7. Fundamentals of electron optics and electron microscopy

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7.1 Electronic waves and electromagnetic waves

- The resolution limit of optical microscopy

Resolution refers to the minimum distance between two object points that can be resolved on an object. The resolution of an optical microscope is:

$$\Delta r_0 \approx \frac{1}{2}\lambda$$

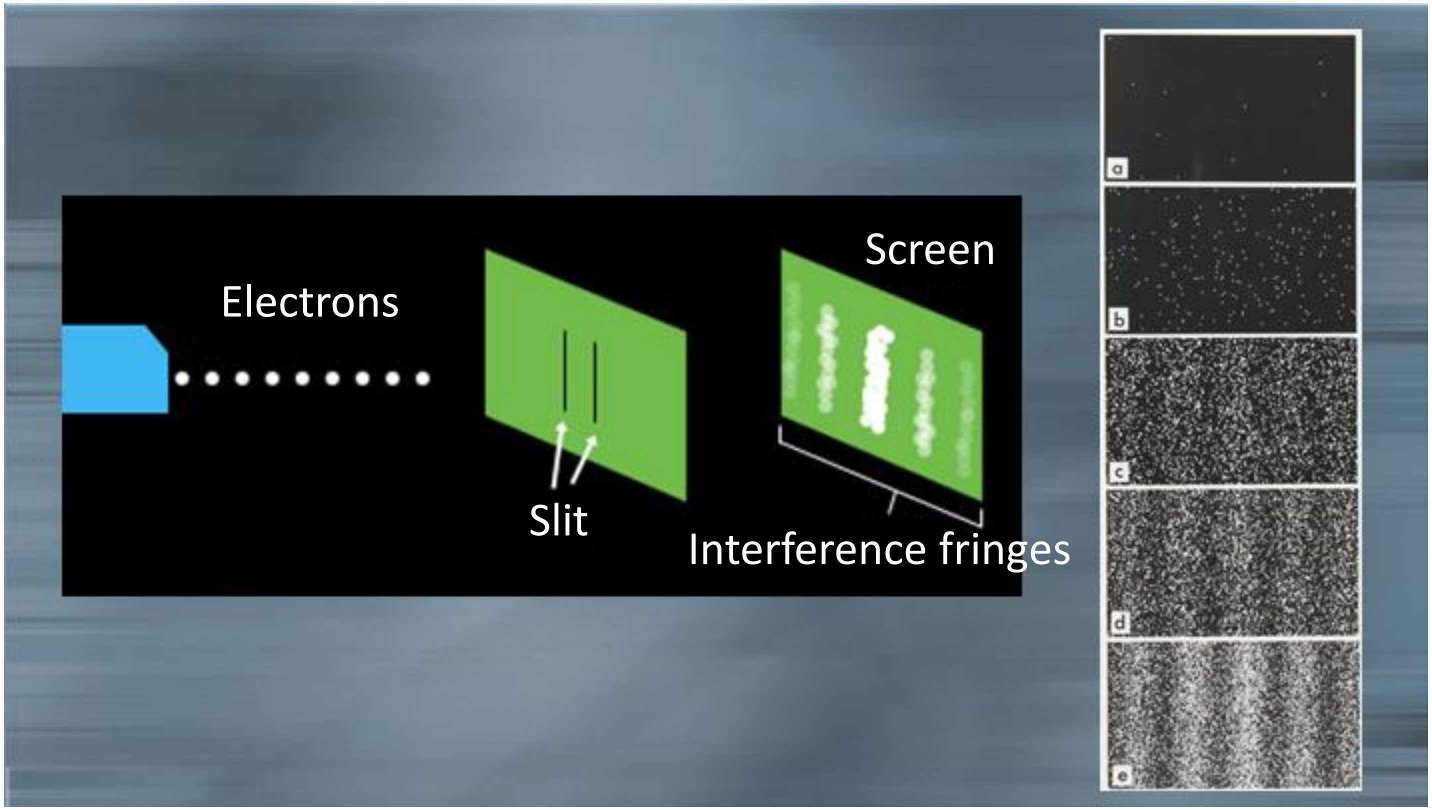
In the formula, λ is the wavelength of the light source.

The resolution of an optical microscope depends on the wavelength of the light source and is approximately half the wavelength. It can be seen that the key to improving resolution is to reduce the wavelength of the light source. In the visible wavelength range, its resolution limit is 200 nm. The microscope light source must first have volatility, and secondly, there must be a device that can focus it.

7.1 Electronic waves and electromagnetic waves

- The resolution limit of optical microscopy

In 1924, electron diffraction experiments confirmed that electrons have wavelengths 100,000 times shorter than the visible light; in 1926, it was discovered that electron waves can be focused using an axially symmetrical non-uniform magnetic field; in 1933, the world's first transmission electron microscope was designed and manufactured.



7.1 Electronic waves and electromagnetic waves

- Wavelength characteristics of electron waves

The wavelength of the electron wave depends on the speed and mass of the electron, that is:

$$\lambda = \frac{h}{mv}$$

In the formula, ***h*** is Planck's constant; ***m*** is the electron mass; ***v*** is the electron's speed, and its relationship with the acceleration voltage ***U*** is:

$$\frac{1}{2}mv^2 = eU$$

$$\downarrow$$

$$v = \sqrt{\frac{2eU}{m}}$$

$$\lambda = \frac{h}{\sqrt{2emU}}$$

7.1 Electronic waves and electromagnetic waves

- Wavelength characteristics of electron waves

If the electron speed is small, its mass is similar to that at rest, $m \approx m_0$; otherwise, m needs to be corrected by Theory of relativity:

$$m = \frac{m_0}{\sqrt{1-(v/c)^2}}$$

In the formula, c is the speed of light. The wavelengths of electron waves under different accelerating voltages are shown in the table.

Wavelengths of electron waves under different accelerating voltages.

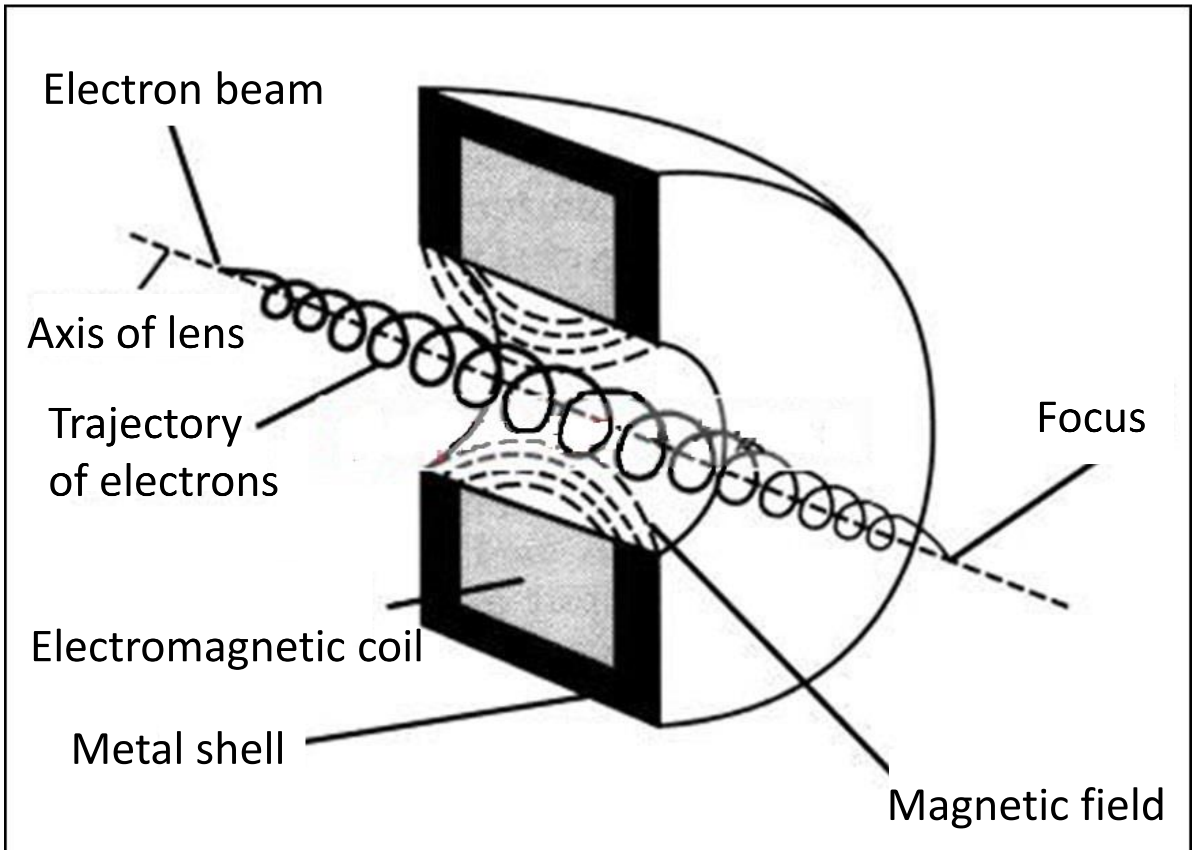
U / kV	λ / nm	U / kV	λ / nm	U / kV	λ / nm
20	0.00859	80	0.00418	200	0.00251
40	0.00601	100	0.00371	500	0.00142
60	0.00487	120	0.00334	1000	0.00087

The wavelength of visible light is 390 ~ 760 nm. Under typical acceleration voltages, the wavelength of electron waves is 5 orders of magnitude smaller than that of visible light.

7.1 Electronic waves and electromagnetic waves

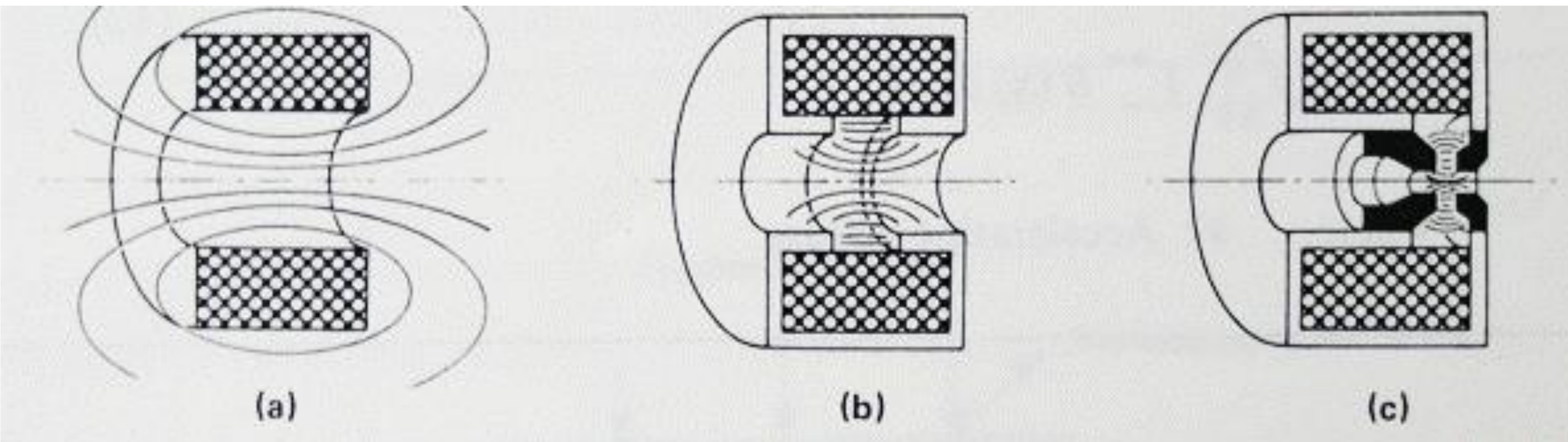
- Electromagnetic lens (focus)

The device in an electron microscope that uses a magnetic field to focus electron waves into an image is called an electromagnetic lens.



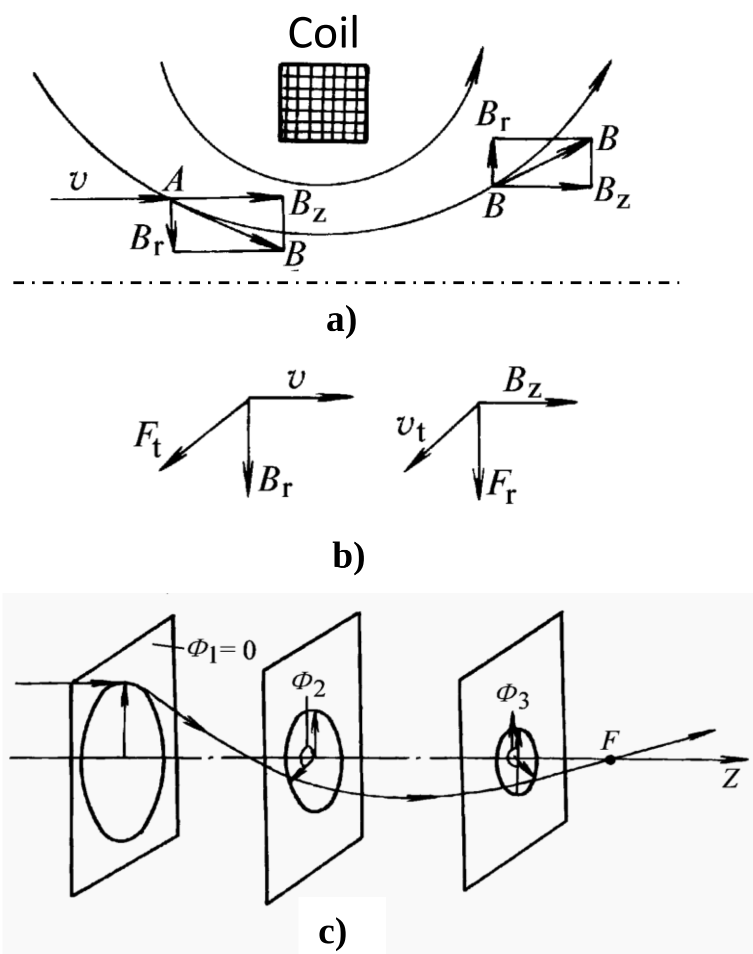
7.1 Electronic waves and electromagnetic waves

- Electromagnetic lens (focus)



7.1 Electronic waves and electromagnetic waves

- Electromagnetic lens (focus)



As shown in Figure, an energized coil is the simplest electromagnetic lens, forming an axially symmetrical and uneven magnetic field with electrons at speed ***v*** entering the lens in parallel.

At point A, the electron is applied by $\boldsymbol{B_r}$, generating a tangential force $\boldsymbol{F_t}$ and obtaining a tangential velocity $\boldsymbol{V_t}$. Under the action of the $\boldsymbol{B_z}$ component, a radial force $\boldsymbol{F_r}$ is formed that makes the electrons approach the main axis, causing the electrons to make a spiral paraxial motion.

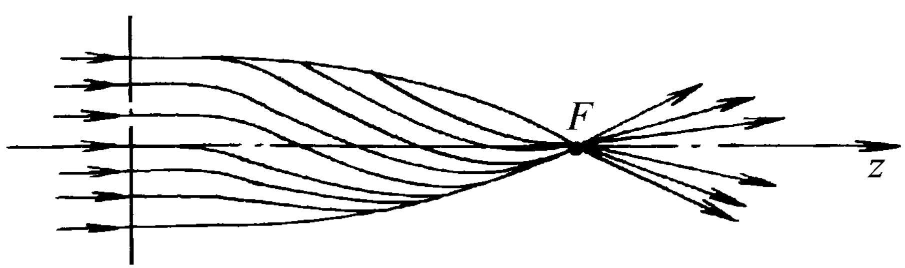
7.1 Electronic waves and electromagnetic waves

- Electromagnetic lens (focus)

The focusing of electron beams parallel to the main axis by an electromagnetic lens is similar to that of a glass lens. The relationship between the object distance L_1, the image distance L_2, and the focal length f is:

$$\frac{1}{f} = \frac{1}{L_1} + \frac{1}{L_2}$$

The magnification M is: $M = \frac{f}{L_1 - f}$



The focal length f can be approximately calculated by the following formula:

$$f \approx K \frac{U_r}{(IN)^2}$$

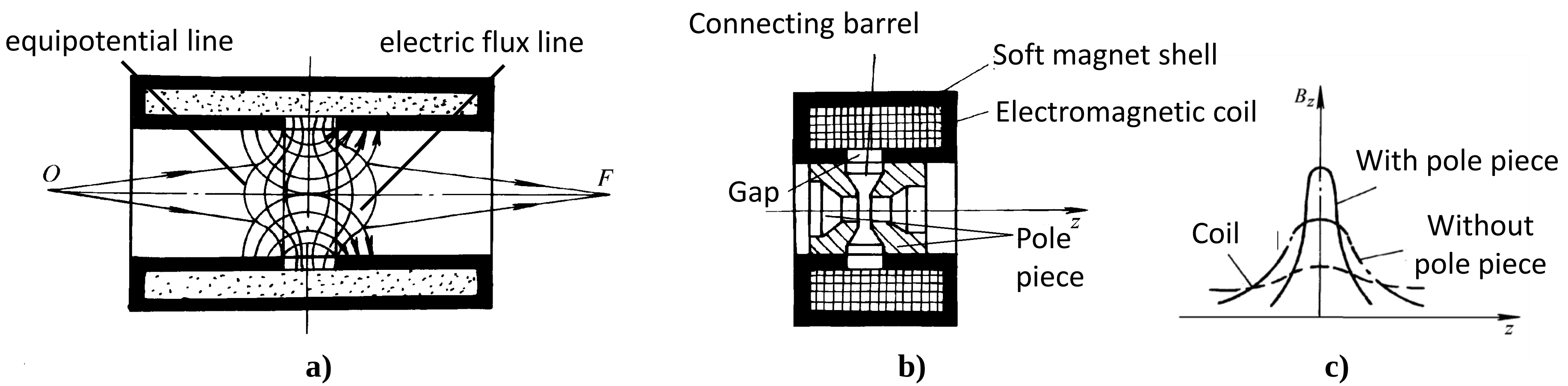
In the formula, K is a constant; U_r is the corrected acceleration voltage; IN is the coil ampere-turns.

7.1 Electronic waves and electromagnetic waves

- Electromagnetic lens (focus)

The focal length of an electromagnetic lens is always positive, and the focal length can be changed by changing the excitation current. The electromagnetic lens is a converging lens with variable focal length or variable magnification.

Figure is a schematic diagram of the electromagnetic lens structure and axial magnetic induction intensity distribution. Adding a short coil with an iron shell and internal pole can significantly change the magnetic induction intensity distribution of the lens.



a) With iron shell; b) With pole piece; c) Magnetic induction intensity distribution

7.2 Aberration and resolution of electromagnetic lens

- Aberration

Electromagnetic lens aberrations are divided into two categories, namely geometric aberration and chromatic aberration.

Aberrations
- Geometric aberration
  - Spherical aberration
  - Astigmatism
- Chromatic aberration

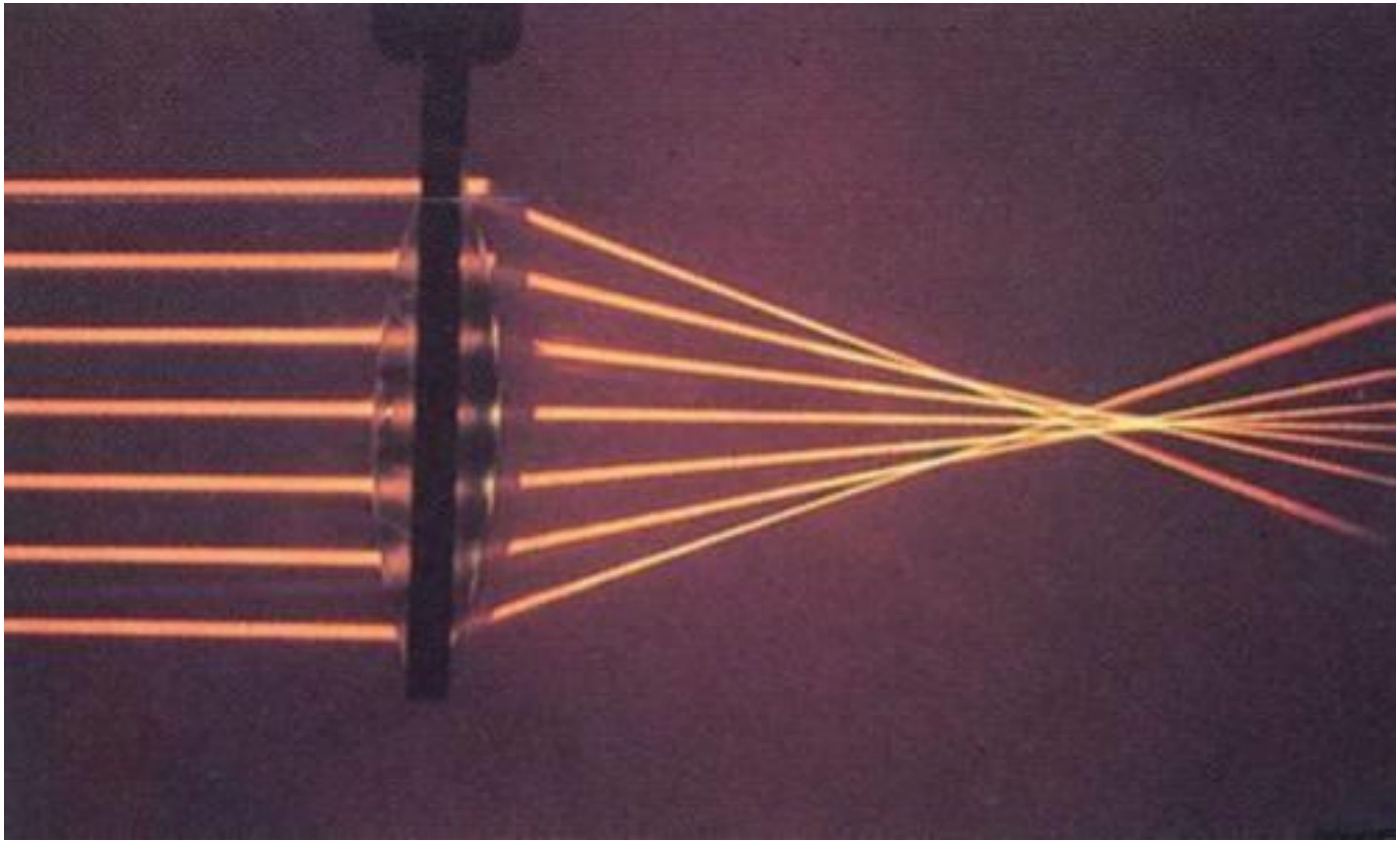
- Geometric aberrations include spherical aberration and astigmatism, also known as aberrations caused by monochromatic light. Spherical aberration is caused by the different refractive abilities of electrons in the lens's center area and edge area; astigmatism is caused by the non-rotational symmetry of the lens magnetic field causing differences in focusing ability at different directions.

- Chromatic aberration is the aberration caused by polychromatic light with different wavelengths. Chromatic aberration is caused by the difference in the ability of the lens to focus electrons with different energies.

7.2 Aberration and resolution of electromagnetic lens

- Spherical aberration

The electromagnetic lens cannot focus the light at one point.



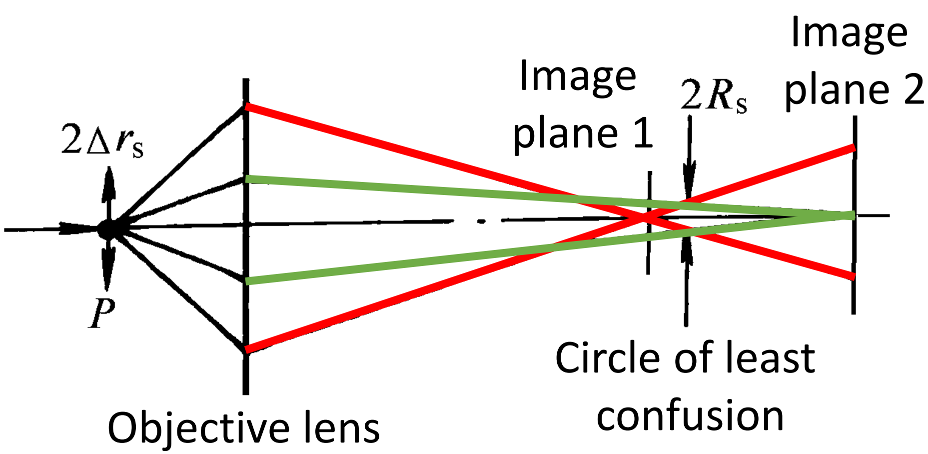
7.2 Aberration and resolution of electromagnetic lens

- Spherical aberration

Spherical aberration is caused by the difference in the refractive capability of electrons between the center area and the edge area of the lens. Use Δr_s to represent the size of the spherical aberration.

$$\Delta r_s = \frac{1}{4} C_s \alpha^3$$

C_S is the spherical aberration coefficient; α is the aperture half angle.

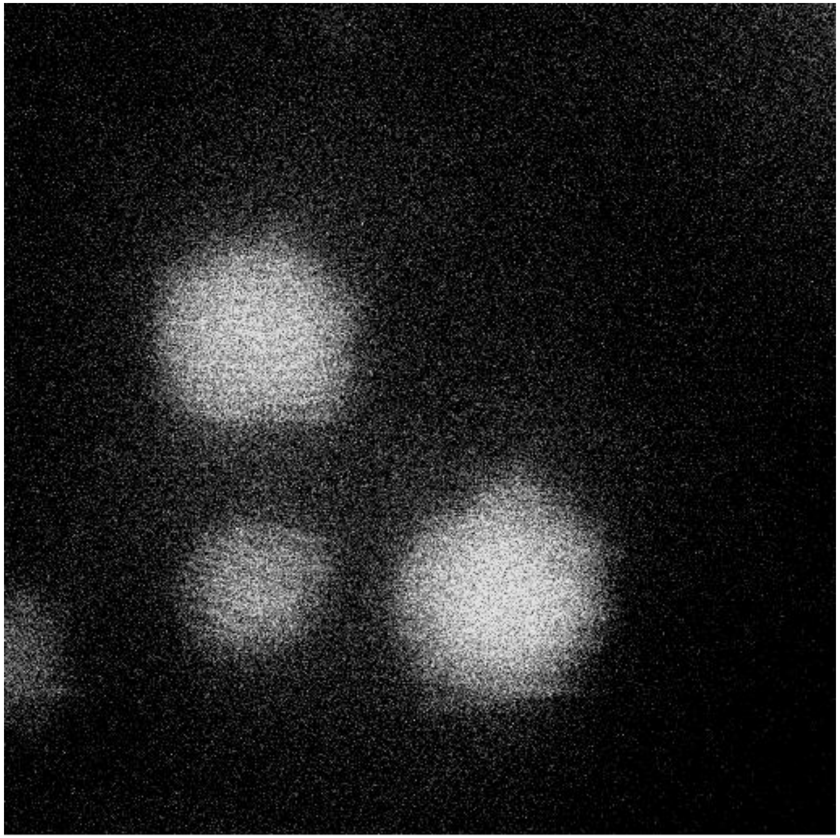


The way to reduce spherical aberration is to reduce C_S and small aperture angle. If the lens magnification is ***M***, the relationship between spherical aberration and the circle of least confusion R_S on the image plane is:

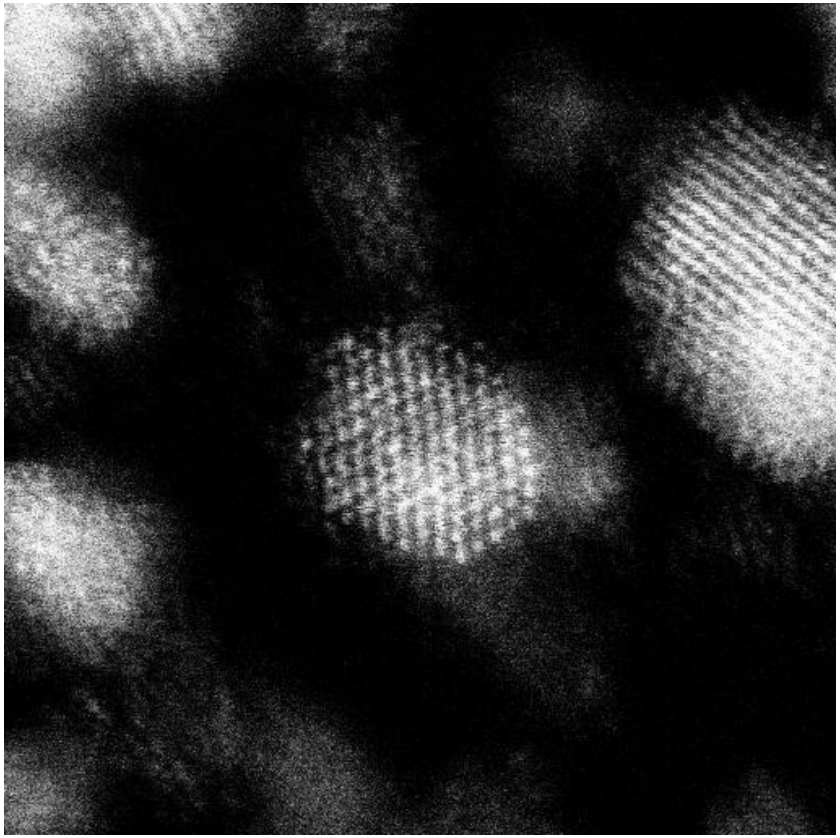
$$\Delta r_s = \frac{R_S}{M}$$

7.2 Aberration and resolution of electromagnetic lens

- Spherical aberration



un-Cs corrector (Cs:1.0 mm)

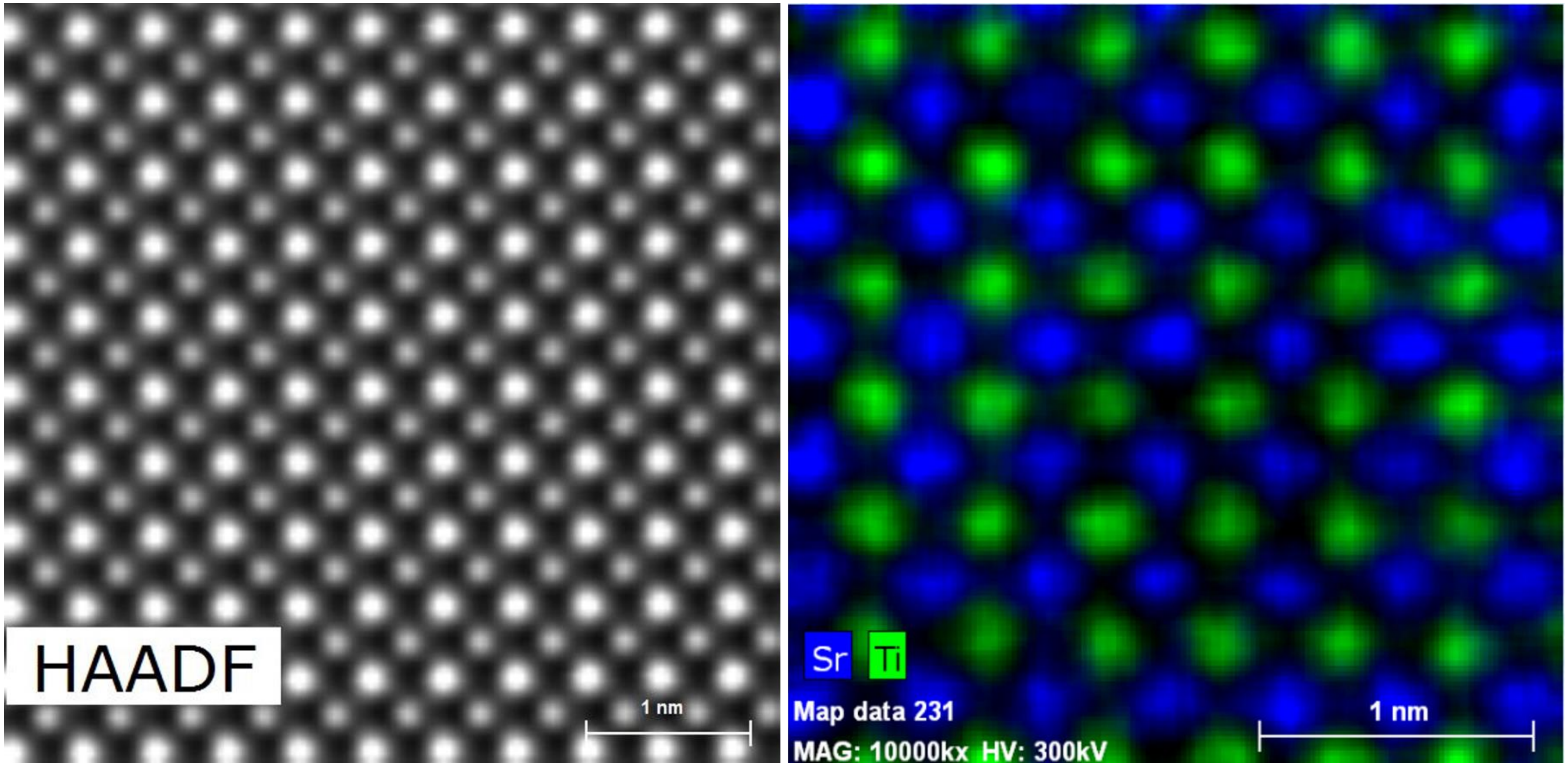


STEM Cs corrector

Dark field image of Pt particles

7.2 Aberration and resolution of electromagnetic lens

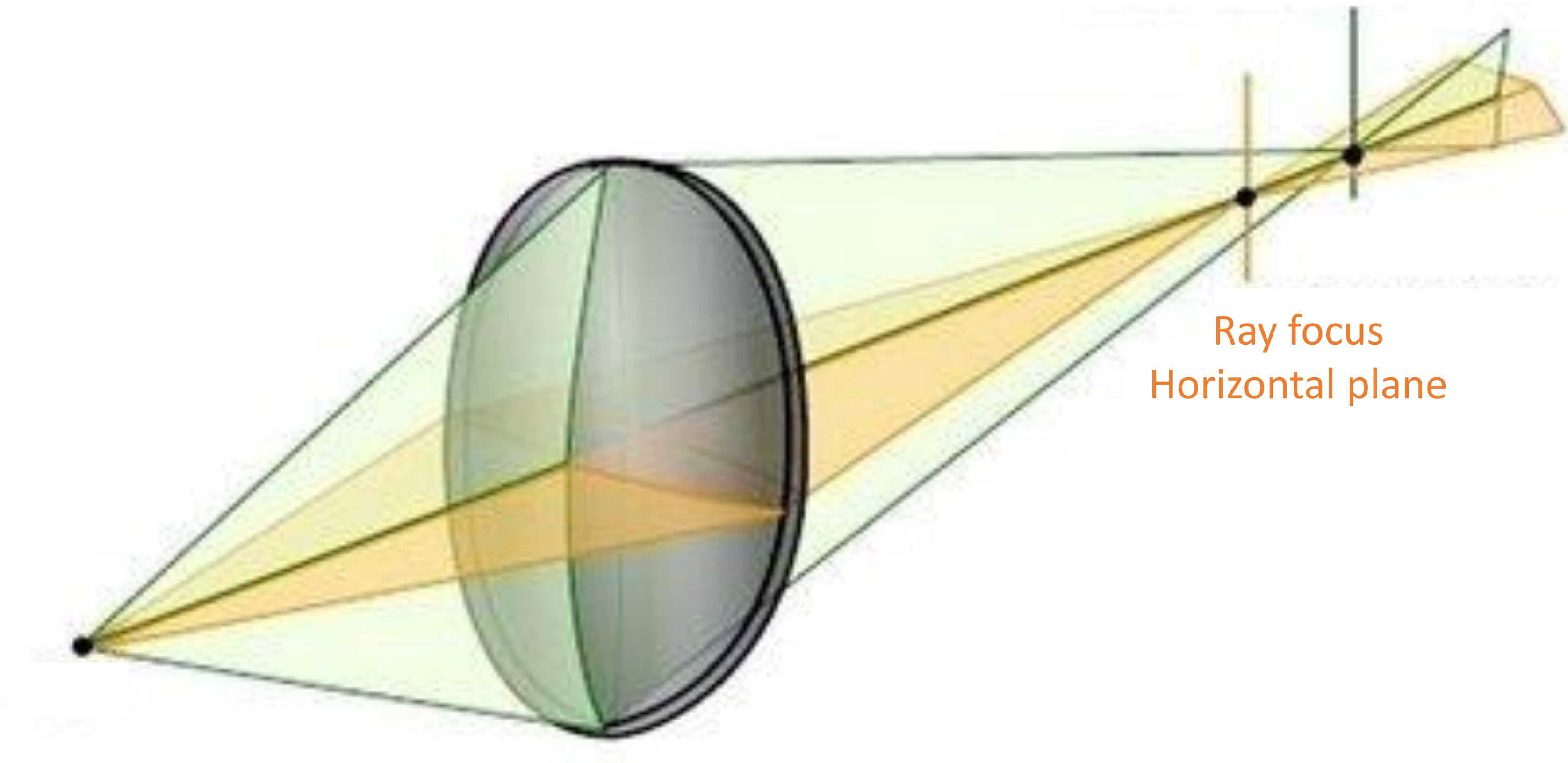
- Spherical aberration



The spherical aberration-corrected scanning transmission electron microscope achieves atomic resolution. Combining energy spectroscopy and energy loss spectroscopy, it can obtain atomic-level composition and structural information of materials. It has become one of the indispensable tools for scientific and technological workers in many fields.

7.2 Aberration and resolution of electromagnetic lens

- Astigmatism



7.2 Aberration and resolution of electromagnetic lens

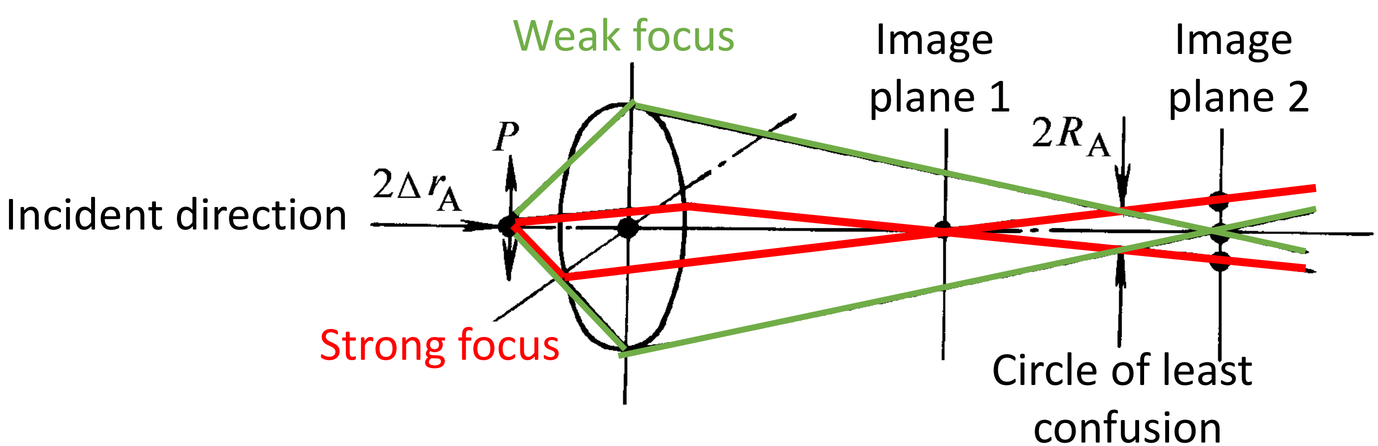
- Astigmatism

Astigmatism is caused by the non-rotational symmetry of the lens' magnetism, which results in differences in focusing ability at different directions. Δr_A is used to represent the size of the astigmatism.

$$\Delta r_A = \Delta f_A \alpha$$

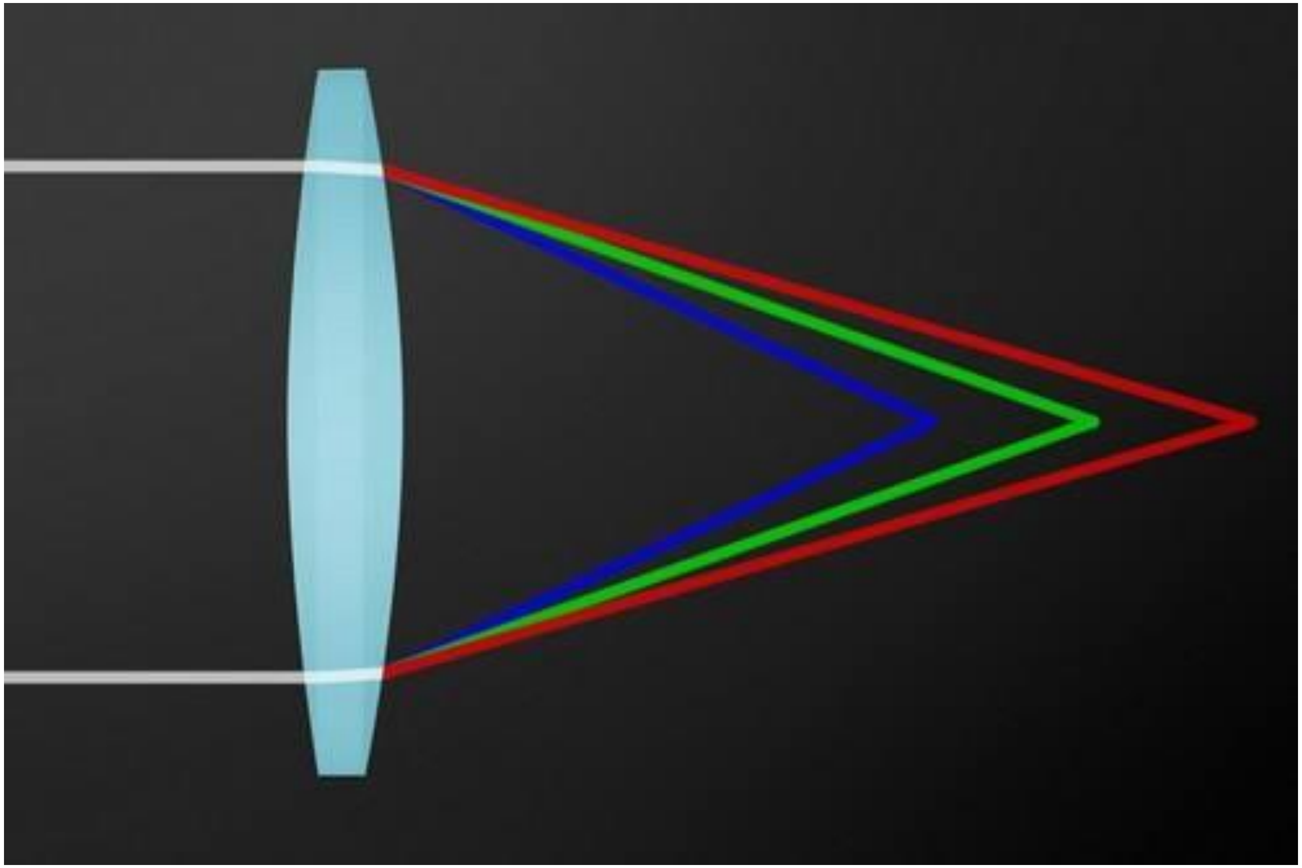
Δf_A is the focal length difference when the magnetic field is non-rotationally symmetric; α is the aperture half angle.

Astigmatism is eliminated by introducing a corrective magnetic field with adjustable intensity and orientation. If the relationship between the lens magnification ***M***, astigmatism and the minimum defocus spot radius R_A on the image plane is: $\Delta r_A = \frac{R_A}{M}$



7.2 Aberration and resolution of electromagnetic lens

- Chromatic aberration



7.2 Aberration and resolution of electromagnetic lens

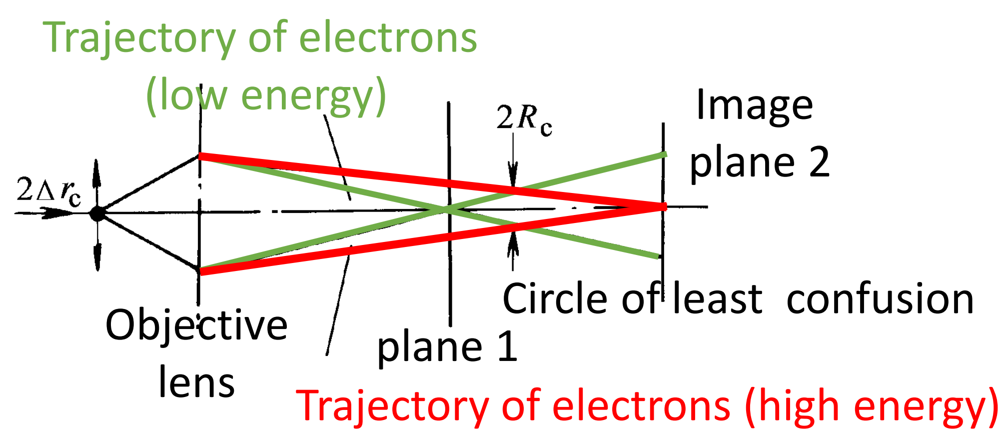
- Chromatic aberration

Chromatic aberration is caused by the difference in focusing ability caused by the non-uniformity of the incident electron wavelength (or energy). Δr_C is used to express the size of the chromatic aberration.

$$\Delta r_c = C_c \alpha \left| \frac{\Delta E}{E} \right|$$

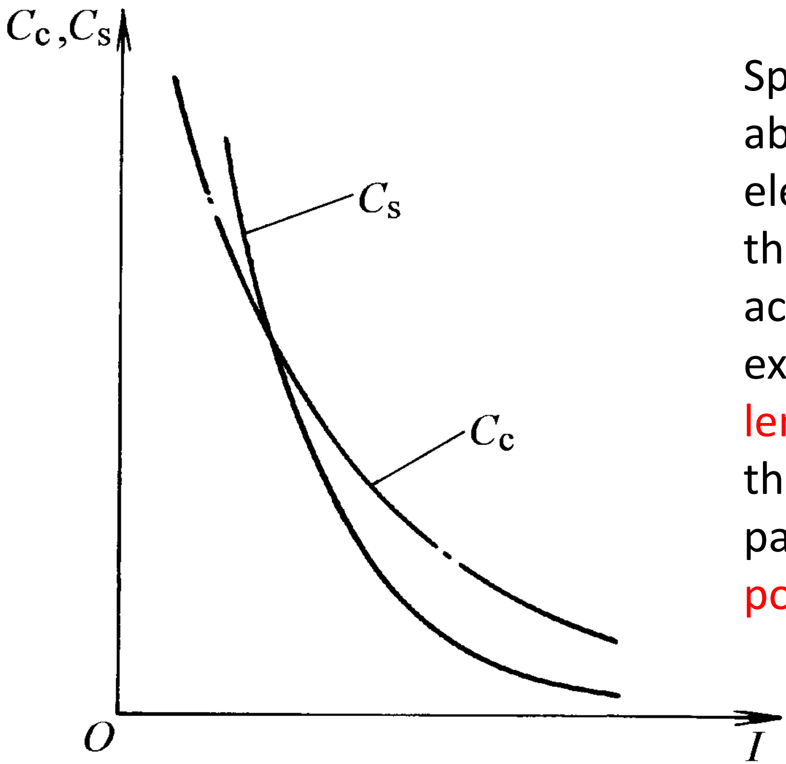
C_c is the chromatic aberration coefficient; $\Delta E/E$ is the electron energy change rate, which depends on the stability of the accelerating voltage and the degree of inelastic scattering of electrons passing through the sample. Chromatic aberration can be reduced by stabilizing the accelerating voltage and monochromator. If the magnification is M, the relationship between chromatic aberration and the minimum defocus spot radius R_C on the image plane is:

$$\Delta r_c = \frac{R_c}{M}$$



7.2 Aberration and resolution of electromagnetic lens

- Chromatic aberration coefficient (C_c) and spherical aberration coefficient (C_s)



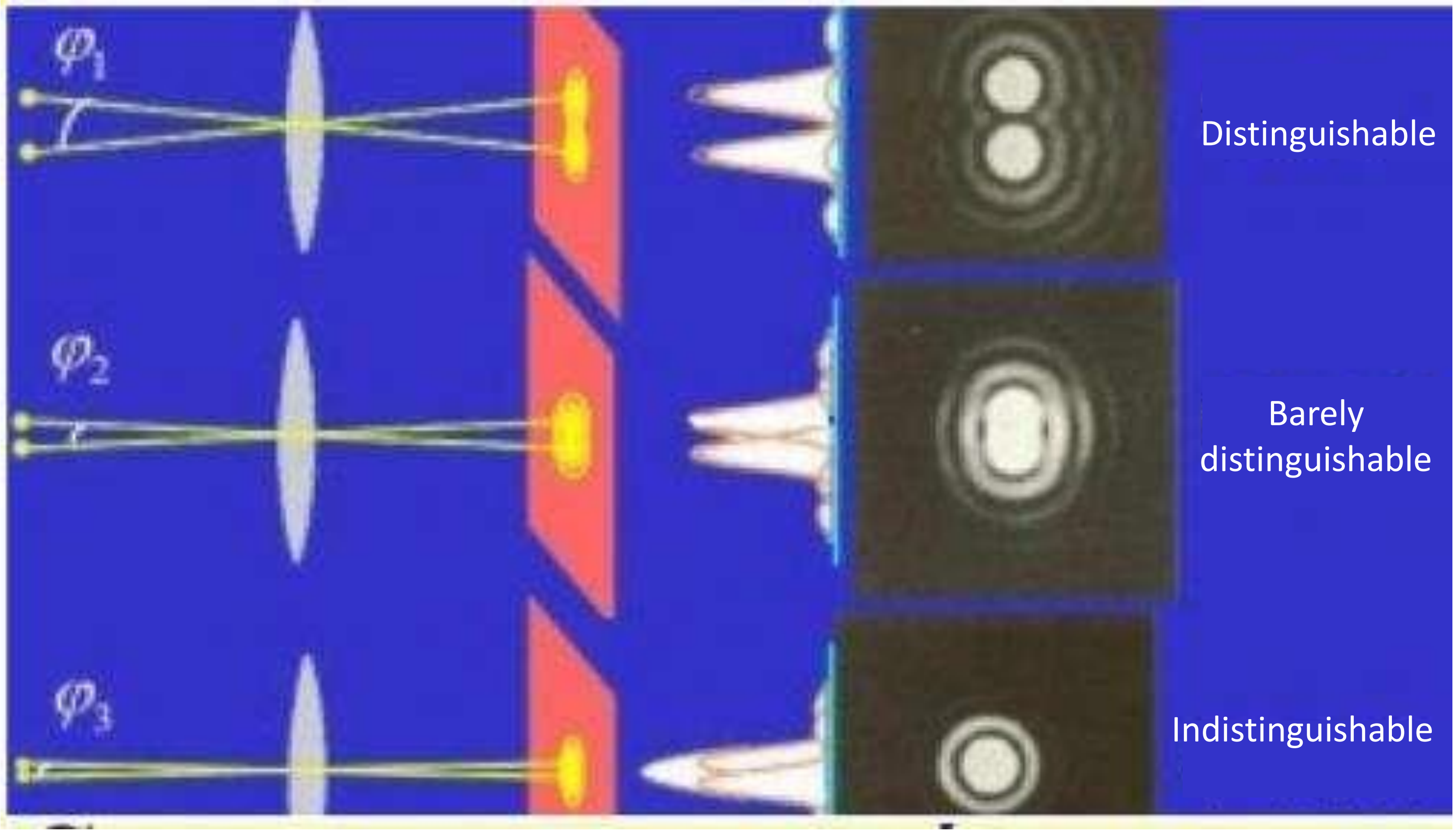
Effect of excitation current on lens spherical aberration coefficient C_s and chromatic aberration coefficient C_c.

Spherical aberration coefficient C_s and chromatic aberration coefficient C_c and the indicators of electromagnetic lens. In addition to being related to the lens structure, pole piece shape and processing accuracy, their value are also affected by the excitation current. C_s and C_c both change with the lens excitation current. To reduce the aberration of the electromagnetic lens, the lens coil should be passed through as large an excitation current as possible.

7.2 Aberration and resolution of electromagnetic lens

- Resolution

The impact of diffraction effects on resolution



7.2 Aberration and resolution of electromagnetic lens

- Resolution

The resolution of electromagnetic lenses is determined by diffraction effects and spherical aberration.

1. The impact of diffraction effects on resolution

The resolution limited by the diffraction effect can be calculated by Rayleigh's formula:

$$\Delta r_0 = \frac{0.61\lambda}{N \sin\alpha}$$

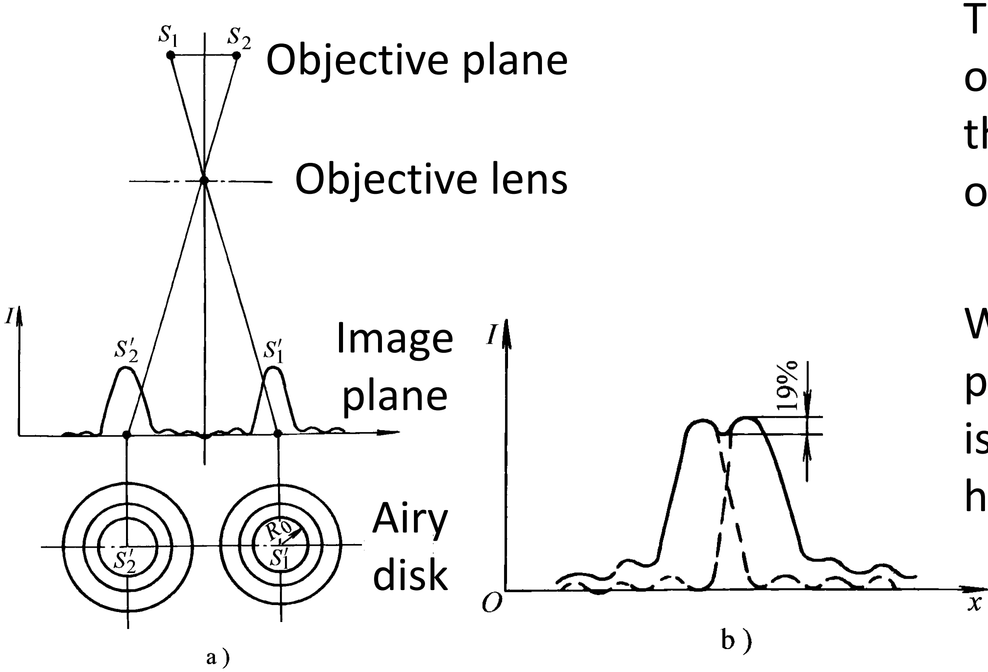
λ is the wavelength; ***N*** is the relative refractive index of the medium; α is the aperture half-angle of the lens. It can be seen that the smaller the wavelength λ and the larger the aperture half angle α, the smaller the resolution Δr_0 limited by the diffraction effect, and the higher the resolution of the lens.

Due to the diffraction effect, the image corresponding to the object point is a circular spot with the brightest center and an alternating light and dark ring around it - the Airy disk.

7.2 Aberration and resolution of electromagnetic lens

- Resolution

1. The impact of diffraction effects on resolution



There are two points $\boldsymbol{S_1}$ and $\boldsymbol{S_2}$ from objective plane that are imaged through the lens. The corresponding two Airy spots on the image plane are $\boldsymbol{S_1}'$ and $\boldsymbol{S_2}'$.

When the intensity difference between the peaks and valleys formed by two Airy disks is 19%, it is the critical value that the human eye can just distinguish.

At this time, the distance between $\boldsymbol{S_1}'$ and $\boldsymbol{S_2}'$ on the image plane is exactly the Airy disk radius $\boldsymbol{R_0}$, and the relationship between Δr_0 and the Airy disk radius $\boldsymbol{R_0}$ is, $\Delta r_0 = R_0 / M$. If the distance between two object points is less than Δr_0, the images of the two object points cannot be distinguished through the lens.

7.2 Aberration and resolution of electromagnetic lens

- Resolution

2. Effect of aberration on resolution

The resolutions defined by spherical aberration, astigmatism and chromatic aberration are Δr_S, Δr_A and Δr_C respectively. Among them, spherical aberration Δr_S is the main factor limiting the lens resolution. The spherical aberration can be made smaller by reducing α, but reducing α makes Δr_0 larger and the resolution decreases.

$$\Delta r_s = \frac{1}{4} C_s \alpha^3 \quad \Delta r_A = \Delta f_A \alpha \quad \Delta r_c = C_c \alpha \left| \frac{\Delta E}{E} \right| \qquad \Delta r_0 = \frac{0.61\lambda}{N \sin \alpha}$$

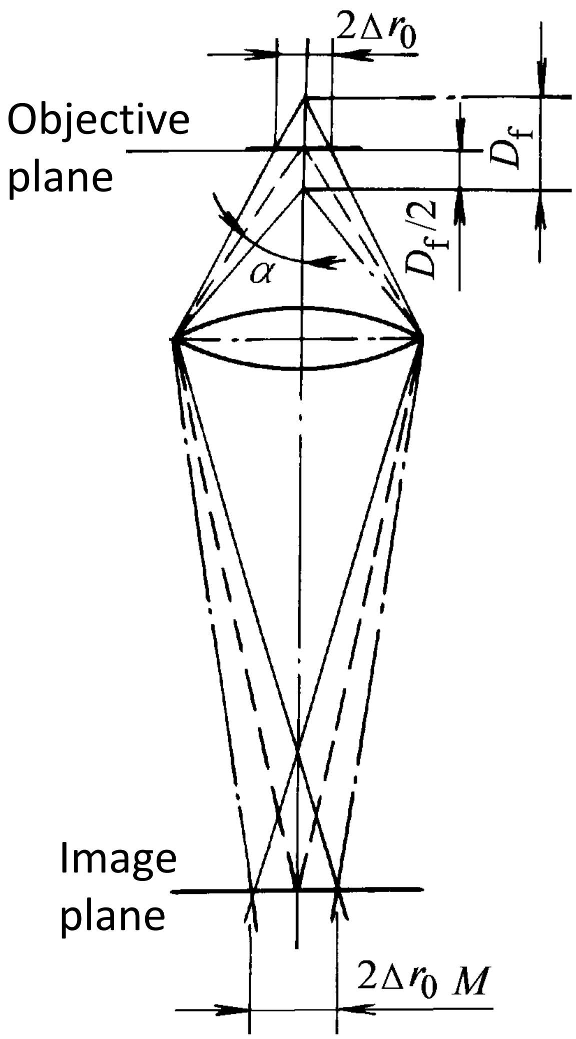
It can be seen that the key is to determine the optimal aperture half angle α_0 so that the effects of diffraction effects and spherical aberration on resolution are equal ($\Delta r_0 = \Delta r_S$), Therefore, we can find $\alpha_0 = 12.5(\lambda / C_S)^{1/4}$

Therefore, the electromagnetic lens resolution is: $\Delta r_0 = A\, \lambda^{3/4} C_S^{1/4}$ where $A \approx 0.4 \sim 0.55$.

The main way to improve the resolution of electromagnetic lenses is to reduce the wavelength of the electron beam (increase the acceleration voltage) and reduce the spherical aberration coefficient.

7.3 Depth of field and focal length of electromagnetic lens

- Depth of field



- The Depth of field is defined as the allowable axial deviation of the lens object plane.
- When the object plane deviates from the ideal position, a certain degree of defocus will occur. If the defocus spot size is not larger than the defocus spot corresponding to $2\Delta r_0$, it will not affect the lens resolution.
- From Figure, we can get the depth of field D_f as:

$$D_f = \frac{2\Delta r_0}{\tan\alpha} \approx \frac{2\Delta r_0}{\alpha}$$

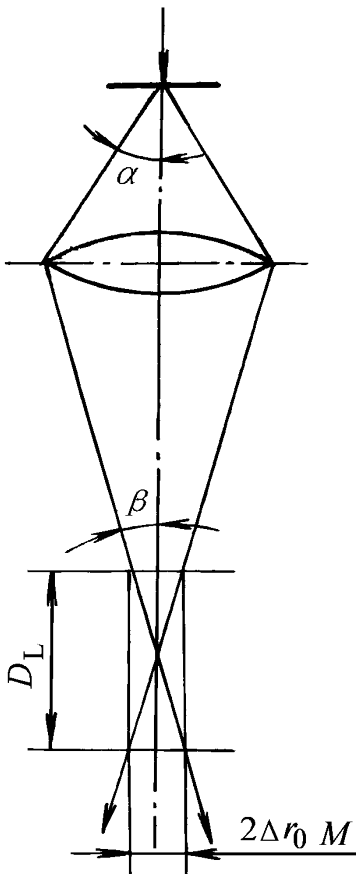
It shows that the smaller the aperture half angle α, the greater the depth of field. If Δr_0 =1 nm, $\alpha = 10^{-2} \sim 10^{-3}$ rad, then D_f = 200~ 2000 nm.

The thickness of the TEM sample is about 200 nm. Within the depth of field of the lens, clear images can be displayed at all levels of the sample.

7.3 Depth of field and focal length of electromagnetic lens

- Focal length

When the image plane moves within a certain range, if the defocused spot is not larger than $2\Delta r_0$, the corresponding defocused spot will have no effect on the lens resolution.



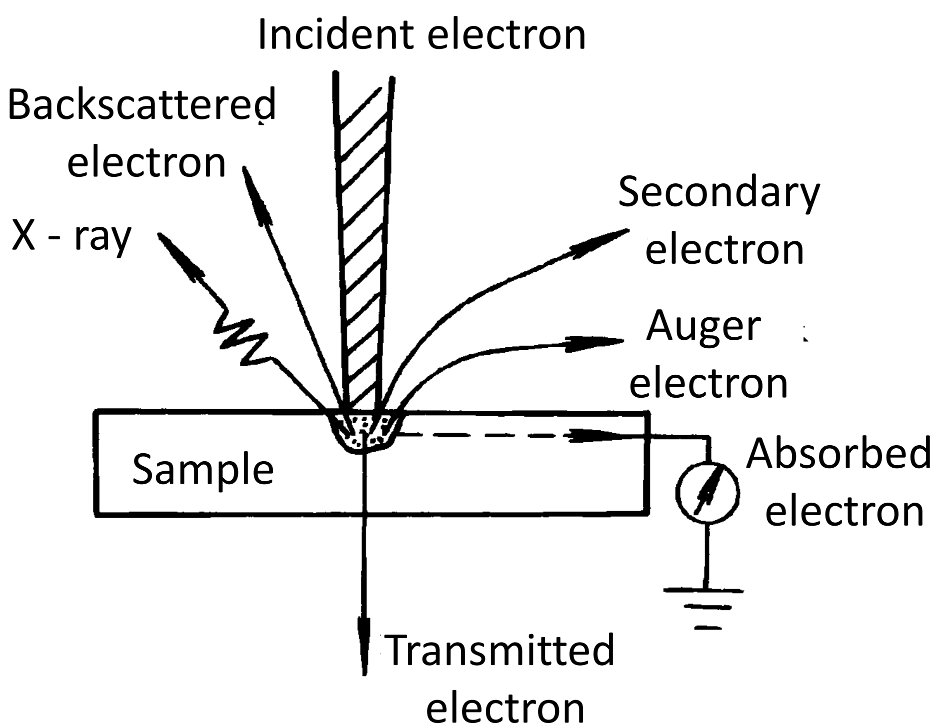
$$D_L = \frac{2\Delta r_0 M}{\tan\beta} \approx \frac{2\Delta r_0}{\alpha} M^2$$

In the formula, $\beta = \alpha / M$, M is the lens magnification. It shows that the focal length D_L increases as α decreases. If $\Delta r_0 = 1$ nm, $\alpha = 10^{-2}$ rad, $M = 200$, then $D_L = 8$ mm.

As long as the image is clear on the fluorescent screen of the transmission electron microscope, it must also be clear on the film or CCD camera, which brings great convenience to the observation and recording of the image.

7.4 The signal generated when an electron beam interacts with a solid sample

The main effect of the sample on the incident electron beam is scattering, which includes elastic scattering and inelastic scattering. The signals generated by this process mainly include backscattered electrons, absorbed electrons and transmitted electrons, as well as continuous X-rays.

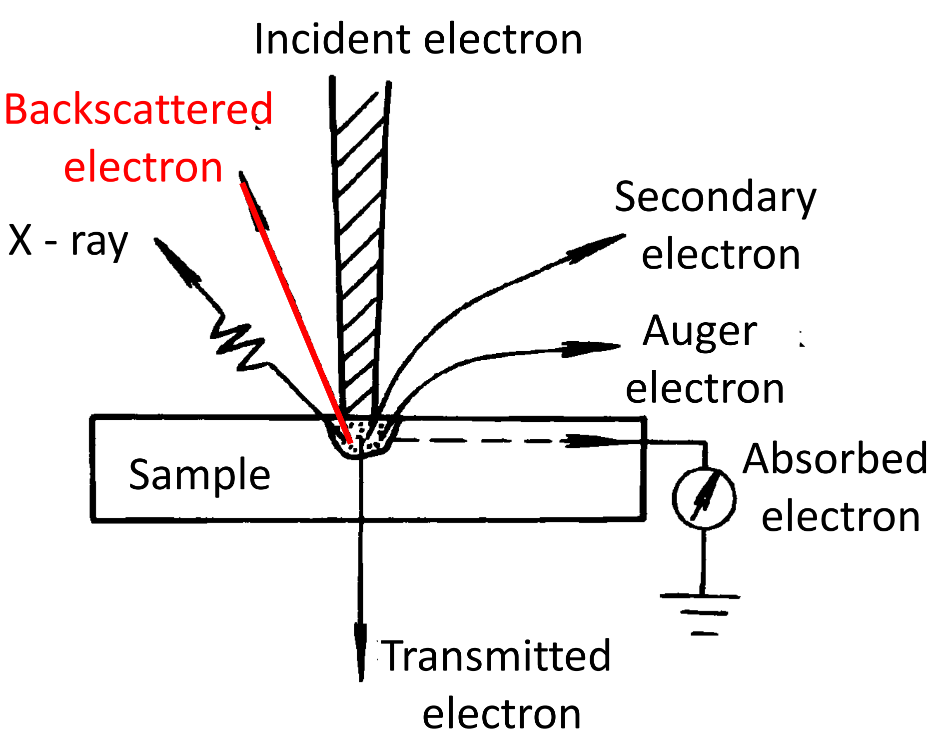


The main effect of incident electrons on the sample is atomic ionization. The signals generated by this effect mainly include secondary electrons, characteristic X-rays and Auger electrons. In addition, there are also signals such as cathode fluorescence.

7.4 The signal generated when an electron beam interacts with a solid sample

- Backscattered electrons

Part of the incident electrons that are scattered by sample atoms with a scattering angle greater than 90° and scatter outside the sample surface are called backscattered electrons, including elastic backscattered electrons and inelastic backscattered electrons.



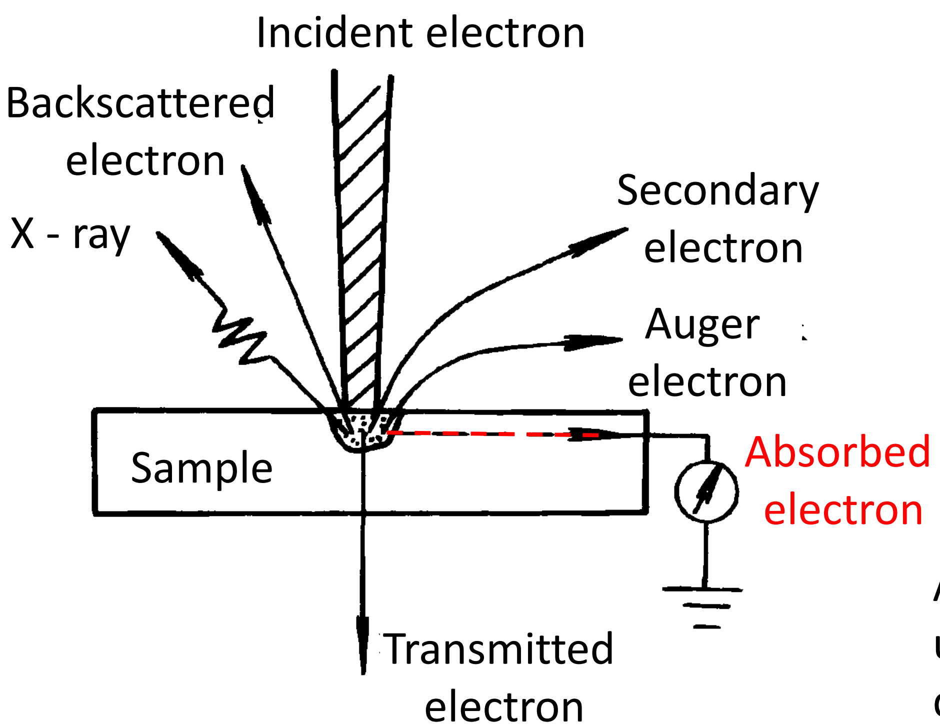
Backscattered electrons are generated in the depth range of a few hundred nanometers at the surface of the sample. The energy range is wide, from tens to tens of thousands of electron volts.

Backscattered electron images are mainly used to qualitatively analyze the composition distribution of materials and display the shape and distribution of phases.

7.4 The signal generated when an electron beam interacts with a solid sample

- Absorbed electrons

After the incident electrons enter the sample, they undergo multiple inelastic scatterings to consume all their energy, and are finally absorbed by the sample. This part of the incident electrons is called absorbed electrons. Absorbed electrons are generated in a depth range of about 1 micron on the surface of the sample.



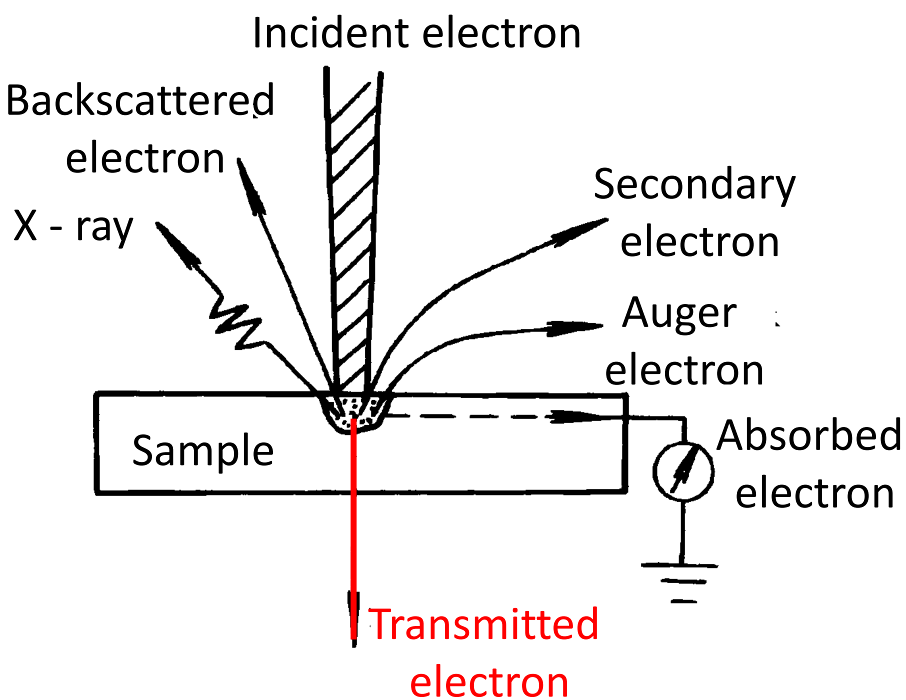
The amount of absorbed electrons decreases as the average atomic number of the sample increases. Because, when the intensity of the incident electron beam is constant, the area corresponding to a large backscattered electron yield absorbs fewer electrons, so the absorbed electron image can also provide atomic number contrast.

Absorption electron images are also mainly used to qualitatively analyze the component distribution of materials and display the shape and distribution of phases.

7.4 The signal generated when an electron beam interacts with a solid sample

- Transmitted electrons

If the energy of the incident electrons is very high and the sample is very thin, some of the electrons will pass through the sample, and this part of the incident electrons is called transmitted electrons. In addition to the elastically scattered electrons with energy equivalent to the incident electrons, there are also inelastically scattered electrons with different energy losses among the transmitted electrons. The energy loss of some electrons has characteristic values, which are called characteristic energy loss electrons.

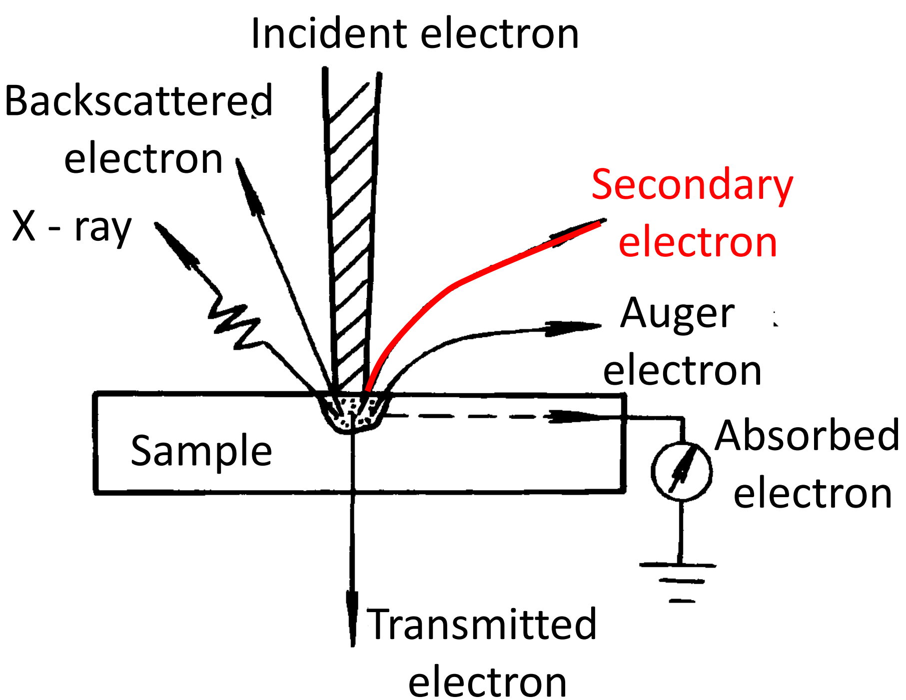


The energy of characteristic energy loss electrons has a corresponding relationship with the atomic number of the element in the sample, and its intensity increases as the content of the corresponding element increases. Using the electron energy loss spectrometer to receive characteristic energy loss electronic signals, qualitative and quantitative analysis of micro-area components can be performed.

7.4 The signal generated when an electron beam interacts with a solid sample

- Secondary electrons

Under the action of incident electrons, the outer valence electrons or free electrons of the sample atoms are knocked out of the sample surface, called secondary electrons, which are generated in the depth range of 5 ~ 10 nm on the sample surface. Secondary electron energy is low, generally no more than 50 eV, most are less than 10 eV.

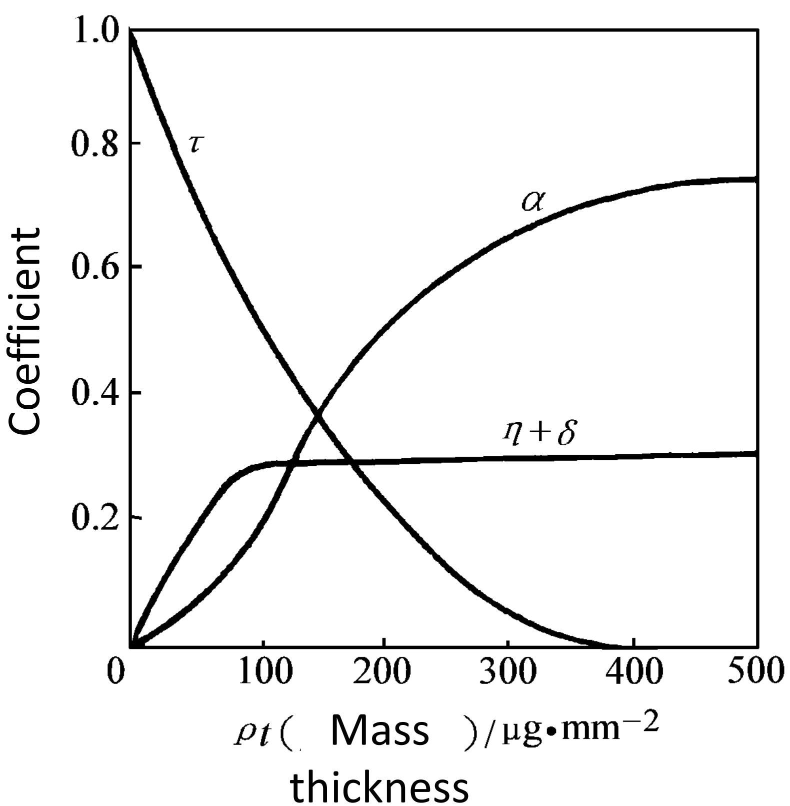


Secondary electron images are mainly used for fracture analysis, microstructure analysis and original surface morphology observation.

7.4 The signal generated when an electron beam interacts with a solid sample

- The electronic signal intensity

The relationship between the above four electronic signal strengths and the incident electron intensity (i_0) should satisfy: $i_b + i_s + i_a + i_t = i_0$.



i_b, i_s, i_a and i_t are the signal intensities of backscattered electrons, secondary electrons, absorbed electrons and transmitted electrons respectively. Divide both sides of the above equation by i_0 to get:

$$\eta + \delta + \alpha + \tau = 1$$

In the formula, η, δ, α and τ are the backscattering, emission, absorption and transmission coefficients respectively. The relationship between the above four coefficients and the sample mass thickness is shown in Figure.

7.4 The signal generated when an electron beam interacts with a solid sample

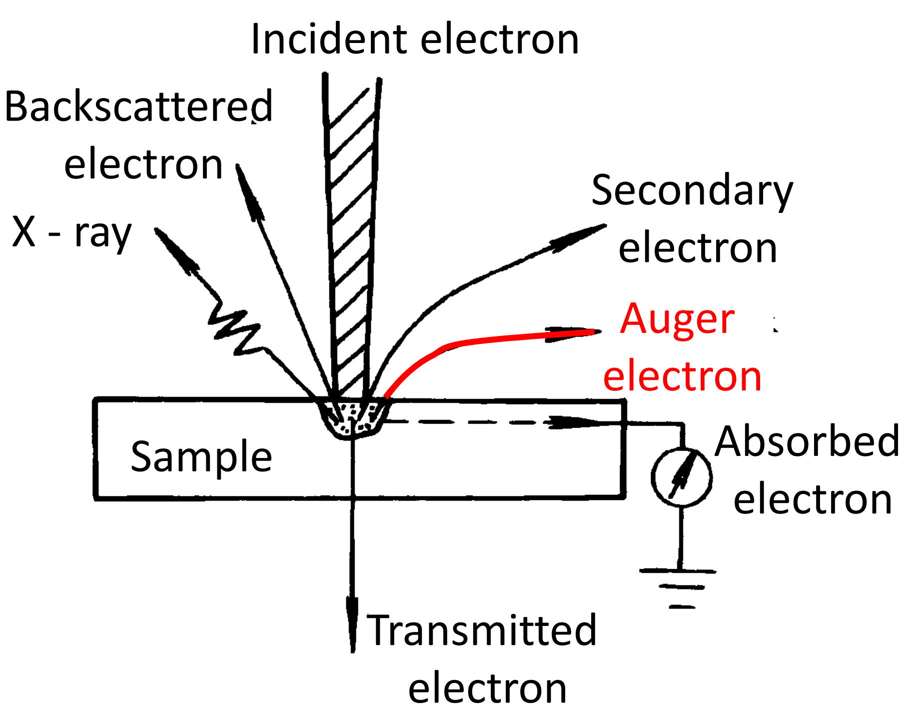
- X-ray

When the energy of the incident electron is enough to knock out the inner electrons of the sample atoms, the atoms are in an excited state with higher energy. The outer electrons will jump to the inner layer to fill the inner vacancies, and the characteristic X-rays will be emitted to release the excess energy generated on the surface of the sample. Depth range of approximately 1 **μm**. Its energy or wavelength has a corresponding relationship with the atomic number of the elements in the sample, and its intensity increases as the content of the corresponding element increases. Characteristic X-rays are mainly used for qualitative and quantitative analysis of material micro-region components.

7.4 The signal generated when an electron beam interacts with a solid sample

- Auger electron

When an atom in an excited state with a higher energy will jump to the inner layer to fill the inner vacancy, the outer electron will not release the excess energy in the form of emitting characteristic X-rays, but will emit another electron from the outer layer outward, which is called Auger electrons are generated in the depth range of about 1 nm on the surface of the sample.



There is a corresponding relationship between its energy and the atomic number of the elements in the sample. The energy is low, generally in the range of 50 ~ 1500 eV, and its intensity increases with the increase in the content of the corresponding element. Auger electrons are mainly used for qualitative and quantitative analysis of the composition of the extreme surface layer of materials.