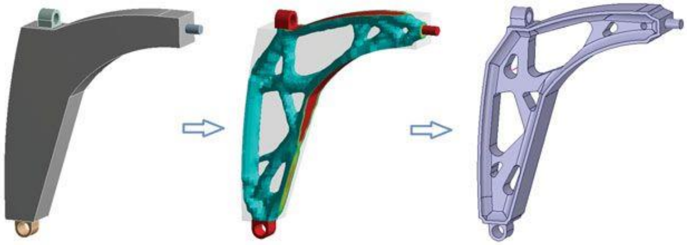
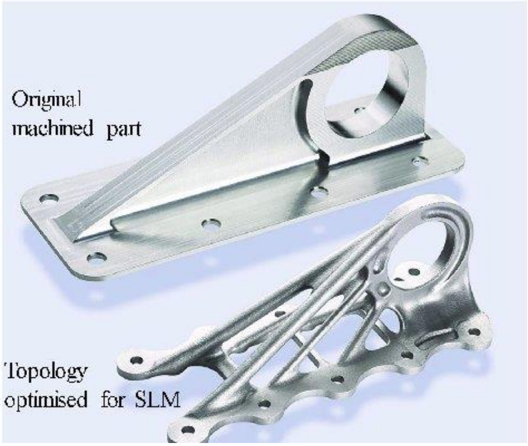
MAEG5160: Design for Additive Manufacturing

Lecture 20: Digital design of lattice and zero-mean curvature structures





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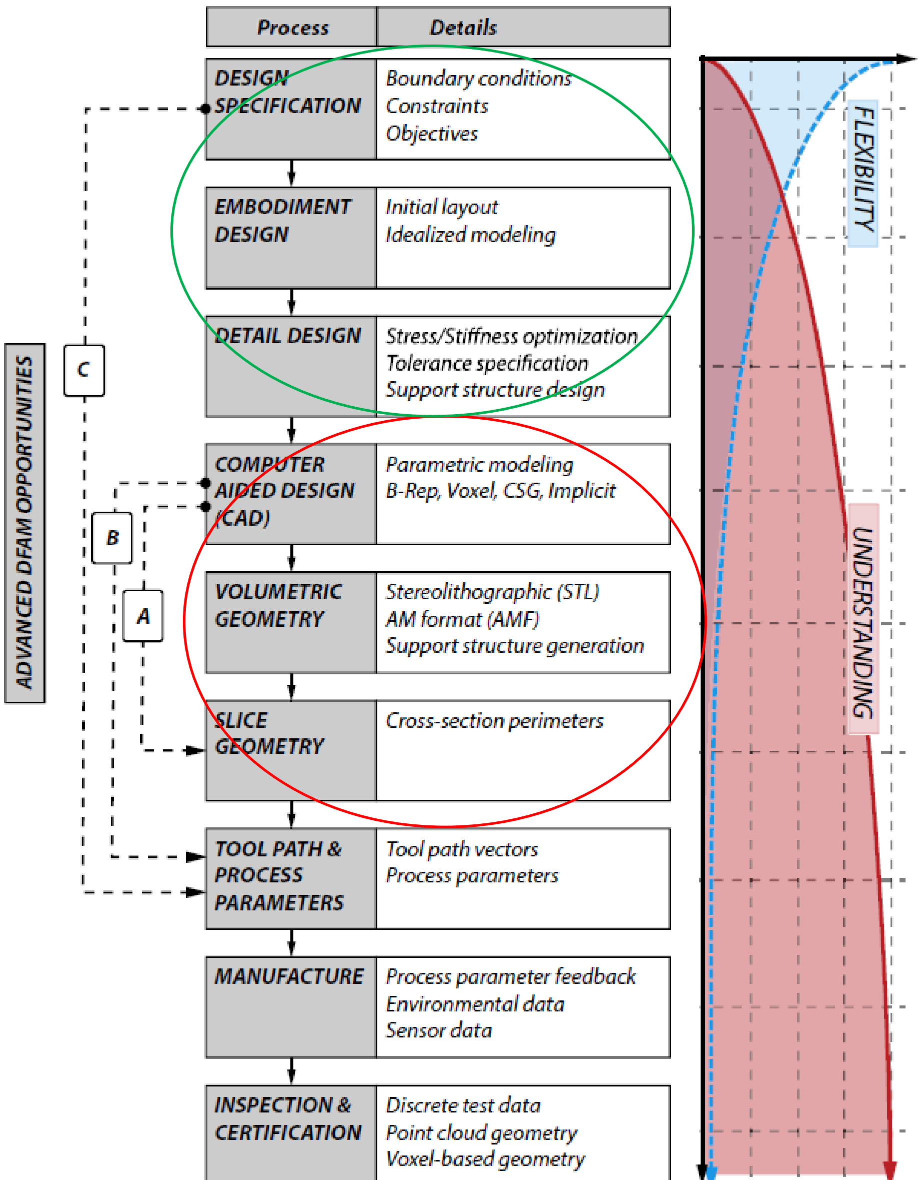
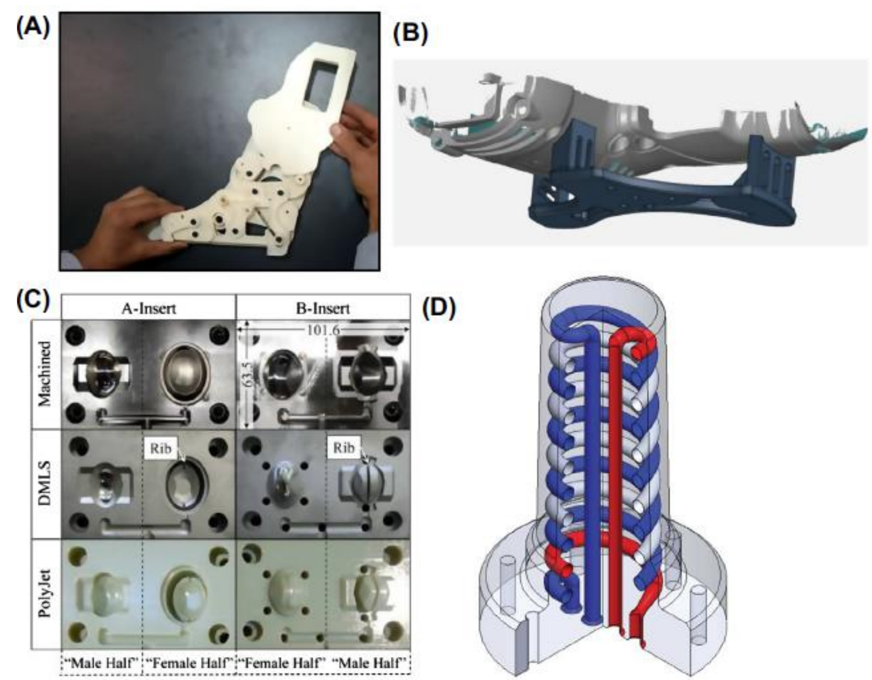


Schematic representation of the AM design process, including summary of digital data and transformations associated with each design phase. Solid arrows indicate the transfer of data. Dashed lines indicate opportunities for enhanced digital DFAM by the feed-forward of digital data



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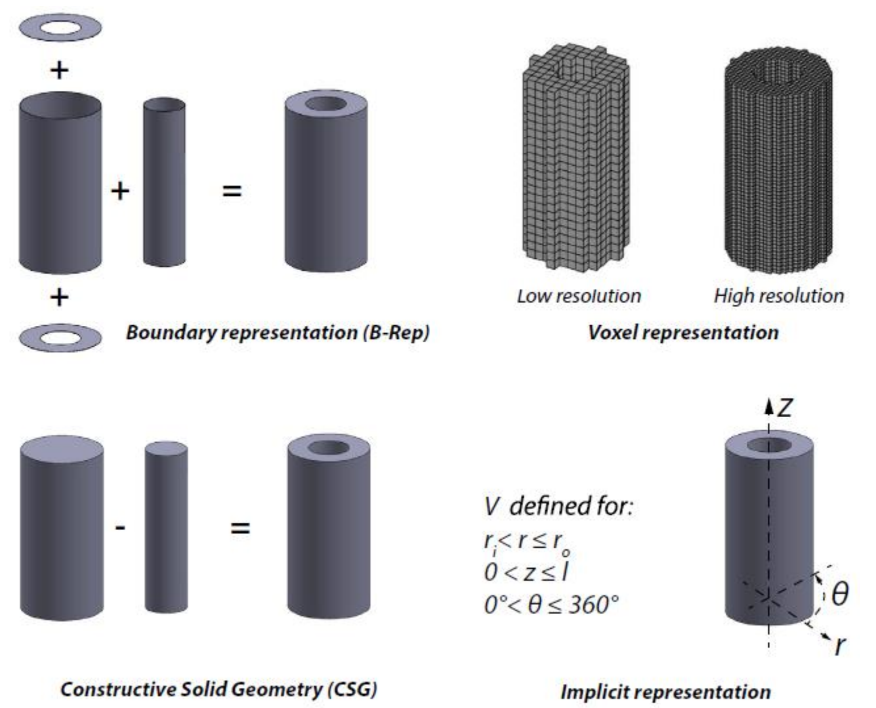
1. Computer Aided Design (CAD)

Computer Aided Design (CAD) data provides unambiguous representations of the geometric envelope associated with a specific intended geometry. Numerous formal protocols for CAD representation exist and are defined for various purposes according to their specific capabilities and attributes. These attributes include the fundamental method of geometric representation, either as a volume or external surface; the representation of geometry as either explicit or implicit data; the associated data storage protocol; and the representation of either toleranced or nominal geometries.

CAD data can be generated according to numerous protocols. These protocols consist of both proprietary and open-source formats and can be classified according to the method of geometry generation. Prominent *explicit methods* for data generation include *Constructive Solid Geometry (CSG)* and *boundary representation (B-rep)*. *Implicit methods* for geometry representation include *voxel methods*, *level sets*, and *scalar fields* that indirectly represent the geometry of interest. Each of these methods has distinct opportunities and challenges for AM application.



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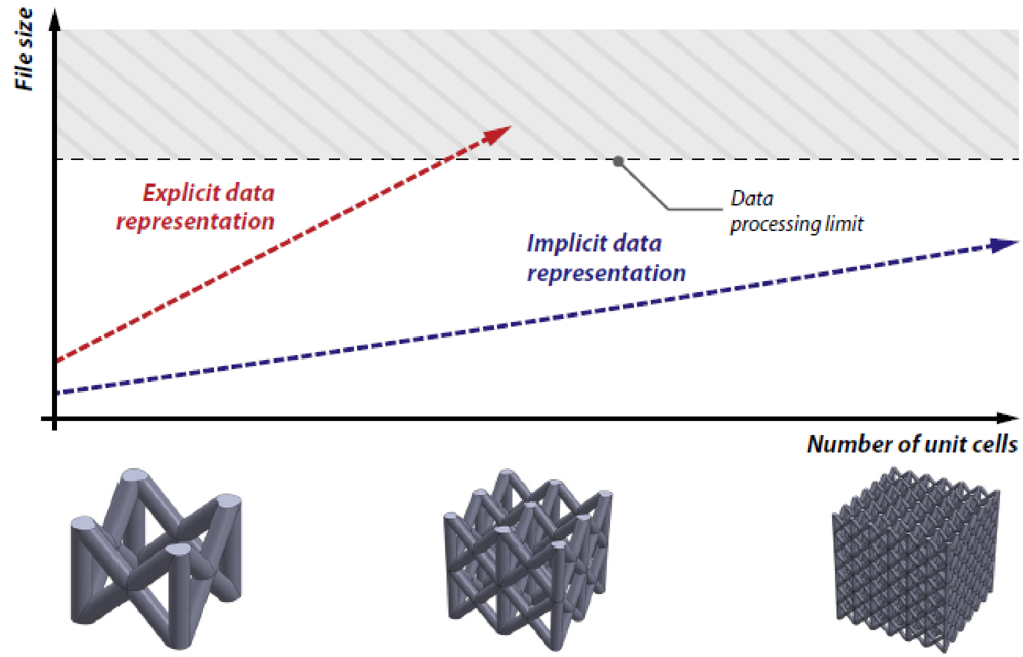
Constructive Solid Geometry (CSG) represents the intended component geometry with Boolean operations applied to primitive geometry structures, such as spheres, cylinders and cubes. These CSG methods are therefore eminently compatible with algorithmic methods, as are required for generative AM design. However, these representations are not necessarily compatible with curvilinear geometries, which are not readily constructed from the available library of primitive structures.

Boundary representations (B-rep) consist of surface elements that interconnect to define the volume of interest. B-rep is robust and flexible, especially for curvilinear geometries including complex variable radius fillets and sweeping blends. Consequently, B-rep is often the preferred data representation used in manual CAD software. The explicit data representation of B-rep and CSG methods can be data-inefficient for repetitive geometries, including for self-tessellated lattice structures and the complex geometry that is often preferred for high-value AM applications. For these scenarios, the necessary file size can rapidly expand beyond the feasible data processing limit.



Schematic representation of explicit and implicit data representation of lattice structure. Data generated by commercially available tools with no data compression for repetition elements.

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This data storage challenge can be potentially offset by efficient CAD protocols that allow geometry to be defined in terms of the repetitions of a unit cell, rather than requiring that this geometry be re-stated for each repetition. This approach is feasible for self-tessellating geometry and can achieve a significant file size reduction. However, this data compression approach may be incompatible with functionally graded or conformal lattice structures as for these structures each lattice cell is geometrically unique, although it may be based on a similar topology. Another approach is to utilize implicit CAD representations to define complex geometry. These implicit representations may provide data size efficiencies for a specific geometry complexity, and consequently such implicit representations are emerging within the commercial DFAM tools.

Topological optimization is an enabling design method for the commercialization of AM technologies. The geometric output of topological optimization methods is typically in the form of a voxel field that implicitly defines the volume of interest by discrete cubic sub-volumes. The application of implicit voxel field data to represent AM geometry presents a research opportunity for the development of data structures that represent topologically optimized geometry with high data efficiency.

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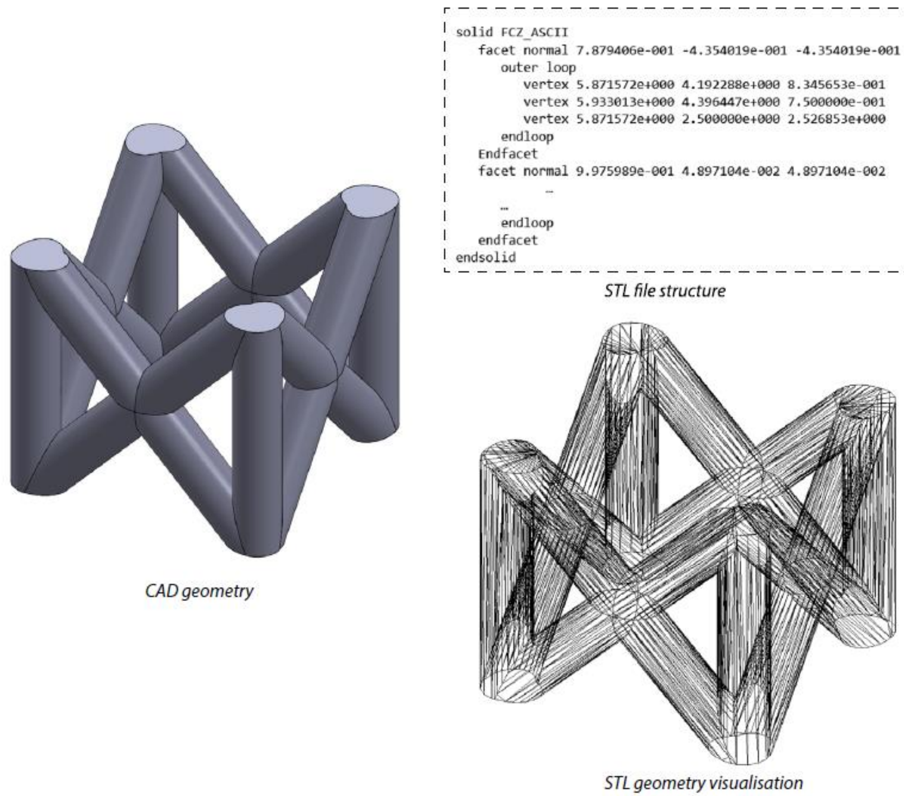
2. Volumetric geometry

2.1 STereolithographic File format (STL)

At the time of writing, the standard volumetric AM format is the stereolithographic file (STL). This format was originally generated by 3D Systems for use with their stereolithographic methods of 3D printing. The STL format represent the surface of the intended geometry with a finite number of adjoining triangular facets that are assigned by a standard format consisting of three vertices per facet and a normal vector specified to define the outward direction of the surface being defined. These facets are presented in unstructured manner with no specific scale data. Facets are defined uniquely and therefore data repetition is inherent in the STL format as facets share vertices and this data is redundantly repeated for each facet. STL data can be stored as either as ASCII or (more efficiently) binary data.



CAD geometry and associated STL format representation in stereolithographic (STL) file format, including a truncated sample of the ASCII STL file used to generate this data.

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Standard STL formatting: ASCII (left) and binary (right). While either file type contains the same information and is acceptable for most AM pre-processors, the binary option offers a significant reduction in file size over ASCII files.

solid (name)	UINT8	*%header*
facet normal $(n_1)(n_2)(n_3)$	UINT32	*%total number of facets*
outer loop	FLOAT32	*%facet normal*
vertex $(v_{1\text{-}1})(v_{1\text{-}2})(v_{1\text{-}3})$	FLOAT32	*%vertex 1*
vertex $(v_{2\text{-}1})(v_{2\text{-}2})(v_{2\text{-}3})$	FLOAT32	*%vertex 2*
vertex $(v_{3\text{-}1})(v_{3\text{-}2})(v_{3\text{-}3})$	FLOAT32	*%vertex 3*
endloop	UINT16	*%free attribute byte count*
endfacet	...	
...	end	
Endsolid (name)		

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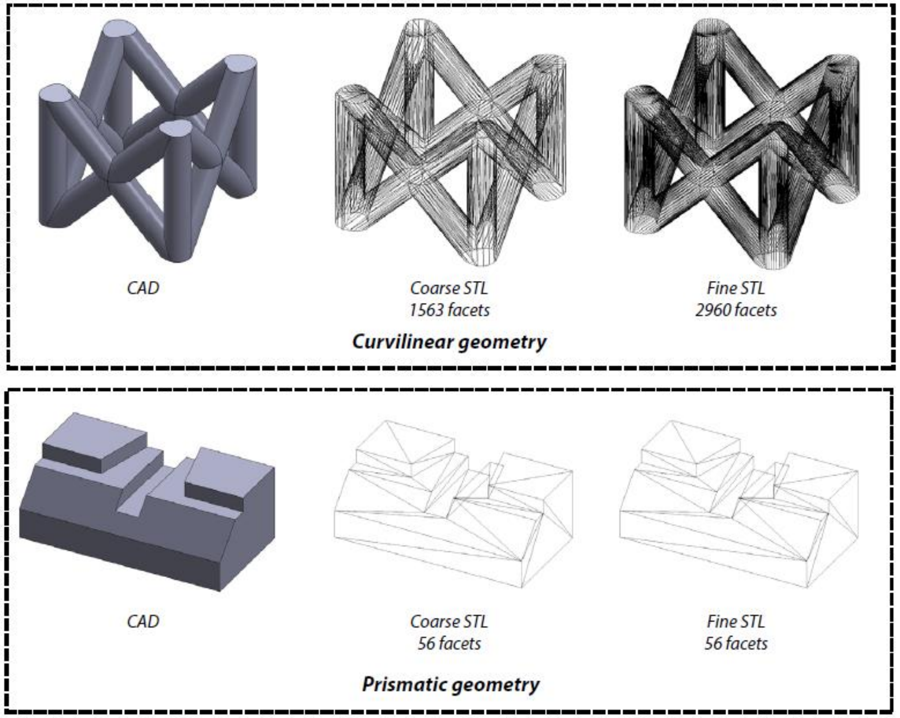
The STL data structure provides a straightforward and technically robust mechanism for defining volumetric data. For prismatic geometries, this structure is computationally efficient, as few facets are required to unambiguously define the component volume. However, the representation of curvilinear geometry with discrete planar facets is a particular challenge associated with the STL format. There exist multiple strategies for measuring and accommodating the associated faceting error, including the angular and chordal deviation associated with a specific facet in representing the intended geometry. The typical response to the challenge of faceting error is to manually prescribe an allowable deviation from the original CAD representation: this is often qualitatively described in CAD export options as, for example, either fine or coarse resolution. It is apparent that an infinite number of planar facets are required to represent a curvilinear geometry without geometric error. It is in response to this challenge that custom AM volumetric data storage formats have been developed.

Representation of curvilinear and prismatic geometries with STL format at coarse and fine resolution. The STL represents prismatic geometry without error, but can suffer from poor data efficiency for the representation of curvilinear geometry.



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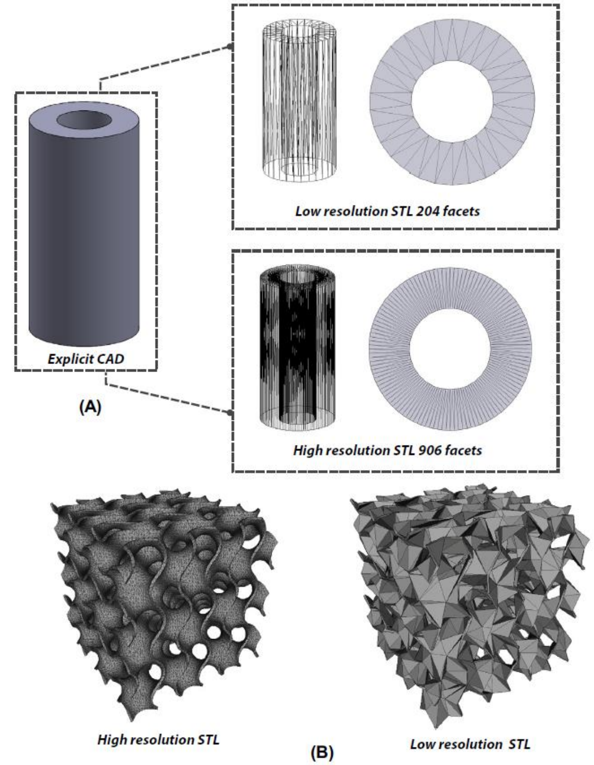
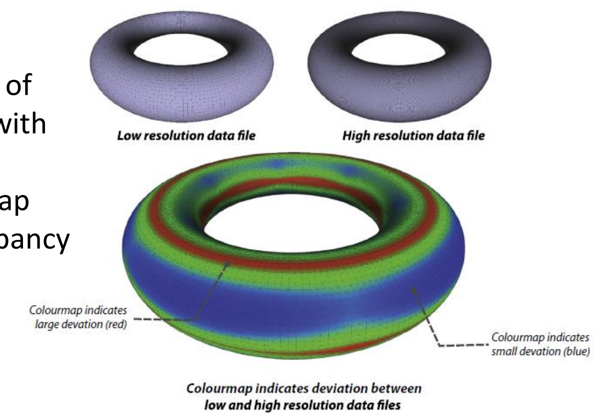
The data storage challenge for the STL format exists only for curvilinear geometries. Unfortunately, many of the geometries of interest to AM generate their technical function and structural integrity by their local curvature. Two examples of this locally curved geometry are the Triply Periodic Minimal Surface (TPMS) and the lattice structure, both of which are of significance to optimal AM design and can be mechanically and aesthetically compromised by under-sampling of the STL, and we will study them in details later this lecture. Under-sampling occurs when the STL facet representation provides insufficient resolution for the specific AM process, resulting in measurable facets in the manufactured part. This insight provides a little-utilized DFAM design opportunity for the optimization of STL data sets - i.e. that the facet frequency and associated geometric error is not technically compromising if its effects on the manufactured geometry are understood and actively accommodated. There are two associated DFAM opportunities: experimentally quantifying the effect of faceting error on the as-manufactured geometry, and utilizing the STL file data format to more efficiently represent the required geometric data.



STL representation of curvilinear geometry with high and low resolution for (A) explicitly defined cylinder and (B) implicitly defined gyroid

STL representations of toroidal structures with high and low facet resolution, colourmap indicates the discrepancy between these data representations



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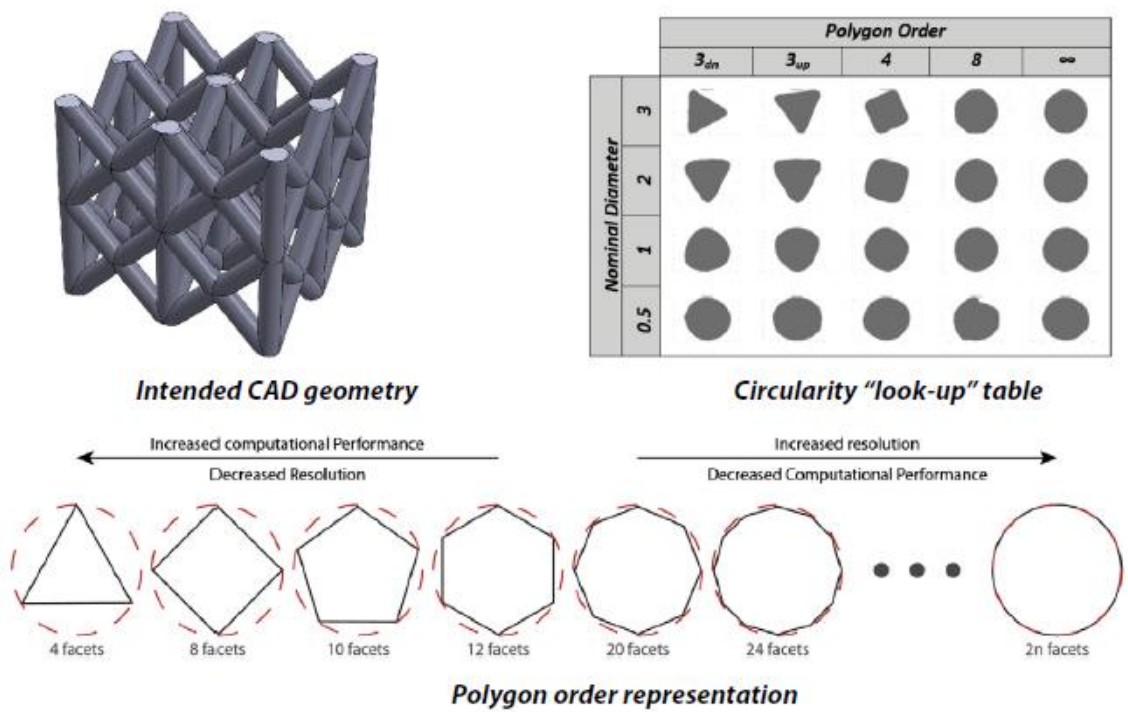
Faceting error incurred by STL data representation can compromise data quality, resulting in visible geometric artefacts and compromised structural response. The specific features of these artefacts are not only dependent on the geometric faceting effect but are also influenced by all phases of the AM design process, including the local geometry, orientation within the build-envelope, material feedstock specifications, prescribed toolpath and process parameters. The practical effect of these errors can be actively mitigated by experimentally quantifying the effect of these variables on the as-manufactured geometry and associated mechanical properties. Various experimental methods exist for quantifying this experimental error, depending on the specific geometry of interest. By way of example, an experimental method to quantify the error introduced in AM cylindrical lattice strut elements is presented below.



Informed STL generation based on intended CAD geometry and experimental data on strut circularity for a given polygon order representation

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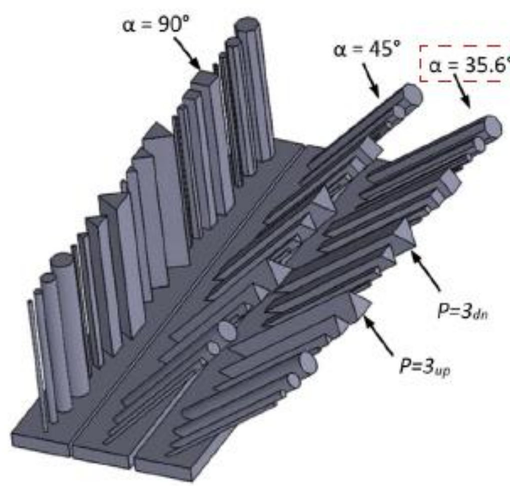
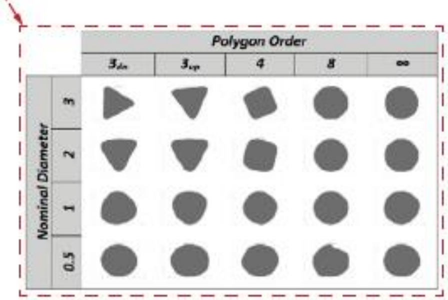
Lattice strut elements are an example of a technically important AM geometry that is challenging to implement in the STL format. Previous figure demonstrates that the digital file size exponentially increases as the allowable error decreases. For this scenario, the relevant functional quality is the associated strut circularity, as measured by the *isoperimetric quotient, Q,* which is the ratio of cross-sectional area, *A*, to the area of a circle with equal perimeter, *p*. It is apparent that, as the number of facets used to represent a strut element increases, the isoperimetric quotient tends to unity e implying that, for a given geometry and material and process parameters, a threshold exists whereby further refinement of the STL data will render no further benefit to technical performance. For a particular AM scenario of interest, the relationship between the STL polygon order and the measured isoperimetric quotient provides an explicit and systematic DFAM tool to quantify the technical limit to the required volumetric data resolution.

$$Q = 4\pi A/p^2$$

A potential experimental approach to quantify the technical limit to the required STL resolution.



Inclination and resolution specimen



Specimen cross section is influenced by local geometry and inclination

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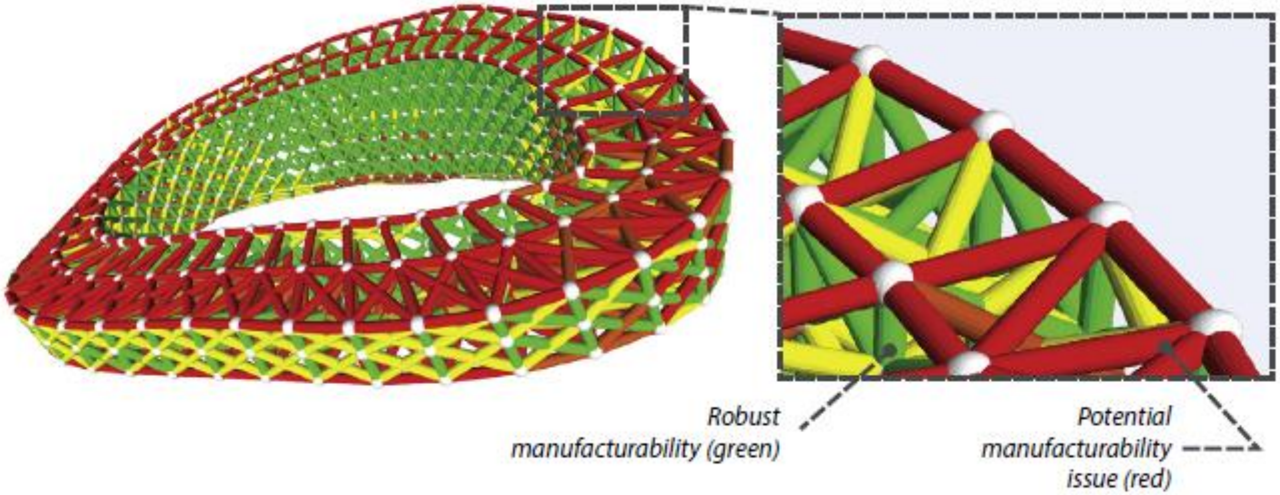
A particular criticism often levelled at the STL file format is a supposed inability to accommodate surface specific data; for example, to identify characteristics such as allowable roughness, intended fit, colour and other surface functions. It is correct that the STL does not explicitly accommodate such data; however, a relatively simple workaround allows this problem to be largely circumvented. As identified in previous Table, the standard binary STL format accommodates a user-definable header (UINT8), as well as a free attribute byte count (UINT16) of data per facet. These user-definable data structures provide an informal mechanism for the accommodation of relevant surface data. For example, the header can be used to specify the global classification of surface functional requirements (such as nominal colour or preferred layer thickness) and this global classification can then be locally modified at local facets by modifying the attribute byte count data. The attribute byte count is specified by an unsigned 16 bit integer (UINT16) and thereby allows 65,536 unique combinations of surface data inputs. Hybrid data, for example integrated colour and allowable roughness, can be specified by a look-up table. A potential limitation of this method is that additional facets may need to be defined to identify surface feature attributes that are incompatible with the facets required for geometric specification.

Informal method to integrate surface specific data (in this case, colour) within the STL format.



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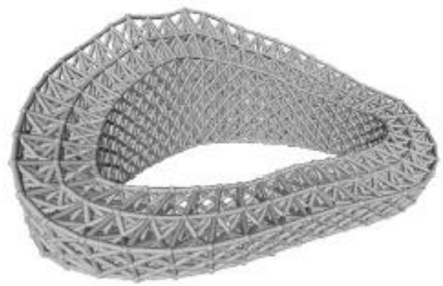
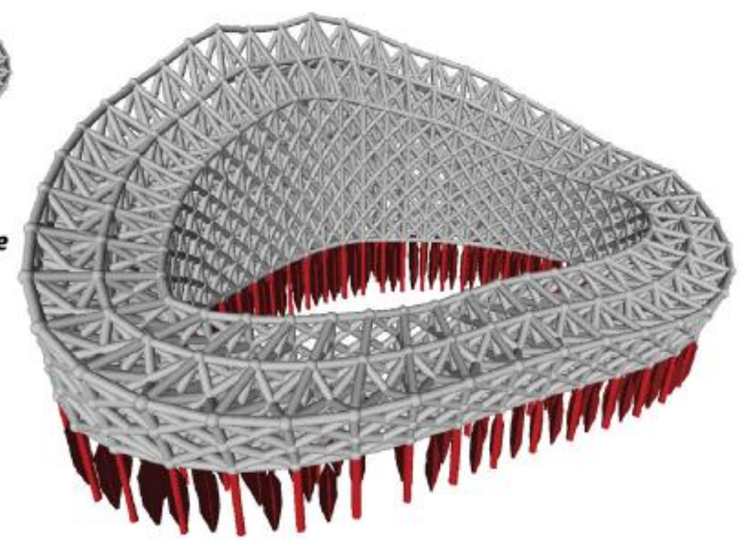
Another often-cited challenge associated with the STL file format is its inability to accommodate multiple volumetric qualities. For example, the STL format cannot formally accommodate build and support materials concurrently within a single data file. This is technically correct; however, informal methods exist to accommodate this challenge. One method is to generate a unique STL file for each unique volume required. The function of these unique files can then be informally identified within the file header data (UINT8, previous table) as to their intended function. Custom scripts or manual processing can then be used to integrate these data sets as required, for example, in the specification of support structures and manufactured geometry with unique process parameter attributes.

Informal method to accommodate distinct volumetric data within the STL format. Support structure and manufactured geometry are informally distinguished by unique STL files that are then integrated with custom scripts.



Manufactured geometry data file

Support structure data file



Integrated data file allowing unique geometry and support processing parametrers

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Overall, the STL format has proven remarkably compatible and resilient to the needs of AM. This is not particularly surprising, as, although the technical requirements for AM systems have considerably evolved, the format was initially developed as a fundamental surface data format for the stereolithographic AM processes. To provide an informed review of the compatibility of the STL format for the requirements of current AM technologies, the associated advantages and disadvantages are enumerated below.

Advantages of the STL file format include:

- the simplicity of data structure definition
- an industry standard input for the slicing phase
- both ASCII and binary versions accommodate readability and storage efficiency, respectively
- compatibility with slicing in any orientation and with any layer thickness
- opportunities for informally accommodating surface and volumetric data.

Disadvantages of STL file format include:

- potentially large file size, especially for curvilinear geometries
- challenges in incorporating additional surface and volumetric functionality, for example, formally accommodating surface and volumetric data
- challenges in accommodating multiple independent volumes, and associated volume specific data
- computational challenges in generating slice data.

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In response to the challenges associated with the available data files, significant research effort has been directed towards the development of novel DFAM formats for surface specification. Of these developed formats, of interest are the Polygon File Format (PLY), Additive Manufacturing Format (AMF) and 3D Manufacturing Format (3MF). These formats were developed to overcome specific limitations of STL including the accommodation of multiple unique volumes, formal definitions of surface attributes, and unlimited character header files and comments. The PLY format was primarily developed to enable the storage of sophisticated data sets generated by 3D scanning. The AMF and 3MF format represent similar efforts to enhance data storage for AM. Industry consortia developed the 3MF format; whereas international standards organizations developed the AMF, which is presented in detail below.

2.2 Additive Manufacturing Format (AMF)

It is apparent that the stereolithographic format is useful, but not necessarily optimal for AM applications. In response, significant research and development effort has been applied to develop custom file formats that are tailored to the specific requirements of current and evolving AM technologies. The AMF is an example of such a DFAM tool. To encourage application and research contributions in the use of AMF, the important AM data structure attributes are described in detail below.

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The AMF format was formally developed based on recommendations by ASTM Committee F42 on Additive Manufacturing Technologies with the following considerations:

- open source to encourage universal application
- technology independence such that the data format can be compatible with any AM technology
- process independence such that the data does not restrict process settings such as slice thickness
- simple data structures such that understanding is promoted, while data repetition is to be avoided
- efficient data storage, including efficient scaling of file size with geometric complexity, increasing AM resolution, periodic geometric structures, and multiple concurrent structures
- efficiency of data read and write operations
- capability to describe geometric units
- backwards compatibility, allowing for example, AMF versions of existing STL data to be generated
- forwards compatibility, allowing AMF to be converted to STL for use in legacy systems
- future-technology-proofing such that future generations of AM technology can be supported by the standard AMF format.

The AMF format was initially released in 2011, and in 2013 the management of AMF became the joint responsibility of ASTM and ISO. AMF is defined in XML format and is compressed according to the ZIP compression protocol. AMF may be defined in binary or ASCII by a data structure including the following five high-level elements:

- object - definition of volumes of specific materials
- material - specific printing material with unique identification
- texture - images or textures for surface mapping with unique identification
- constellation - hierarchical combination of objects in a relative pattern
- metadata - additional information on objects to accommodate future AM technology needs.

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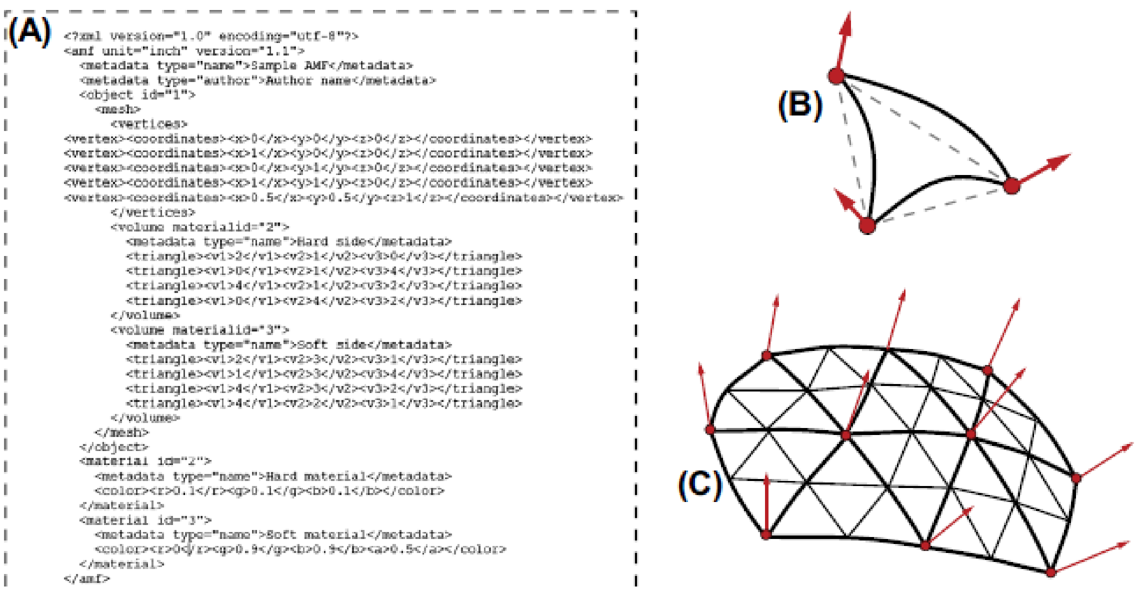
Geometry is then specified for each object as a face-vertex polygon mesh as follows:

- vertices - sequentially listed (from zero) in order of their definition to define the object mesh
- coordinates - a child of the vertices defines the position of each vertex in Cartesian 3D space
- volume - encapsulates an enclosed volume of vertices (vertices can be shared, but volumes must not overlap)
- triangle - a child element of a volume that specifies triangles by reference to three vertices.

The triangle geometry interlocks to define a continuous surface but differs from the STL facet. Each triangle is numbered from zero in the order of definition and is defined according to the right-hand rule in clockwise order when viewed from outside of the surface; this method eliminates the need for the specific surface normal of STL. Furthermore, these triangles may be curved to increase the associated geometric resolution.



(A) Sample additive manufacturing format (AMF) data file, (B) curved patch with tangent normals, (C) recursively defined self-tessellation.

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Additional attributes of interest to the AMF format include the explicit accommodation of: colour data, texture data, graded materials, periodic structures, multiple independent geometries, comments and unlimited metadata for the accommodation of future AM digital requirements.

	STL	*PLY*	*3MF*	*AMF*
Formal method for specifying surface attributes	N	Y	Y	Y
Informal method for specifying surface attributes	Y	–	–	–
Formally accommodates surface curvature	N	N	Y	Y
Accommodates periodic structures	N	N	Y	Y

Table above summarizes the various attributes of AM surface data formats considered. It is apparent that the AMF format formalizes many of the limitations of STL format. However, the STL does provide the capacity for many of these attributes in an informal manner. The AMF format provides higher storage efficiency for curvilinear geometry, in particular by allowing the formal definition of curved triangular patches. However, this advantage is tempered by the inherent computational challenges associated with characterizing the intersections of this curved patch with layerwise slices. Intersections can be identified by recursively representing the patch by successively smaller sub-patches up to the required resolution, although this may incur high computational cost at the slice data phase. Associated DFAM tools are emerging in response to the need for advanced understanding and utilization of second-generation AM data structures such as the AMF.

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Although AMF has addressed some of the challenges associated with AM data formats, it is apparent that open DFAM research questions remain regarding volumetric data storage, and that these challenges must be intelligently accommodated by the designer in order to enable commercially robust design outcomes.

Despite formally defined specifications for AM data formats, it is commonly observed in commercial practice that volumetric digital data is corrupt, incorrect or incomplete; potentially resulting in flawed outcomes in the following design phases. In response to this potential risk, numerous commercial and open-source DFAM tools have been developed that attempt to identify and correct these flaws; which typically include incorrectly defined surface normals, invalid facet definitions, and nonmanifold (watertight) surface definitions. These tools include methods that range from autonomous systems that attempt to algorithmically resolve these flawed surface representations, to manual methods that allow the designer to provide input on the preferred technical response. Although technical DFAM tools exist, the challenges associated with ensuring robust surface data representations and the associated tools to correct these identified flaws remains an open research question.

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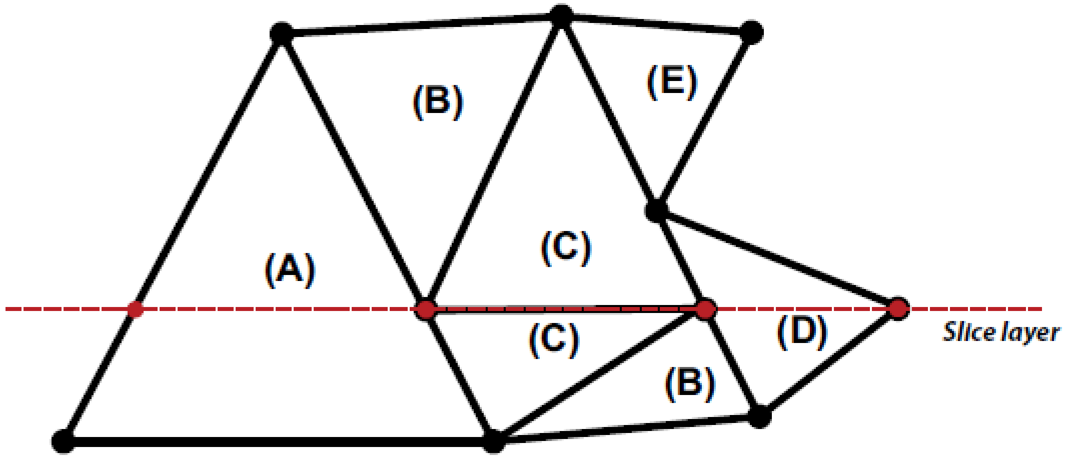
3. Slice Geometry

AM technologies typically fabricate the intended three-dimensional component volume by the addition of materials within consecutively offset layers. The slice geometry phase refers to the digital generation of component perimeters at these consecutive layers.

A brute force slicing approach for STL/AMF can be implemented as follows. A plane is defined at an offset from the build platen that corresponds to the local layer thickness. For this defined plane, each triangular facet is sequentially examined, and recursively generated AMF facets must be explicitly assessed (these can be either unpacked within a reference data repository or can be calculated on the fly for each facet of interest). For both the STL and AMF formats, each facet is identified by three spatial coordinates. If these coordinates are ordered according to their height from the build platen (Z-value), the existence of an intersection and the associated segment is then readily characterized. Identifying the intersection of each facet with the plane of interest allows a set of polygons to be generated that defines the associated layer slice.



Possible intersection scenarios for slice layer and facet intersection: (A) slice plane intersects two facet edges, (B) plane intersects one vertex only, (C) plane is collinear with facet edge, (D) plane intersects one vertex and one facet edge, (E) no intersection.

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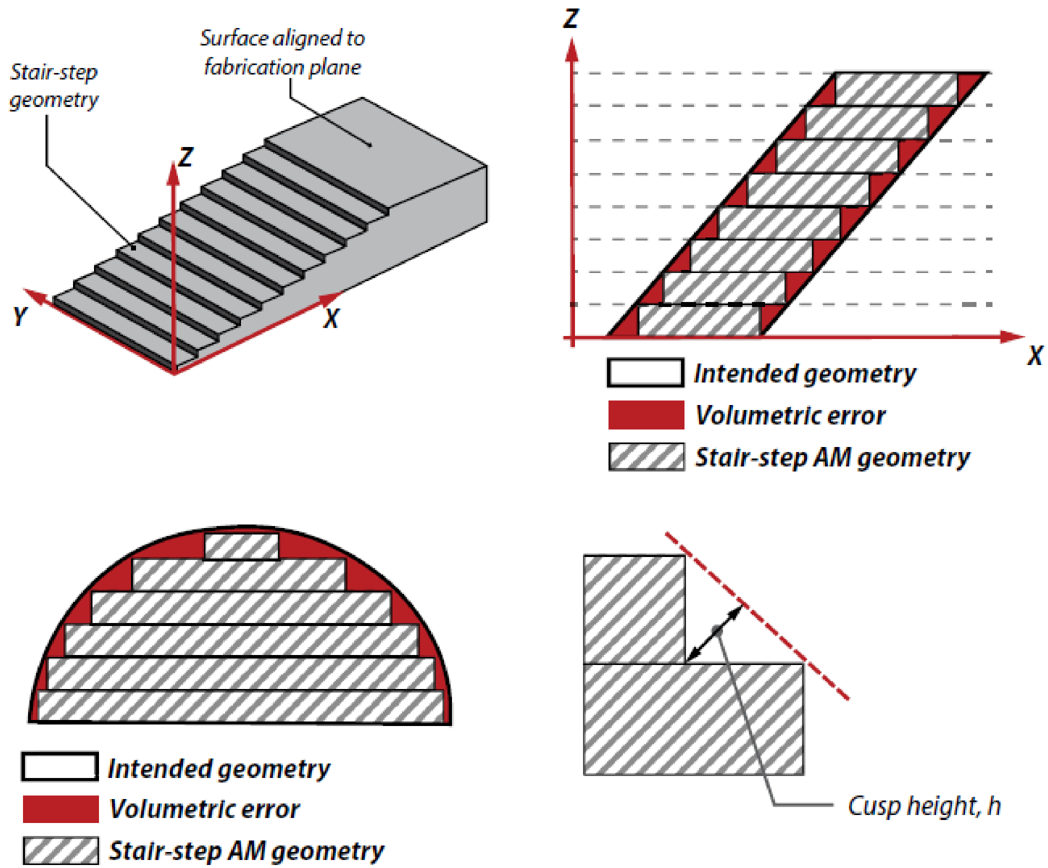
This brute-force approach is mathematically robust but can become computationally challenging for relatively large input data files. An alternative approach is to pre-process the associated facets such that they are then categorized according to which layers they are associated. This pre-processing requires an initial processing step but can reduce the overall computation time for large data sets. Such data management tools for slicing optimization remain an open DFAM research opportunity. For those seeking to enhance the computational performance of slicing operations, it is important to note that, computationally, *the surface volume data file (either STL or AMF) can be processed in parallel for each layer, as the associated slice data is independent of other layers*.

The slicing of a continuous geometry into a finite number of prismatic volumes inherently introduces geometric error, specifically known as the stair-step effect. The stair-step effect is especially significant as the inclination angle, a, becomes increasingly acute, and is zero for structures that are either horizontal or vertical. Metrics introduced to assess the error associated with a slicing operation include the cusp height and volumetric error. Cusp height error is measured as the maximum distance measured by a normal chord between the sliced geometry surface and the design surface. Alternatively, volumetric error may be computed between the sum volume of all associated prismatic sections and the intended component volume. Depending on the specific scenario, either of these slicing error estimates may be more appropriate.



Stair-step effect associated with a specific inclination angle (upper), and associated error measures, including volumetric error and cusp height.

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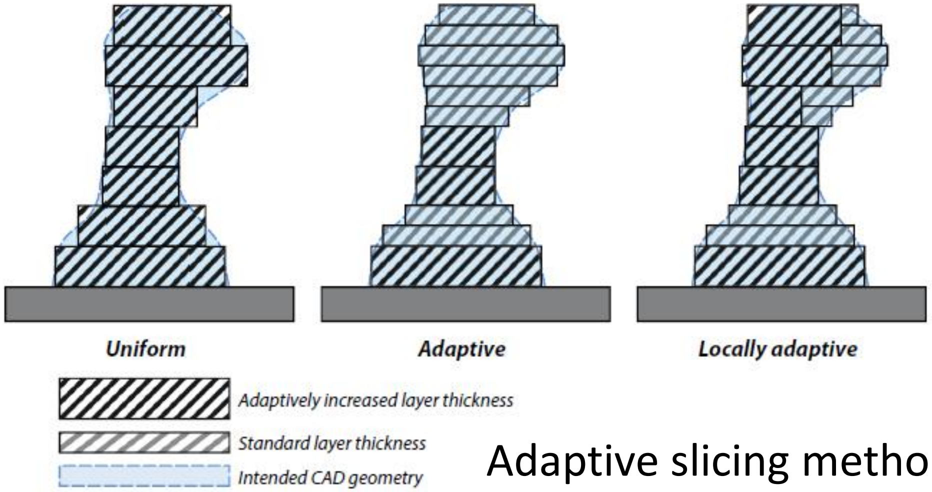
DFAM methods have been proposed to mitigate the measured slice error. For example, *adaptive slicing* refers to the slicing of AM part geometry with slice thickness varying according to allowable values of local slice error. Adaptive slicing requires that the feasible slice thicknesses for the intended AM technology be known a priori; however, it can provide significant technical advantages including a reduction in manufacturing time without compromising slice error, as well as a potential for reduced slice file size. *A variant of adaptive slicing allows the processing parameters to be locally modified* such that the externally facing surfaces to be fabricated with a smaller slice thickness that the internal core structure; this method can potentially allow further reduced manufacturing time without compromising surface error.

A slice data attribute that is often overlooked is the resolution of the curve-fitting polygon used to represent the slice data. The rationale for overlooking this attribute is likely due to the significantly higher resolution inherent in the XY-plane for AM systems. However, for high-resolution volumetric data with small feature size, the number of facet interactions, and consequently the number of data points within the slice polygon, can become large. In practice the resolution of this polygon may exceed the manufacturable resolution of the intended AM technology; for these scenarios it is useful to reduce the polygon resolution according to the AM manufacturability to avoid excessive file size. Alternatively, the polygon can potentially be replaced with a spline or other curvilinear geometry format, with the objective of reducing the data file size required to achieve an allowable error in the manufactured geometry. These methods are compatible with advanced design methods that omit traditional phases for AM slicing, especially those that generate slice data directly from the original CAD.



Adaptive slicing methods to mitigate slice error

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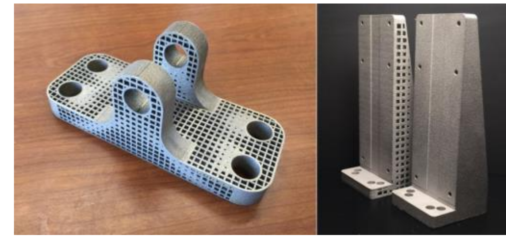
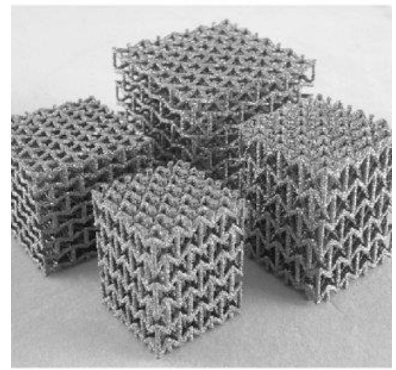
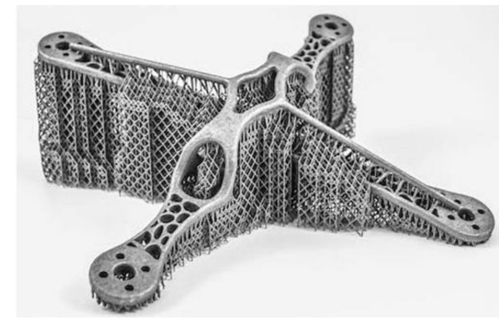
Classification of Cellular Materials

Open (pore)

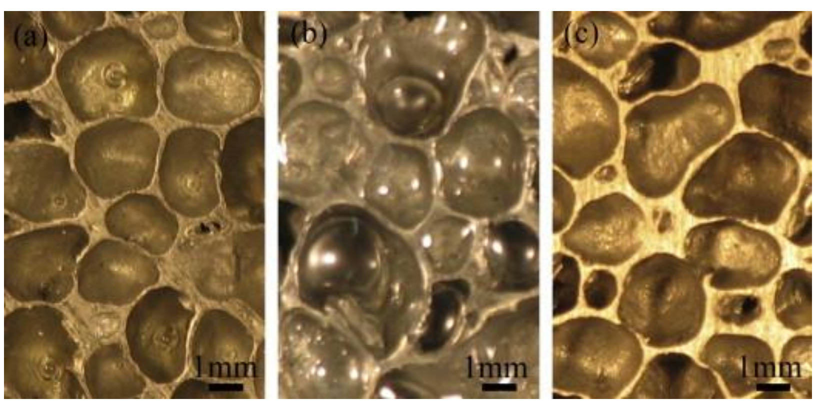


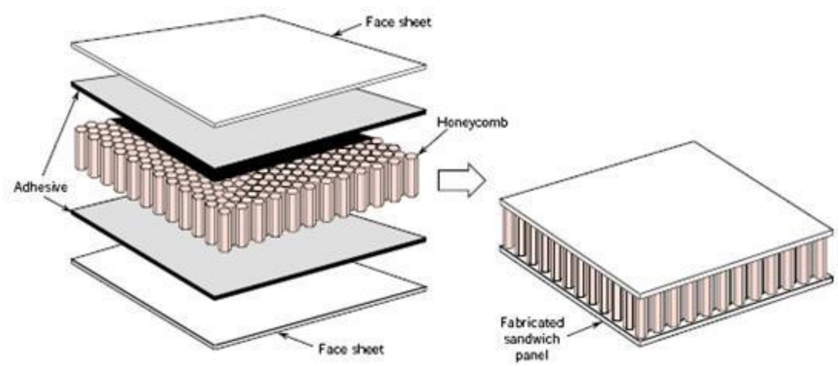




Random

Regular



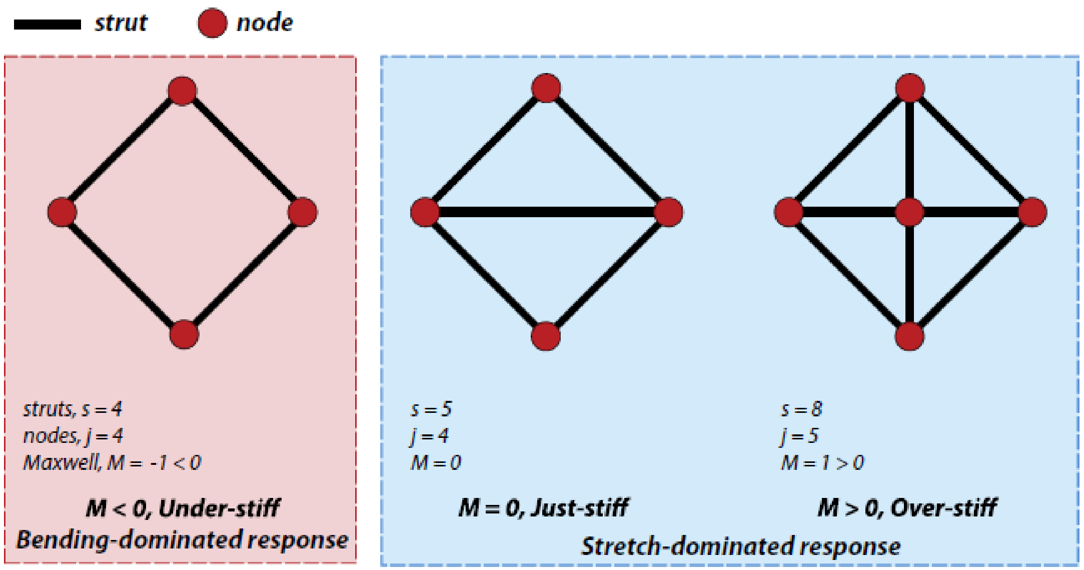


Close

Lecture 20: Digital design of lattice and zero-mean curvature structures

4. Lattice structures

Lattice structures refer to the open-celled arrangement of strut elements with defined connectivity at specified nodes. These lattice arrangements are readily tessellated to fill space and allow highly efficient paths for the grounding of external and internal loads. Consequently, lattice structures are often observed in both naturally occurring biological systems such as the cellular structures of trabecular bone, and in engineered structural systems, for example three-dimensional truss structures used to efficiently ground loads in civil engineering structures. Lattice structures may have either stochastic or periodic arrangements. Biological lattice systems generally feature stochastic arrangements, whereas engineered lattice structures are typically periodic; however engineered structures with stochastic and hierarchical attributes are emerging within the research literature and in commercial applications.



Schematic representation of potential planar lattice configurations, Maxwell number, M and associated response type, either bending or stretch-dominated

Lecture 20: Digital design of lattice and zero-mean curvature structures

4.1 Maxwell stability criterion

Triangulated truss structures (known as space-frames in 3D space) provide a fundamentally efficient structural system. Such truss structures are typically designed to ground internal and external loads by a combination of tensile and compressive axial loading within the associated strut elements. For external loads to be robustly equilibrated by internal forces within the truss structure, a sufficient number of strut elements with appropriate nodal connectivity must exist. Maxwell proposed a set of mathematical conditions that must be satisfied for the loads be grounded in a mechanically robust manner. The Maxwell stability criterion is stated such that there are 2 free equilibrium-equations (3 for 3D space) associated with each node, *j*, and that each strut, *s*, represents an unknown equilibrating force; furthermore, there are 3 external forces (6 for 3D space), resulting in the inequalities

$$M = s - 2j + 3, \text{ for planar (2D) truss systems}$$

$$M = s - 3j + 6, \text{ for 3D space-frames}$$

For scenarios where M < 0, there are insufficient struts to equilibrate the external forces, and the under-stiff truss system becomes a mechanism. For M = 0, the strut elements are arranged such that the strut loading is determinant for any external loading; such structures are defined as just-stiff and contain no structurally redundant strut elements. Additional strut elements can further increase the Maxwell number, M > 0, making the truss over-stiff.

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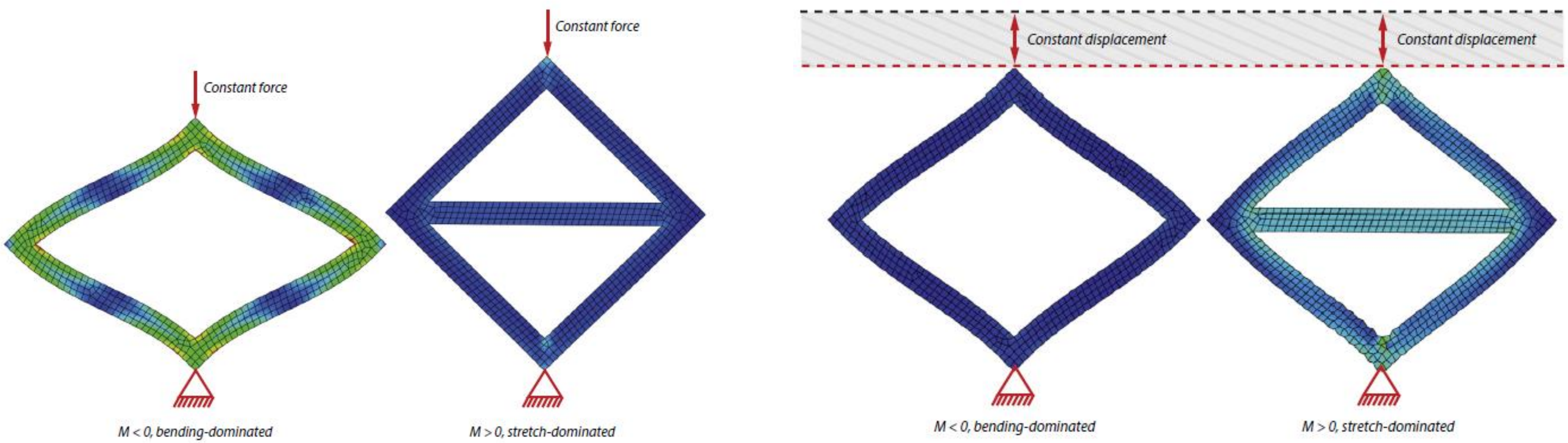
4.2 Observed mechanical response (unit cell)

For AM lattice structures, the analogy of truss structures breaks down, as the nodes are not pin-jointed (as in a typical truss system) but are of a continuous structure that can accommodate the transfer of bending across node elements, i.e. the end-fixity is encastre or built-in. Despite the imperfect analogy, the Maxwell stability criterion enables a profound insight into the mechanical response of proposed lattice topologies. For AM lattice structures that are under-stiff, lattice deformation is resisted by bending at the nodes. Due to the large strains induced by the associated bending stresses, these structures are highly compliant, and are known as bending-dominated ($M < 0$). Conversely, for structures that are either just-stiff or over-stiff, loads are grounded by tension and compression of the strut elements only, and there is no bending applied across the node. These structures are known as stretch-dominated ($M>=0$) and display relatively high stiffness.



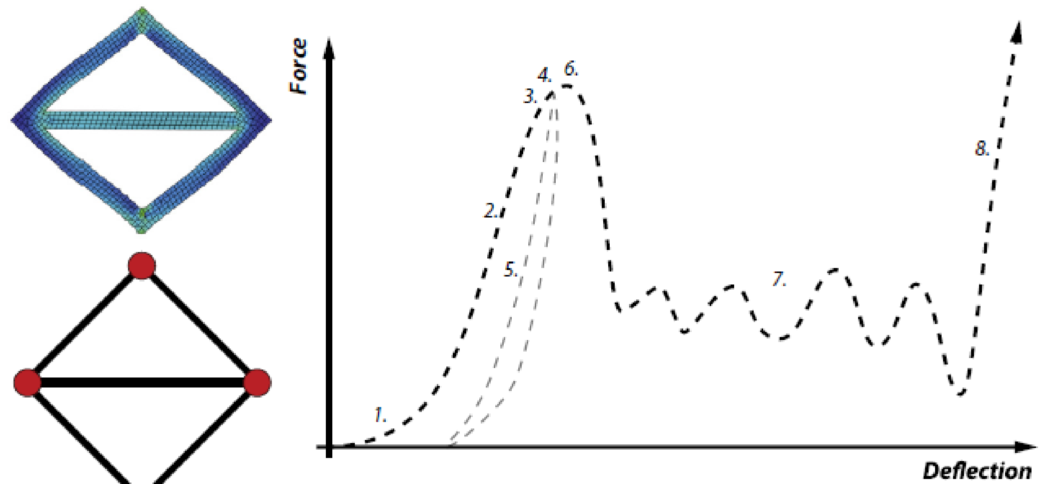
Mechanical response to a constant force (left) and displacement (right) in bending-dominated and stretch-dominated Structures. Colour indicates local stress

Lecture 20: Digital design of lattice and zero-mean curvature structures

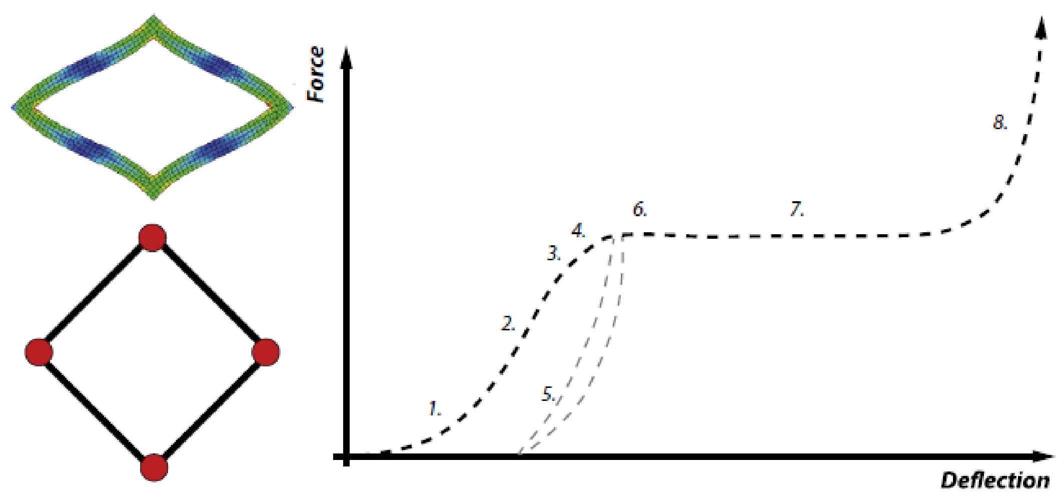
4.3 Observed mechanical response (lattice system)

Lattice system typically consists of an assemblage of individual unit cells. The mechanical response of such a lattice system is dependent on the structural response according to the Maxwell stability criterion. The overall behaviour of AM lattice systems has been documented both experimentally and mechanistically and may involve the following observed responses:

1. Initial plastic consolidation
2. Linear elastic response
3. Non-linear elastic response
4. Yield strength, or elastic limit
5. Unloading Modulus
6. Ultimate compressive strength
7. Crushing strength
8. Densification



Stretch-dominated response of lattice system



Bending-dominated response of lattice system

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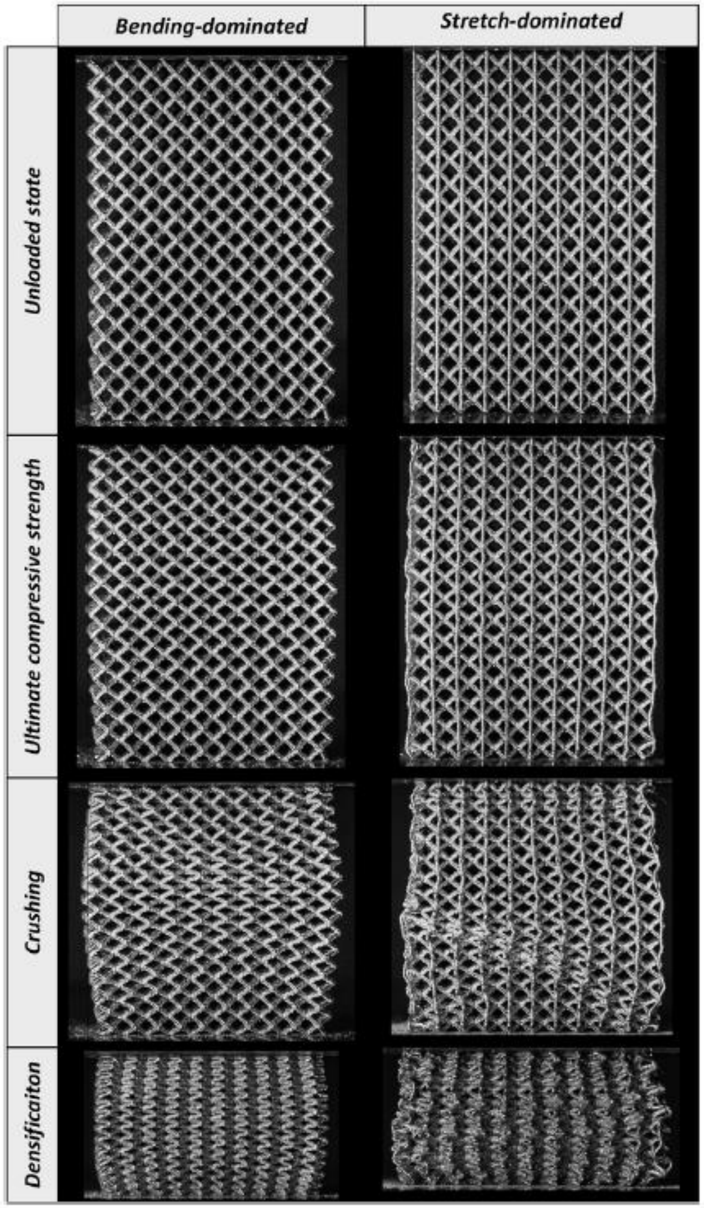

Observed mechanical response of lattice structure displaying bending-dominated (left) and stretch-dominated response. Specimens fabricated by Selective Laser Melting (SLM) with Inconel 625; a highly ductile material that provides insight into the full range of potential lattice deformation response.

These observations apply to lattice structures that are distinctly within either stretch- or bending-dominated regimes. Additional complexity occurs when structures display transitional behaviour between these distinct regimes. These observations are mostly experimental in nature but present a commercially valuable research opportunity for the design of AM structures with engineered mechanical response that would be technically infeasible with traditional manufacturing methods. Cost-effective design of such structures requires further understanding within both experimental and numerical research disciplines.

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4.4 Prediction of AM lattice response

Numerous predictive models have been established to relate the lattice topology and material properties to the observed mechanical properties, most notably the seminal Gibson-Ashby model. This fundamental model is derived from first principles based on insights into cellular geometry and associated mechanical failure modes, resulting in predictive relationships of the type defined below. Specifically, these relationships predict the lattice mechanical response, P, to be proportional to some known response of the solid material P_s, and the ratio of lattice to solid material density, ρ^*/ρ_s, to the power of some exponent, *n*, and scaled by some proportionality constant, *C*. The predicted exponent depends on whether the structure is bending- or stretch-dominated as summarized in next Table for a number of pertinent scenarios.

$$P^* = C\, P_S \left(\frac{\rho^*}{\rho_S}\right)^n$$

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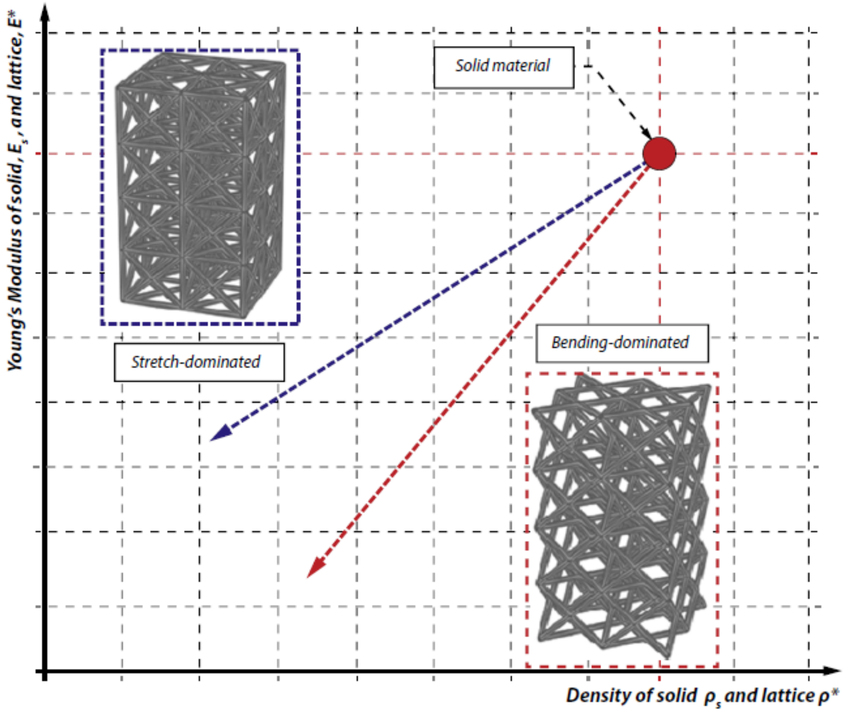
Response type	Mechanical property	Relationship
Bending-dominated	Modulus (E^*)	$E^* = CE_s\left(\frac{\rho^*}{\rho_s}\right)^2$
	Plastic collapse $\left(\sigma^*_{pl}\right)$	$\sigma^*_{pl} = C\sigma_{y,s}\left(\frac{\rho^*}{\rho_s}\right)^{1.5}$
	Elastic collapse $\left(\sigma^*_{el}\right)$	$\sigma^*_{el} = CE_s\left(\frac{\rho^*}{\rho_s}\right)^2$
Stretch-dominated	Modulus (E^*)	$E^* = CE_s\left(\frac{\rho^*}{\rho_s}\right)^1$
	Plastic collapse $\left(\sigma^*_{pl}\right)$	$\sigma^*_{pl} = C\sigma_{y,s}\left(\frac{\rho^*}{\rho_s}\right)^1$
	Elastic collapse $\left(\sigma^*_{el}\right)$	$\sigma^*_{el} = CE_s\left(\frac{\rho^*}{\rho_s}\right)^2$

A selection of Gibson-Ashby relationships for bending-dominated and stretch-dominated mechanical response.



Gibson-Ashby model illustrating Young's Modulus for bending and stretch-dominated structures with reference to the solid material.

Figure above provides further insight into the range of mechanical responses to be expected based on Gibson-Ashby relationships. Specifically, the Maxwell stability criterion can be used to directly characterize the lattice response as being either bending- or stretch-dominated. From this understanding, the Gibson-Ashby model can be utilized to predict the specific mechanical response of the associated lattice system. These predictions are compatible with the experimentally observed mechanical response of AM lattice structures; However, variability exists due to simplifications inherent in the Gibson-Ashby model and variation between the idealized and as manufactured AM lattice geometry. Due to the potential for variability in Gibson-Ashby model predictions, experimental results or validated numerical predictions are required for the design of high-value AM lattice applications.

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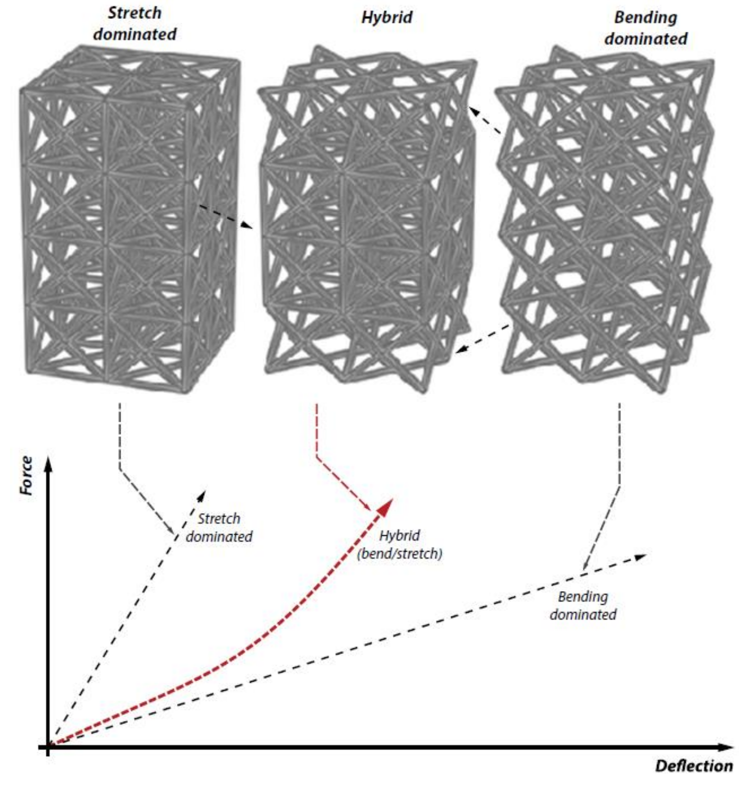
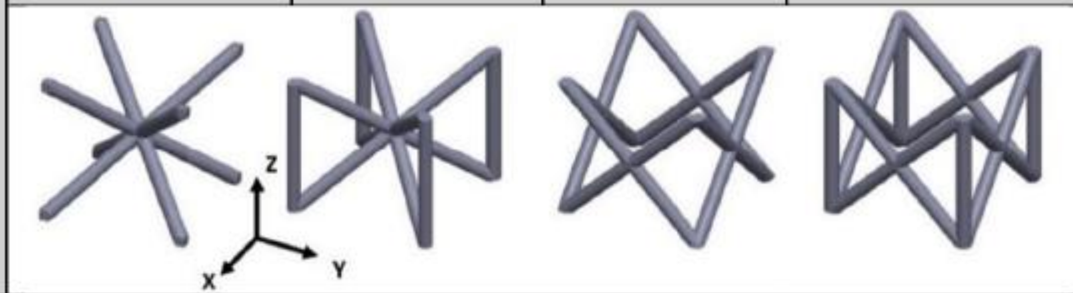
4.5 Hybrid Lattice Design

The nuanced response of practical lattice systems, whereby the local stresses and observed deformation are a function of both the Maxwell number and the associated loading type, enables the design team to make informed and robust engineering design decisions in response to the technical requirements of the AM design project. This insight enables, for example, the specification of functional AM lattice structures that include under-stiff elements to avoid inducing excessive forces in response to the initial displacement and include over-stiff elements to avoid undue deflection.

The strategic functionalization of stretch-dominated and bending-dominated lattice structures enables the manufacture of hybrid lattice systems with bespoke mechanical response.



Lecture 20: Digital design of lattice and zero-mean curvature structures

Cell type	Body Centred Cubic	Body Centred Cubic (Z Struts)	Face Centred Cubic	Face Centred Cubic (Z Struts)
	BCC	BCCZ	FCC	FCCZ
				
Struts, *s*	8	12	16	20
nodes, *j*	9	9	12	12
Strut inclination angles, α	35.3°	35.3°, 90°	45°	90°,45°
Maxwell number, *M*	-13	-9	-14	-10
Strut aligned to load direction	NO	YES	NO	YES

Abbreviated list of candidate lattice unit cells and associated Maxwell number, indicating the existence of strut elements with alignment to loading in the Z-direction, and the inclination, a, of strut elements with respect to the X-Y plane.

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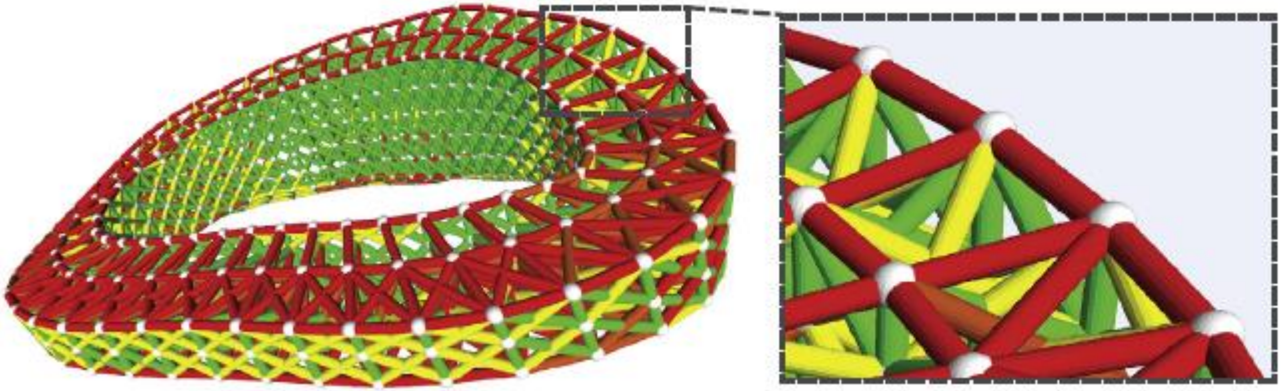
4.6 Conformal lattice structures

Periodic lattice structures provide a basis for the design of engineered lattice structures that are conformal to the required 3D geometry. There are infinite combinations of unit cells that can tessellate to fill space. However, hexahedral (cubic) unit cells are often specified in commercial engineering practice as they are readily defined, provide reasonable flexibility in the design of various lattice structures, have a reasonably well characterized mechanical response, and can be conformal to fill space. An abbreviated list of cubic lattice cells is presented in the previous slide. As the number of cells increases, so does the associated Maxwell number, implying that additional cells provide a stiffening effect, as does the existence of struts aligned to the loading direction. These cubic cells can be permuted to alternative geometry that is conformal in space and can be combined in homogeneous or inhomogeneous distributions of lattice cell elements to manipulate mechanical response as required.

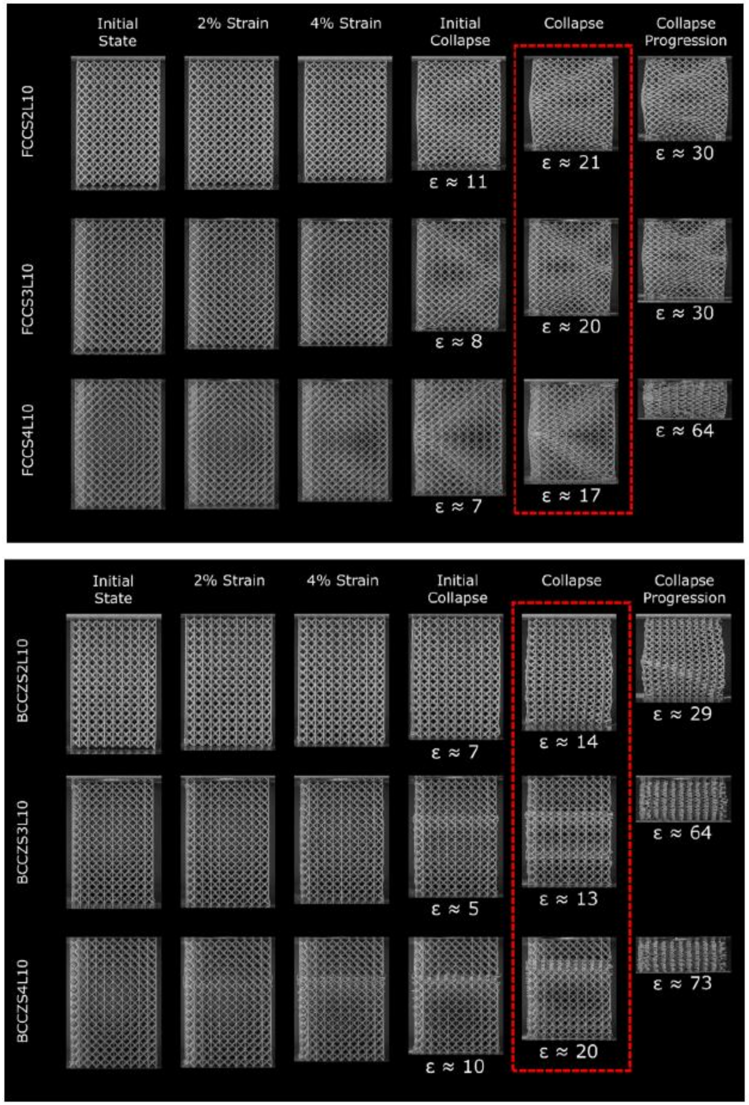
Opportunities to geometrically permute the cubic unit cell to provide conformal geometry, including toroidal and conformally manufactured geometry associated with a bespoke patient-specific medical implant.



Lecture 20: Digital design of lattice and zero-mean curvature structures

4.7 Experimental observation of AM lattice response

Fundamental predictive models of lattice mechanical response such as the as the Gibson-Ashby model provide useful insight into the anticipated mechanical response of a proposed AM lattice system. However, the mechanical response of as-manufactured AM lattice systems is complex and recourse to experimental data is required to provide confidence for the design of high-value applications. For example, the mechanical response of Inconel 625 lattice structures provides a useful basis for experimental investigation. The high ductility of these Inconel structures allows the effect of lattice topology and associated geometry to be characterized for strain up to full-densification. Such data can be used for the validation of proposed numerical models as well as for the identification of unexpected failure modes. For example, a potentially unexpected transition is observed for scenarios that would nominally display stretch-dominated and bending-dominated response and vice versa. This phenomenon appears to be due to a transition between local buckling and local crushing behaviour. This observation provides evidence that topology selection (in this case BCZ and FCC) provides a coarse mechanism for the tuning of mechanical response, whereas local geometry (in this case cell size) provides a fine tuning mechanism. The development of custom DFAM tools to enable these tuning mechanisms for commercial lattice design remains an open research question.



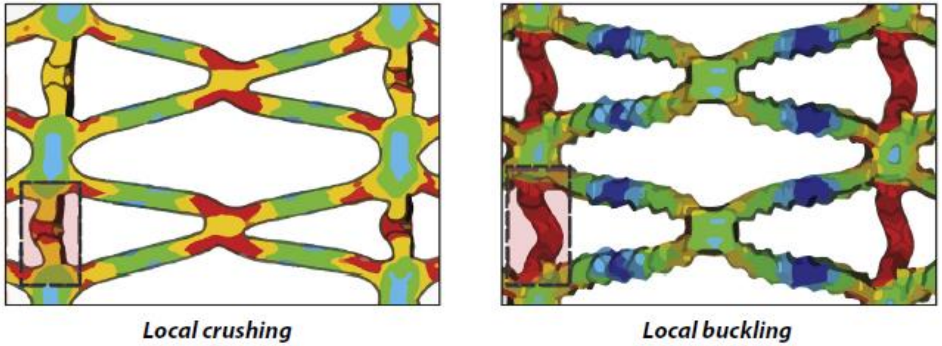
Deformation behaviour of FCC (upper) and BCZ (lower) topologies for cell size of 2, 3 and 4 mm. Red highlighted data identifies that cell size can induce a transition between bending-dominated and stretch-dominated failure mechanisms

Lecture 20: Digital design of lattice and zero-mean curvature structures

4.8 Numerical analysis of AM lattice

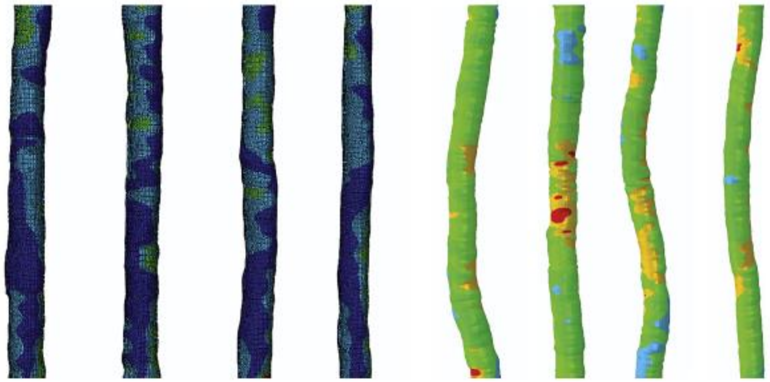
Robust numerical analysis of AM lattice structures can enable profound insight into the fundamental basis for experimentally observed mechanical response. As an example of this opportunity, the lattice elements of previous figure that observed unexpected transitional behavior are analyzed numerically to reveal the effects of local buckling and crushing as being a driving mechanism for this observed transition (figure below).

Numerical analysis applied to generate insight into the experimentally observed failure mechanisms of previous figure. In this case, for a constant topology (BCZ), local geometry initiates a transition between crushing and local buckling.



Despite the opportunities for numerical analysis to add value to AM research and design, there exist a number of technical challenges to its robust and timely application. These challenges provide a roadmap to the commercially useful DFAM research contributions and include: 1. the curse of dimensionality; 2. scale effects; 3. Materials data availability; 4. Geometrical data acquisition

A series of pseudo-random strut elements (left) generated by statistical analysis of micro CT imaging of the AM strut elements of figure in the previous slide. Such methods enable the generation of plausible representations of complex lattice systems that display the statistically variable deformation behavior expected of as-manufactured strut elements (right), but without the measurement costs of explicit manufacture and imaging.



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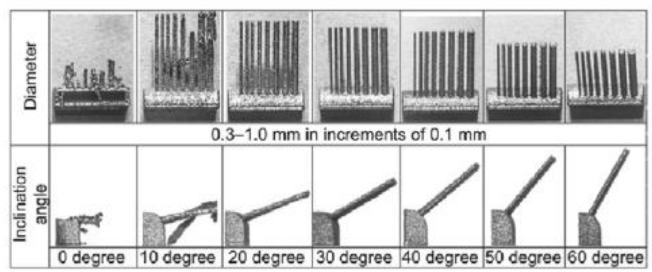
4.9 Lattice manufacturability

Struts in the loading direction can ground loads with high efficiency, resulting in a stretch-dominated response for scenarios that are actually under-stiff according to a fundamental Maxwell analysis. This observation can be useful for the design of lattice structural elements that are technically challenging for AM; for example, horizontally-oriented lattice struts can be challenging to manufacture; these challenging struts can potentially be omitted while maintaining stretch-dominated response if the loading direction is dominantly aligned to the remaining strut elements.

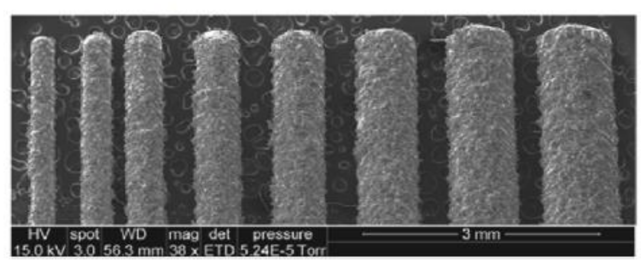
Accommodating these technical lattice design constraints requires formal definitions of AM manufacturability. The specific requirements of these definitions vary according to the specific AM hardware and intended application, but typically include assessment of the effect of inclination angle, a, and specimen diameter, d, on local geometry and associated internal defects; effect of horizontal span parallel to the plate; and the local artefacts generated by this geometry for specific process parameters and material input. These effects are typically quantified by a DFAM design table that characterizes the observed AM manufacturability and allows robust design decisions to be confidently made, as well as providing a basis for the expert systems required for generative design.

Schematic representation of potential definitions of AM lattice manufacturability and the associated DFAM design table



Inclination angle specimens

Horizontal span specimens



SEM examination of local artefacts

Material		Ti6Al4V									
Diam. (mm)		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Angle (degrees)	0	L	L	L	L	L	L	L	L	L	L
	10	L	L	M	M	M	M	M	L	L	L
	20	L	L	H	H	H	H	H	H	H	H
	30	L	M	H	H	H	H	H	H	H	H
	40	L	H	H	H	H	H	H	H	H	H
	50	L	H	H	H	H	H	H	H	H	H
	90	L	H	H	H	H	H	H	H	H	H

DFAM manuacturability table

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5. Triply Periodic Minimal Surfaces (TPMS)

5.1 Physical and mathematical definitions

Soap bubble films are a familiar example of a minimal surface. These surfaces take up their complex forms in response to their physical obligation to balance overall surface tensions. In doing so they generate surfaces that minimize local surface area and balance curvature; specifically, that any infinitesimal surface area cut from a minimal surface results in the smallest possible physical area for the given surface boundary. Furthermore, the curvature along principal curvature planes, k_1 and k_2, is equal and opposite at every point; the mean local curvature, H, is zero; and the Gauss curvature, K, is less than zero.

$$H = (k_1 + k_2)/2 \quad \text{Mean curvature, } H = 0 \text{ for minimal surface}$$

$$K = k_1 k_2 \quad \text{Gauss curvature, } K < 0 \text{ for minimal surface}$$

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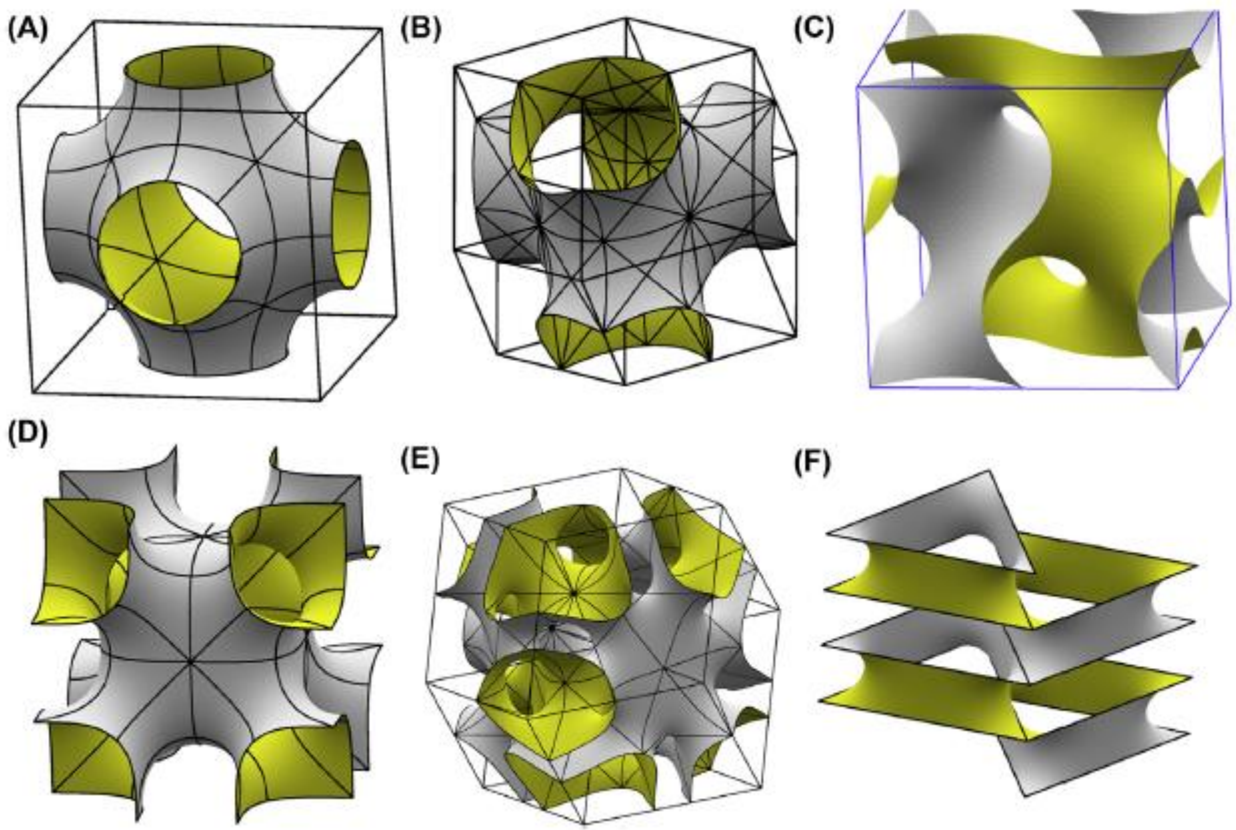

Range of sample unit cells from the range of known TPMS: (A) Schwarz' P Surface cubic unit cell, (B) Schwarz' D Surface rhombic dodecahedron unit cell, (C) Schoen's Gyroid Surface cubic unit cell, (D) Schoen's I-WP Surface, (E) Schoen's Complementary D Surface with rhombic dodecahedral unit cell, (F) Schwarz' H Surface with triangular prism unit cell. TPMS provide the useful engineering quality of filling 3D space with various crystalline symmetries. Labyrinthine domains are colour coded

Lecture 20: Digital design of lattice and zero-mean curvature structures

To be useful for engineered applications, it is desirable that the minimal surface be predictable in filling 3D space. Minimal surfaces that fill space according to crystallographic symmetries are defined as Triply Periodic Minimal Surfaces (TPMS); these minimal surfaces include many variants of technical interest and are finding increased application in the design of high value AM engineering structures.

Of the minimal surfaces shown previously, the Diamond (D), Gyroid (G) and Primitive (P) surfaces have cubic crystalline symmetry, and have been shown to be approximated by level-set equations in local Cartesian coordinates, X, Y, Z, according to a specified iso-value, t. The fundamental D, G, and P surfaces correspond to an iso-value of zero; however, the iso-value may be modified to generate TPMS variants, including structures that are representative of trabecular bone.

$$\cos(Z)\sin(X+Y)+\sin(Z)\cos(X-Y)=t \quad \text{Diamond (D) surface}$$

$$\sin(X)\cos(Y)+\sin(Y)\cos(Z)+\sin(Z)\cos(X)=t \quad \text{Gyroid (G) surface}$$

$$\cos(X)+\cos(Y)+\cos(Z)=t \quad \text{Primitive (P) surface}$$

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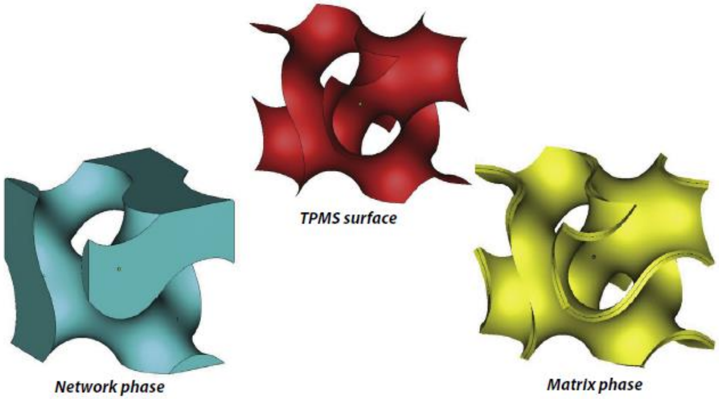
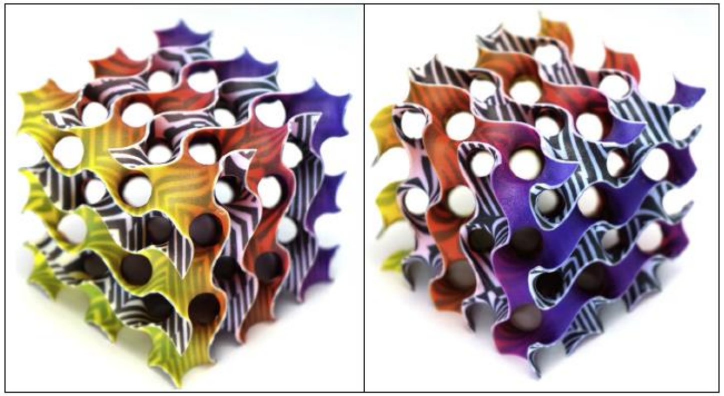


Schematic representation of the TPMS surface and derived network and matrix phase structures.


Material Jetting AM technology applied to visualise the complexity of the TPMS gyroid structure, including colorized labyrinthine domains.

The TPMS are defined by a level-set relationship in terms of the iso-value, t. To generate a physical structure with tangible volume, these level-set relationships can be thickened by some finite distance about t = 0. Alternatively, these equations isolate the bounding volume into labyrinthine domains that can provide a basis for the Boolean selection of a physical volume. In the first case, these domains are in intimate proximity, but remain physically separated without enfolded voids. These structures, are termed matrix phase, can be utilized in the fabrication of heat exchangers and osmotic devices, whereby fluids are physically (or functionally) isolated; or in the design of systems with high mechanical efficiency. The second case, known as network phase systems, is an active research area that shows promise in generating mechanical systems that efficiently ground external loads, especially for fatigue-limited scenarios where geometric stress concentrations are to be avoided. Of the known TPMS permutations, the Gyroid, identified by Schoen in 1970 has captured much engineering interest due to its remarkable geometric characteristics and mechanical properties. The Gyroid has no reflection symmetry or linear features, enabling the mitigation of stress concentrations when subject to mechanical loading: this outcome is particularly important for weak-link failure modes such as fatigue.

Lecture 20: Digital design of lattice and zero-mean curvature structures

5.2 Technical application of TPMS

Minimal surfaces are known to be important biological constructs that are regularly observed in nature. The manufacture of minimal surfaces for engineering applications has traditionally been highly challenging; however, AM technologies are eminently compatible with the associated geometric complexity. Consequently, the technical application of minimal surfaces fabricated with AM technologies has exponentially increased. The following examples are offered as a non-exhaustive summary of the current state of the art of TPMS application enabled by AM; however, we can expect this unique application area to manifest in profound and surprising research outcomes and commercial offerings.

The existing literature in this space is rapidly expanding, particularly in the application of TPMS to biological systems; an outcome that is to be expected given the observed instances of TPMS in naturally occurring biological systems. Biological areas of interest include the application of TPMS to the generation of biocompatible engineered structures with mechanical properties compatible with naturally occurring human bone. The research summarized identifies that both EBM and SLM systems are capable of generating TPMS structures that match the observed stiffness of human trabecular bone. In addition to the application of metallic AM systems, polymeric AM provides an opportunity for the design of bio-implants that can be functionally graded to engineer a biological response or to facilitate interaction with host biological systems.

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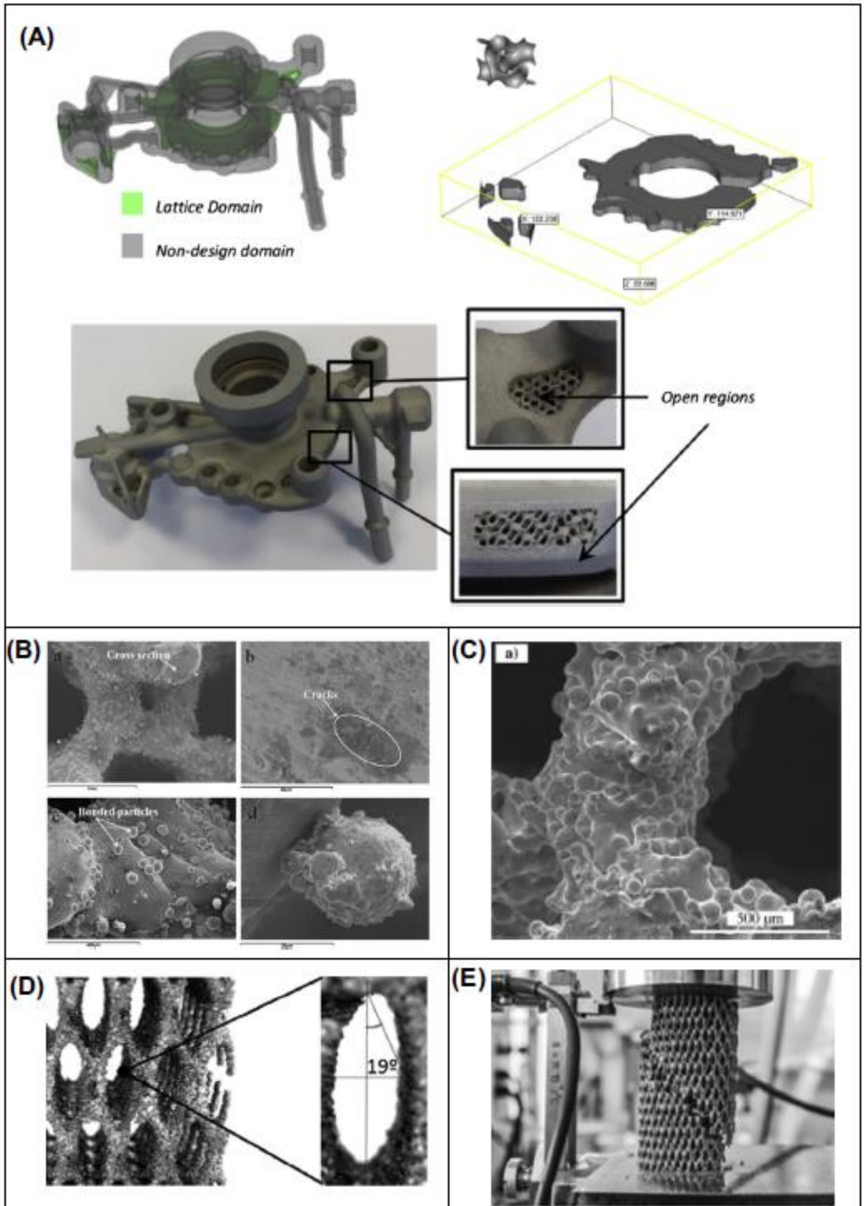
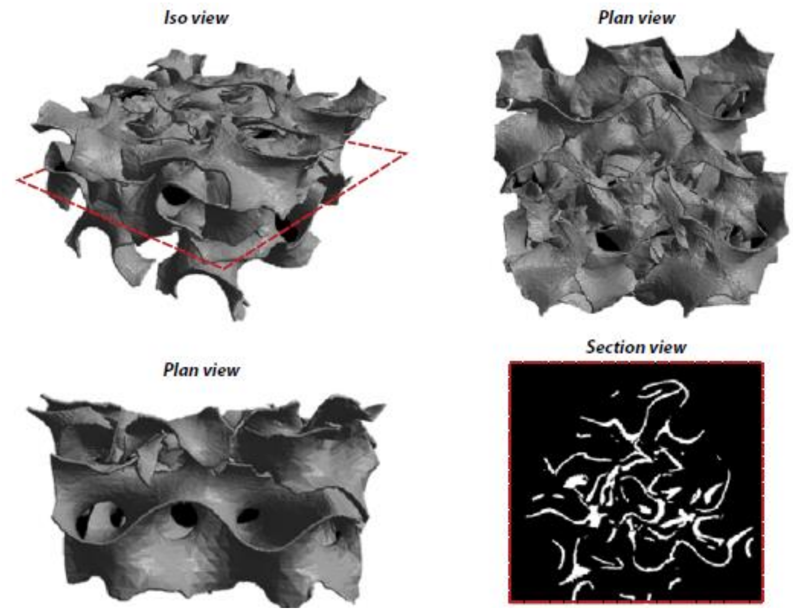


Reconstructed micro-CT images of plastically deformed network Gyroid manufactured with SLM from titanium (Ti64) powder, including an internal cross-section view of the deformed structure. Reference data of this type is much needed to further understand fundamental deformation mechanisms and to verify numerical simulation results.



(A) Gyroid supported internal lattice structures, (B) observation of as manufactured SLM titanium specimens, (C) local geometric artefacts including attached particles result in anisotropic response, (D) effect of local geometry on AM manufacturability and associated mechanical properties, (E) mechanical response (compression and torsion) of cylindrical EBM specimens.

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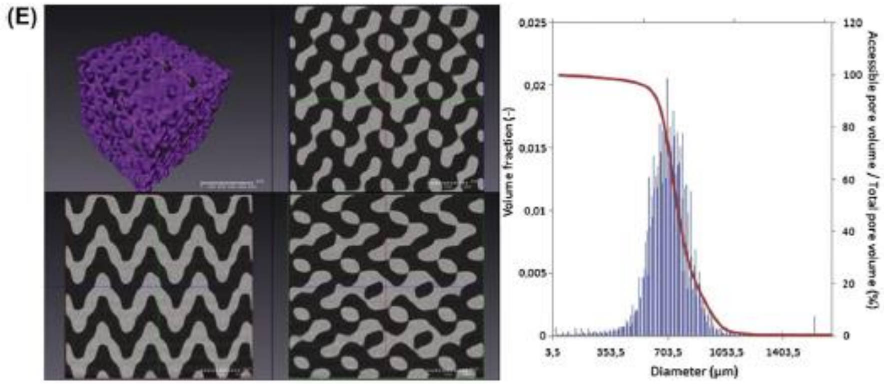
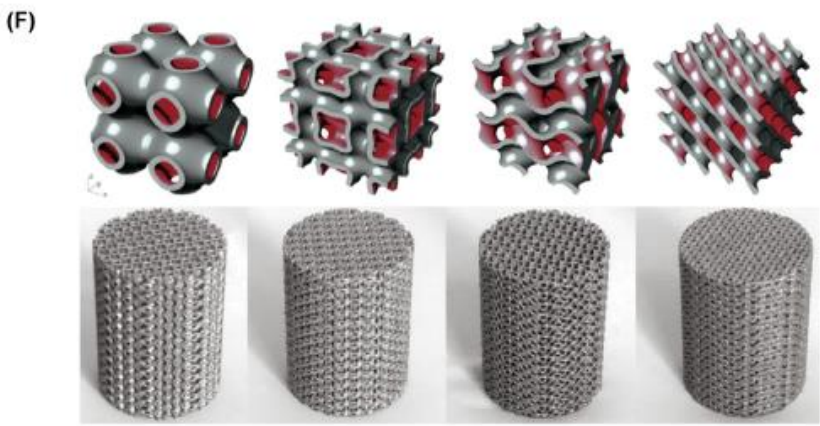
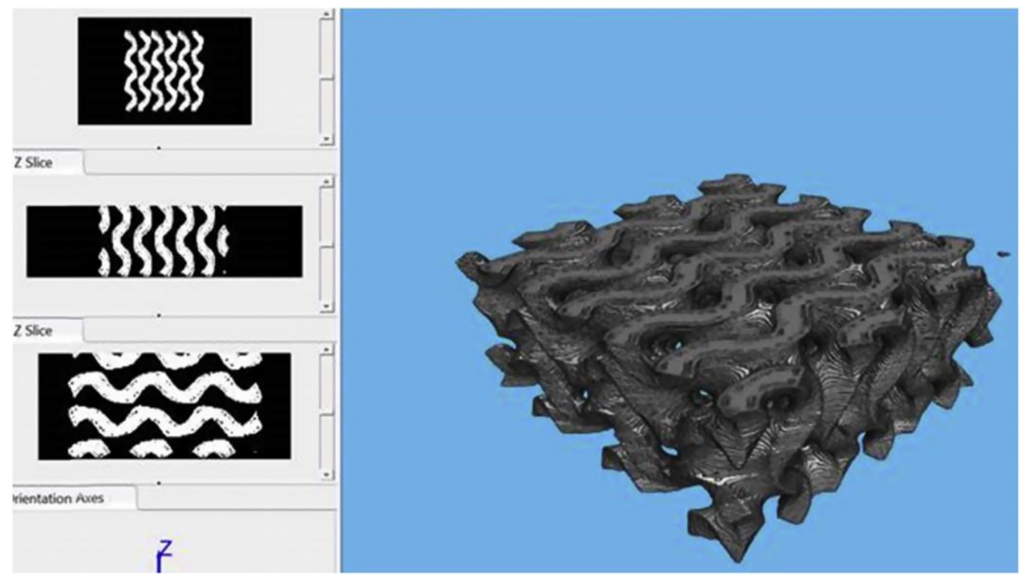
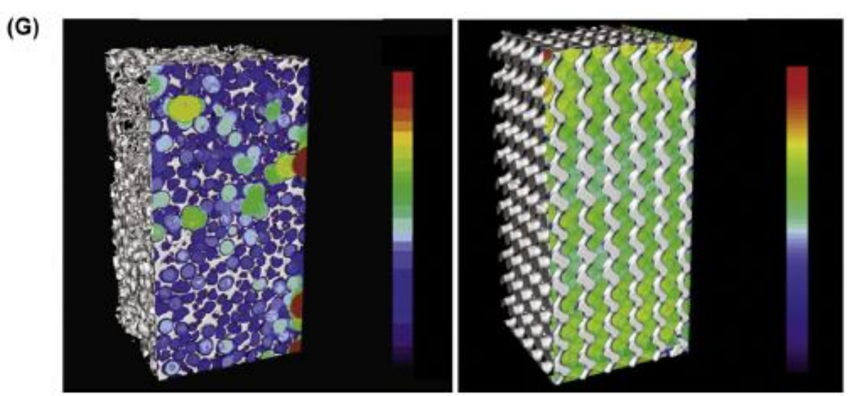


Custom DFAM tool to predict the influence of tool path on manufactured part quality. Left: cross-sections identifying local porosity. Right: 3D visualization of as-manufactured AM part including predicted regions or porosity. These predictive DFAM tools enable pre-production planning of oncological and structural components prior to manufacture.







(E) TPMS bio-scaffold manufactured by SLA and quantified in terms of the as-manufactured morphology, (F) morphology and mechanical response of TPMS specimens manufactured by SLM, (G) permeability simulation of traditional (left) versus SLA manufactured specimens (right)

Thank you for your attention