

- Sid-inclination method

Because measuring the full shape of the diffraction peak requires a certain scanning range, and the counting tube cannot receive the diffraction lines parallel to the sample surface. When the shape of the workpiece is complex, if it is necessary to measure the tangential stress at the corner, the change in azimuth angle will be limited by the shape of the artifact, as shown in the figure. This gave rise to the Sid-inclination method.



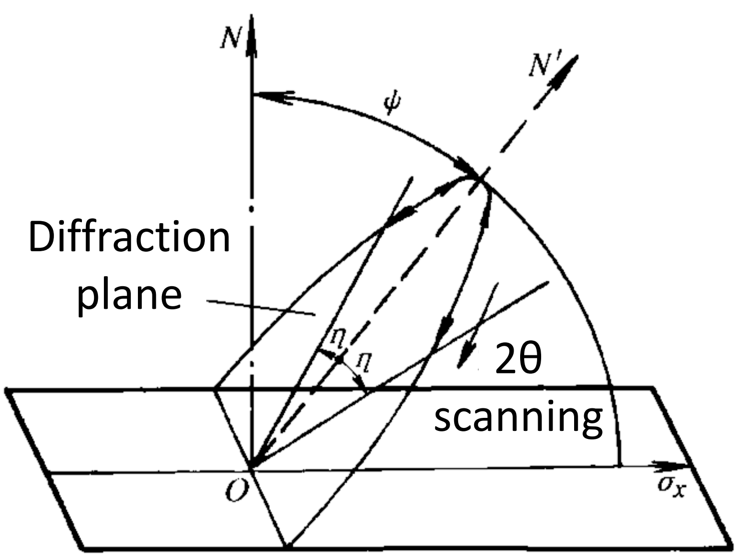
Stress measurement at artifact corners

6.3 Macroscopic stress measurement method

- Sid-inclination method

The Sid-inclination method has the following characteristics:

- The measurement direction plane of the Sid-inclination method is perpendicular to the scanning plane.
- The change of angle ψ is not limited by the diffraction angle, but only depends on the spatial shape of the test piece. For flat specimens, the variation range of ψ is theoretically close to 90°.



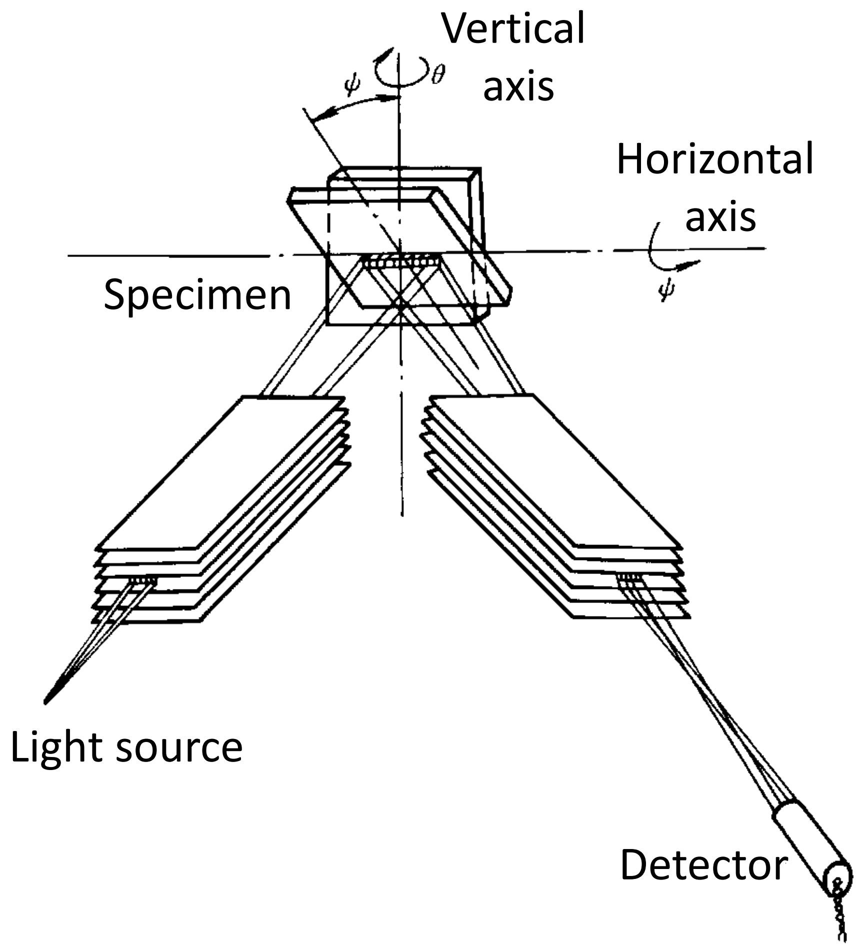
b)

- The method of determining ψ azimuth by the Sid-inclination method belongs to the fixed ψ method.
- The method of selecting the azimuth angle can still use the two-point method and $\sin^2\psi$.

The Sid-inclination method has the advantages of being able to measure the residual stress on the surface of complex-shaped artifacts with high measurement accuracy.

6.3 Macroscopic stress measurement method

- Sid-inclination method



As shown in the figure, the tilting device has two axes, and the specimen holder can rotate around the horizontal axis to achieve changes in azimuth angle ψ.

The sample holder and counting tube are scanned (θ - 2 θ) together around the vertical axis (diffractometer axis) to measure the diffraction angle.

6.3 Macroscopic stress measurement method

- Sid-inclination method

Example: The data for measuring the axial stress of the carbon/aluminum composite wire aluminum-coated layer using the $\sin^2\psi$ method of the side tilt method are listed in Table. Using CuK_α radiation, the aluminum {422} surface was measured.

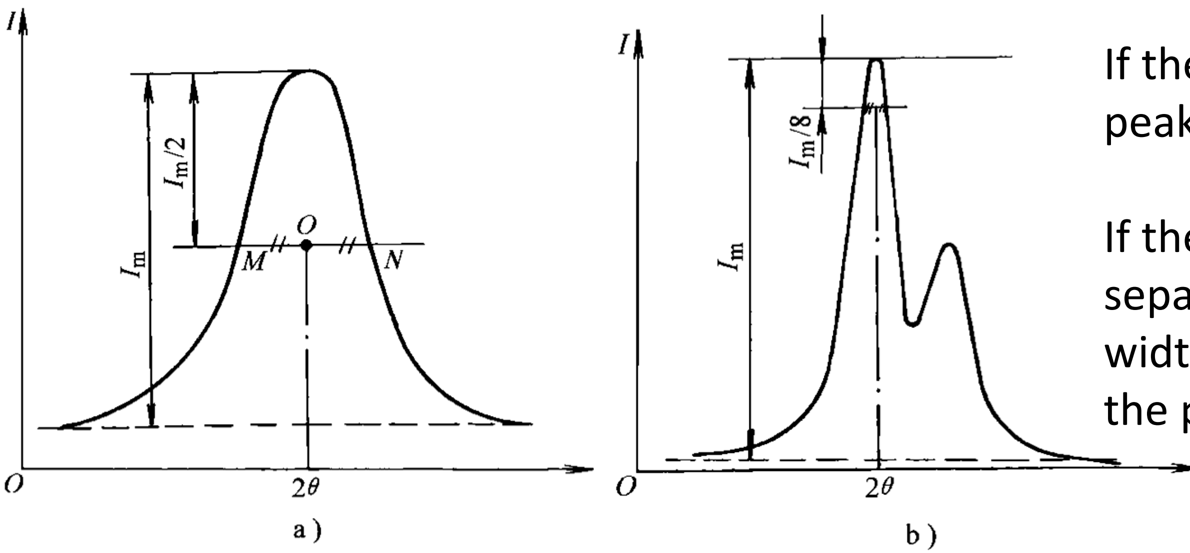
No	$\psi/(°)$	$\sin^2\psi$	$2\theta/(°)$	$2\theta\sin^2\psi/(°)$	$\sin^4\psi$
1	0	0	137.49	0	0
2	25	0.1786	137.45	24.5486	0.0319
3	35	0.3290	137.40	45.2046	0.1082
4	45	0.5	137.30	68.65	0.25
Σ		1.0076	549.64	138.4032	0.3901

$$M = \frac{n\sum(2\theta'_{\phi\psi i}\sin^2\psi_i) - \sum\sin^2\psi_i\sum 2\theta'_{\phi\psi i}}{n\sum\sin^4\psi_i - (\sum\sin^2\psi_i)^2} \longrightarrow \boldsymbol{M} = -0.3752°$$

6.4 Problems in X-ray Macroscopic Stress Measurement

The measurement accuracy of macroscopic stress depends on the accurate measurement of 2θ angle, and the change of 2θ in adjacent ψ orientations is only on the order of 0.1° or even 0.01°. The following peak fixing method can be used to accurately measure the peak position.

- Half height-width and 1/8 height-width method



If the $K_{\alpha1}$ and $K_{\alpha2}$ overlap, use the peak value at half maximum width.

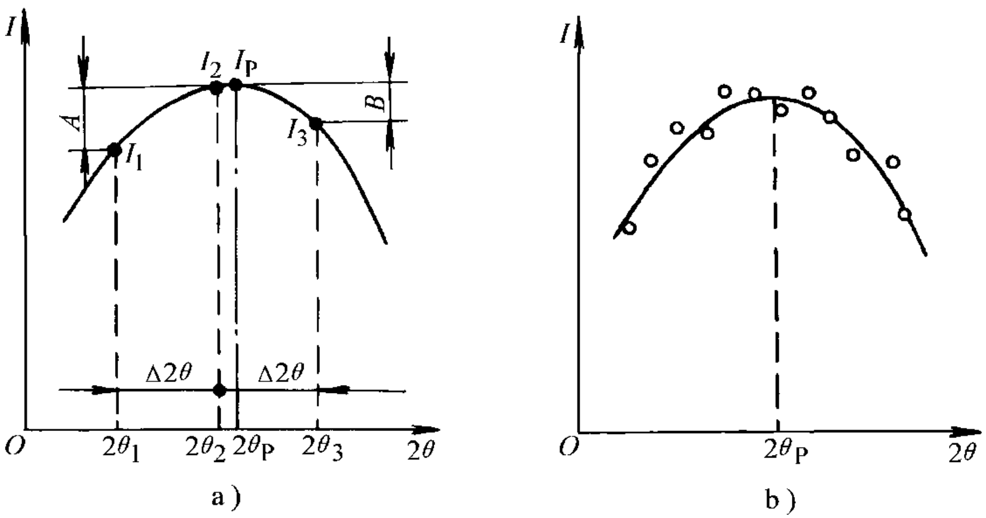
If the $K_{\alpha1}$ and $K_{\alpha2}$ lines are separated, use 1/8 the height and width of the $K_{\alpha1}$ line to determine the peak.

This method is suitable for situations where the peak shape is relatively sharp.

6.4 Problems in X-ray Macroscopic Stress Measurement

- Parabolic method

When the peak shape is diffuse, the half-maximum width method is likely to cause large errors, and a parabola can be used to determine the peak, as shown in Figure.



$$I = a_0 + a_1(2\theta) + a_2(2\theta)^2$$

a_0, a_1, a_2 are constants.

The diffraction angle $2\theta_P$ corresponding to the maximum intensity value I_P should satisfy:

$$dI/d(2\theta)=0$$

$$2\theta_P = -a_1/2a_2$$

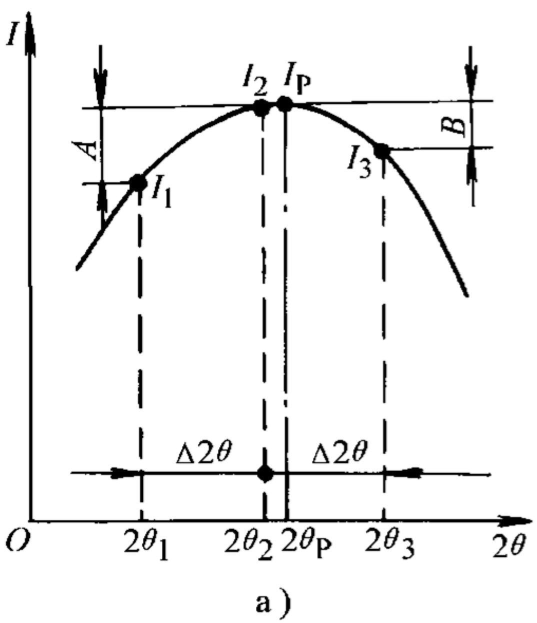
$2\theta_P$ is the peak position

6.4 Problems in X-ray Macroscopic Stress Measurement

- Parabolic method

1). Three-point parabola method

Pick three points at the top of the peak with an intensity greater than 85% I_P, and make the two $\Delta 2\theta$ equal, and substitute the test values I_1, I_2, I_3 and the corresponding 2θ into the formula $I = a_0 + a_1(2\theta) + a_2(2\theta)^2$



$$I_1 = a_0 + a_1(2\theta_1) + a_2(2\theta_1)^2$$

$$I_2 = a_0 + a_1(2\theta_2) + a_2(2\theta_2)^2$$

$$I_3 = a_0 + a_1(2\theta_3) + a_2(2\theta_3)^2$$

Solve for constants a_1, a_2, a_3

$$2\theta_P = 2\theta_1 + \Delta 2\theta \frac{2I_2 - 3I_1 + I_3}{2(I_3 - I_1)}$$

6.4 Problems in X-ray Macroscopic Stress Measurement

- Parabolic method

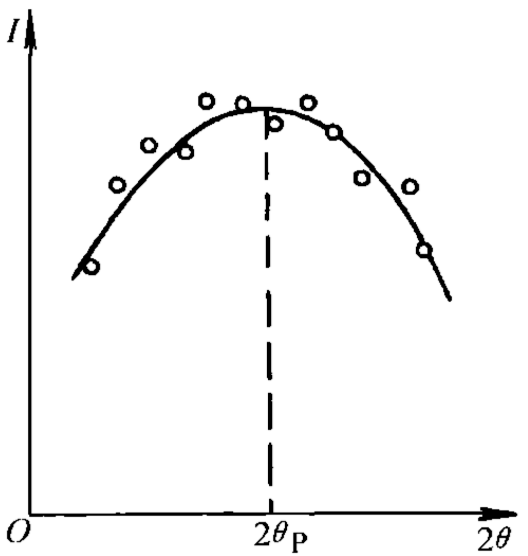
2). Parabolic fitting method

In order to improve the accuracy of peak fixation, multiple points (n≥5) can be selected, and the peak position can be determined using curve fitting method.

Assume that the optimal value of intensity at each point $2\theta_i$ is I_i, which satisfies Equation

$$I = a_0 + a_1(2\theta) + a_2(2\theta)^2 .$$

If the actual measured value of intensity is I'_i, the only difference between the actual measured value of each point and the optimal value is the sum of the squares of v_i.



b)

$$\sum_{i=1}^{n} v_i = \sum_{i=1}^{n} \left[I'_i - a_0 + a_1(2\theta_i) + a_2(2\theta_i)^2 \right]^2$$

According to the principle of least squares:

$$\frac{\partial \sum v_i^2}{\partial a_0} = 0 \qquad \frac{\partial \sum v_i^2}{\partial a_1} = 0 \qquad \frac{\partial \sum v_i^2}{\partial a_2} = 0$$

Solve for constants a_1, a_2

6.4 Problems in X-ray Macroscopic Stress Measurement

- Parabolic method

3). Intensity correction

When using a parabola to determine peaks, long-term timing counting or large-scale counting is required to obtain accurate intensity values, and the following formula needs to be used for correction.

$$I' = I'' / L_P A \quad \text{(Iso-inclination method)}$$
$$I' = I'' / L_P \quad \text{(Sid-inclination method)}$$

I' is the corrected intensity value, I'' is the actual measured value, L_P is angular factor, A is the absorption factor ($A = 1 - \tan\psi \cos\theta$).

6.4 Problems in X-ray Macroscopic Stress Measurement

- Determination of stress constant K

Crystals are anisotropic. When calculating elastic stress using a certain crystal plane strain, it is necessary to measure the elastic properties of the selected crystal plane. Methods as below:

Use the same plate as the material to be tested to make an equal-strength beam without residual stress, and apply a known and changeable unidirectional tensile stress σ to the equal-strength beam on a diffractometer or stress meter. Under uniaxial stretching conditions, according to formula, there is:

$$\frac{\partial \varepsilon_{\psi}}{\partial \sin^2 \psi} = \frac{1+\nu}{E}\sigma$$

M is the slope of ε_{ψ} changing with $\sin^2\psi$, that is:

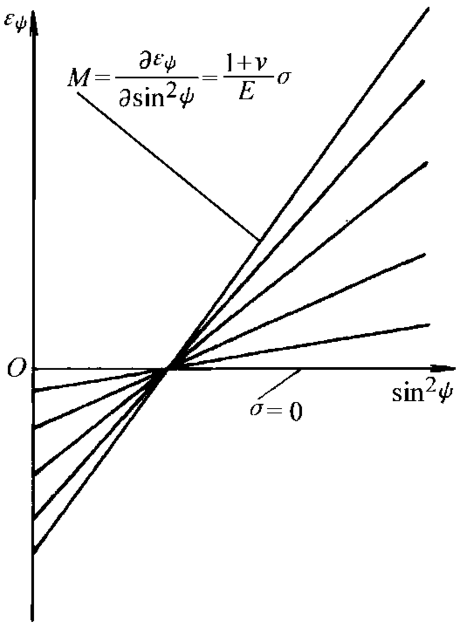
$$M = \frac{1+\nu}{E}\sigma$$

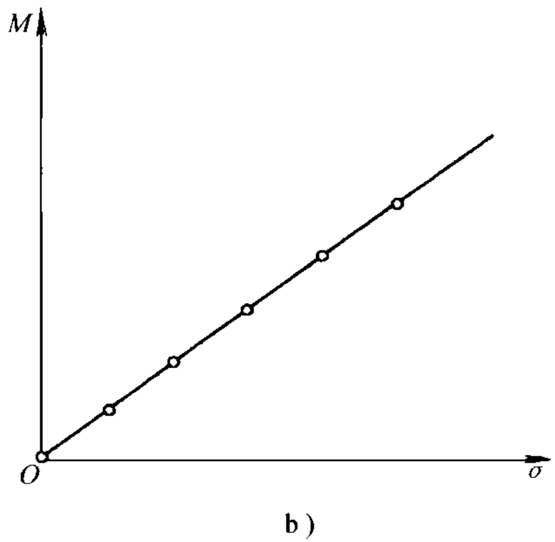
M also changes linearly with σ.

6.4 Problems in X-ray Macroscopic Stress Measurement

- Determination of stress constant K

$$M = \frac{1+\nu}{E}\sigma \quad \xrightarrow{\text{Derivative of } \sigma} \quad \frac{\partial M}{\partial \sigma} = \frac{1+\nu}{E}$$





Apply different stresses to beams of equal strength, measure the strains in different directions in the measurement direction plane, and substitute the above equations to calculate the X-ray elastic constant.

$$\frac{S_2}{2} = \frac{1+\nu}{E}$$

Determination of X-ray elastic constant

a) ε_ψ-$\sin^2\psi$ relationship under different stress

b) M-σ relationship

6.4 Problems in X-ray Macroscopic Stress Measurement

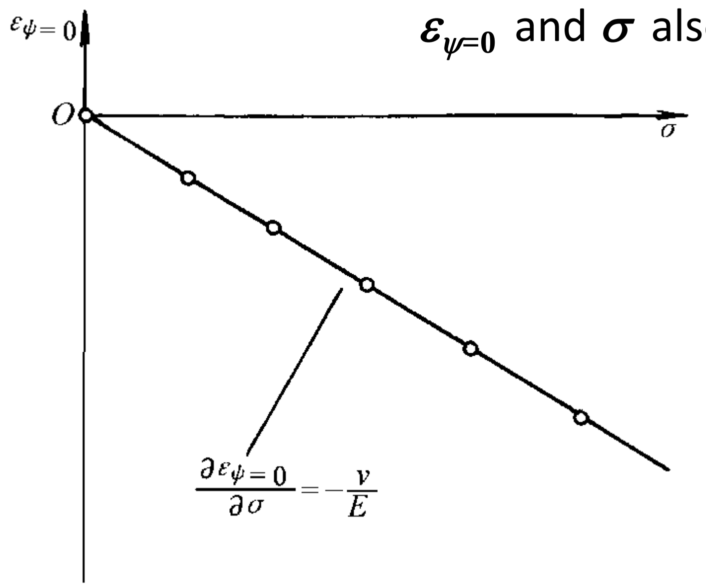
- Determination of stress constant K

Under uniaxial stretching conditions, we have:

$$\varepsilon_{\psi} = \frac{1+\nu}{E}\sigma\sin^2\psi - \frac{\nu}{E}\sigma$$

If $\psi = 0$ $\qquad \varepsilon_{\psi=0} = -\frac{\nu}{E}\sigma$

$\varepsilon_{\psi=0}$ and σ also have a linear relationship, see the figure.



The slope ***S1*** is called the elastic constant.

$$\frac{\partial\varepsilon_{\psi=0}}{\partial\sigma} = -\frac{\nu}{E} = S_1$$

The stress constant K can be calculated from the elastic constant corresponding to the {***h k l***} crystal plane, the X-ray wavelength and the diffraction angle θ_0 when the {***h k l***} crystal plane is stress-free.

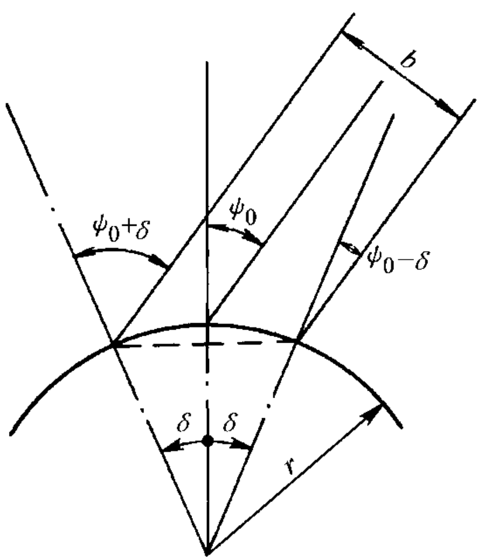
6.4 Problems in X-ray Macroscopic Stress Measurement

- Factors Affecting Macroscopic Stress Measurement Accuracy

1. The influence of diffractive crystal planes

The selection principle is that for strong diffraction lines in the high-angle area, the stress error caused by the measurement accuracy of 2θ will be reduced. The range of 2θ is 143 ~ 163°, or 110° ~ 170°.

2. Effect of specimen condition



Effect of specimen surface curvature

Surface oil stains, oxide scales and processing marks all have an impact on stress measurement. Especially the surface curvature.

ψ changes continuously at different positions on the surface. When calculating stress, $\sin^2\psi$ should be averaged value, without considering absorption, we can get:

$$\overline{\sin^2\psi} = \int_{\psi_0-\delta}^{\psi_0+\delta} \frac{\sin^2\psi}{2\delta} \mathrm{d}\psi = \frac{1}{2} - \frac{1}{4\delta} \cos 2\psi_0 \sin 2\delta$$