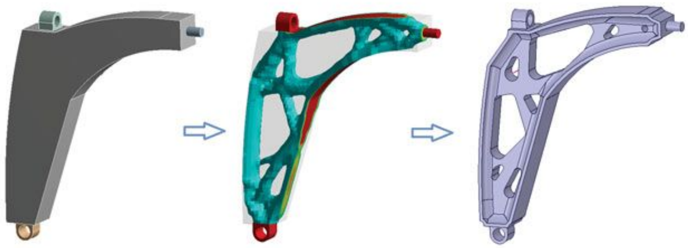
香港中文大學
The Chinese University of Hong Kong

MAEG5160: Design for Additive Manufacturing

Lecture 8: Topology Optimization (TO) by distribution of isotropic material (continue_2)



Prof SONG Xu

Department of Mechanical and Automation Engineering,
The Chinese University of Hong Kong.

Topology Optimization (TO) by distribution of isotropic material

Variations of the conditions

1. Multiple loads

The framework described for minimum compliance design for a single load case generalizes easily to the situation where design for multiple load conditions is formulated as a minimization of a weighted average of the compliances for each of the load cases. We here obtain a simple multiple load formulation as:

$$\min_{u^k \in U, E} \sum_{k=1}^{M} w^k l^k(u^k)$$

$$\text{s.t.} : a_E(u^k, v) = l^k(v), \quad \text{for all } v \in U,\ k = 1, \dots, M\ ,$$

$$E \in \mathsf{E}_{\text{ad}}\ ,$$

for a set $w^k, f^k, t^k, l^k,\ k = 1, \dots, M$, of weighting factors, loads and tractions, and corresponding load linear forms given as

$$l^k(u) = \int_\Omega f^k u \, \mathrm{d}\Omega + \int_{\Gamma_T^k} t^k u \, \mathrm{d}s\ ,$$

for the M load cases we consider.

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In this formulation the displacement fields for each individual load case are independent, thus implying that the multiple load formulation for the displacement based case has the equivalent form

$$\max_{E \in \mathsf{E}_{\mathrm{ad}}} \min_{\substack{\hat{u}=\{u^1,\ldots,u^M\} \\ u^k \in U,\ k=1,\ldots,M}} \left\{ \int_\Omega W(E,\hat{u})\mathrm{d}\Omega - l(\hat{u}) \right\},$$

$$W\left(E, \hat{u} = \{u^1, \ldots, u^M\}\right) = \frac{1}{2}\sum_{k=1}^{M} w^k E_{ijpq}(x)\varepsilon_{ij}(u^k)\varepsilon_{pq}(u^k),$$

$$l\left(\hat{u} = \{u^1, \ldots, u^M\}\right) = \sum_{k=1}^{M} w^k l^k(u^k).$$

Likewise, we have a stress based formulation

$$\min_{E \in \mathsf{E}_{\mathrm{ad}}} \min_{\substack{\operatorname{div}\sigma^k + f^k = 0 \text{ in } \Omega, \\ \sigma^k \cdot n = t^k \text{ on } \Gamma_T^k \\ k=1,\ldots,M}} \left\{ \frac{1}{2}\int_\Omega \sum_{k=1}^{M} w^k C_{ijpq}\sigma_{ij}^k \sigma_{pq}^k \mathrm{d}\Omega \right\}.$$

Similarly, for use of an algorithm like MMA, the sensitivity of the weighted average of compliances just becomes the weighted average of the sensitivities of each of the compliances. Also, the similarity of the iterations in MMA and in the optimality criteria method remains. Finally, it may be remarked that the inclusion of extra load cases is very cheap since the stiffness matrix already has been factorized.

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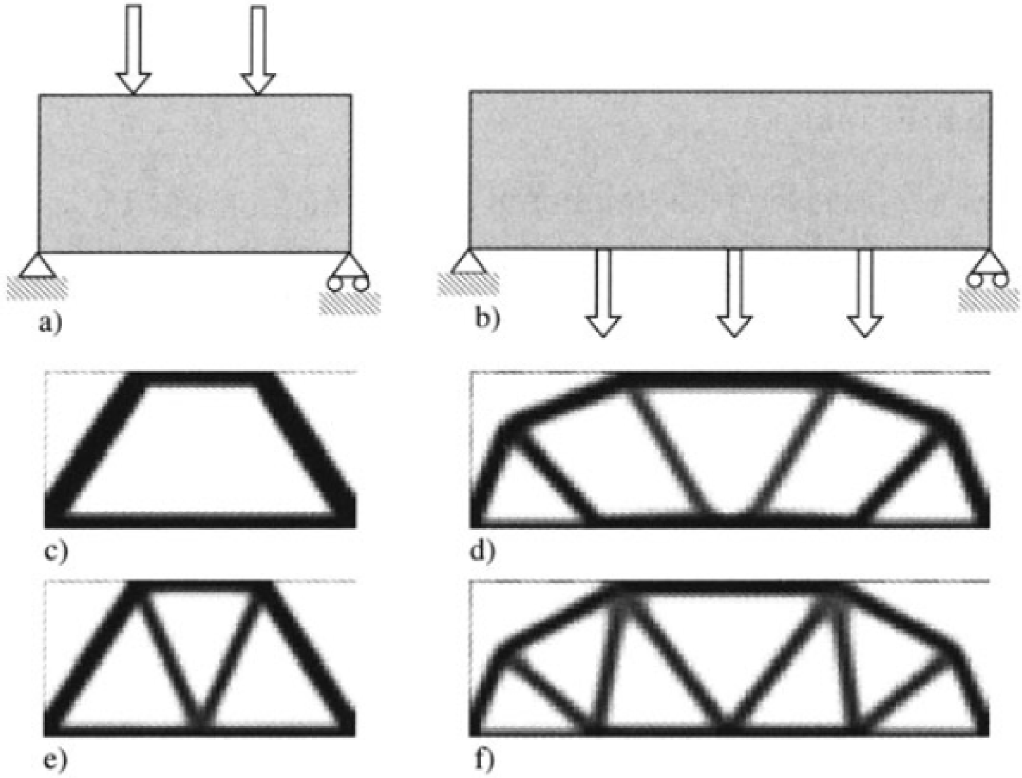
For the stiffness modelled as in the SIMP model, the optimality criteria method developed for the single load case generalizes directly and we obtain an update scheme for ρK at iteration step K which is exactly the same as previous single load, but with a modified "energy" expression

$$B_K = \Lambda_K^{-1} p \rho(x)^{(p-1)} E^0_{ijnm} \sum_{k=1}^{M} w^k \varepsilon_{ij}(u_K^k) \varepsilon_{nm}(u_K^k)$$



Example of differences in using one or more load cases. a) and b) Design domains. c) and d) Optimized topologies for all loads in one load case. e) and f) Optimized topologies for multiple loading cases. It is seen that single load problems result in instable structures based on square frames whereas multi load case problems results in stable structures based on triangular frames.

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2. Variable thickness sheets

For planar problems, the stiffness tensors given by the SIMP method reduces to the setting of the well-known variable thickness sheet design problem if we set p=1; in this circumstance the density function *P* is precisely the thickness *h* of the sheet. The minimum compliance problem then becomes

$$\min_{u,h} \quad l(u)$$

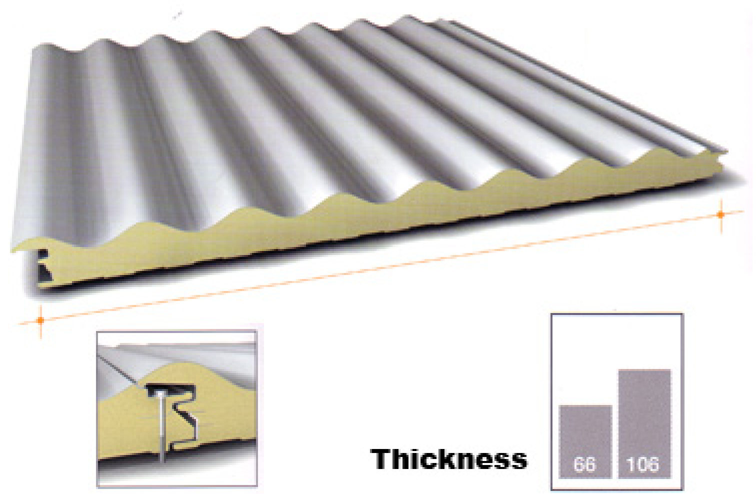
$$\text{s.t.} : a_h(u,v) \equiv \int_\Omega h(x) E^0_{ijkl} \varepsilon_{ij}(u) \varepsilon_{kl}(v) \mathrm{d}\Omega = l(v), \ \text{for all}\, v \in U \ ,$$

$$\int_\Omega h(x) \mathrm{d}\Omega \leq V, \ h_{\min} \leq h \leq h_{\max} < \infty \ .$$

Problem (1.36) can also be written as (cf., (1.6))

$$\min_{\substack{h \in L^\infty(\Omega), \\ h_{\min} \leq h \leq h_{\max} < \infty \\ \int_\Omega h(x)\mathrm{d}\Omega \leq V}} \quad c(h)$$

$$c(h) = \max_{v \in U} \left\{ 2l(v) - \int_\Omega h(x) E^0_{ijkl} \varepsilon_{ij}(v) \varepsilon_{kl}(v) \mathrm{d}\Omega \right\} \ .$$



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As the stiffness is linear in *h*, the compliance c is convex, as it is given as a maximization of convex functions. Also, the complete problem statement above is a convex-concave saddle point problem that (as noted earlier) lends itself to a complete FE convergence analysis within the framework of the theory developed for the Stokes' flow problem. The variable thickness sheet design problem also corresponds very closely to truss design problems in the sense that the stiffness of the structure as well as the volume of the structure depend linearly on the design variable for both models. This implies that a discrete version of the problem can be solved using some very efficient algorithms that have been developed for truss topology design. These algorithms do not require that $h_{min} > 0$, and the setting thus allows for a prediction of the optimal topology of the sheet without the ambiguity inherent in the chosen value of h_{min}; this is especially important here as we do not force the design towards a 0-1 design. The linear dependence of the stiffness on the design function h has an even more significant implication for the continuum problem, as one can prove existence of solutions. Thus there is no need for restriction methods or the introduction of materials with micro-structure (this holds for minimization of compliance and optimization of the fundamental frequency). Finally, we remark that the variable thickness sheet problem also plays a significant role when considering optimal design within a completely free parametrization of the stiffness tensors over all positive definite tensors in 2-D as well as 3-D. Here the problem form arises after a reduction of the original full formulation.

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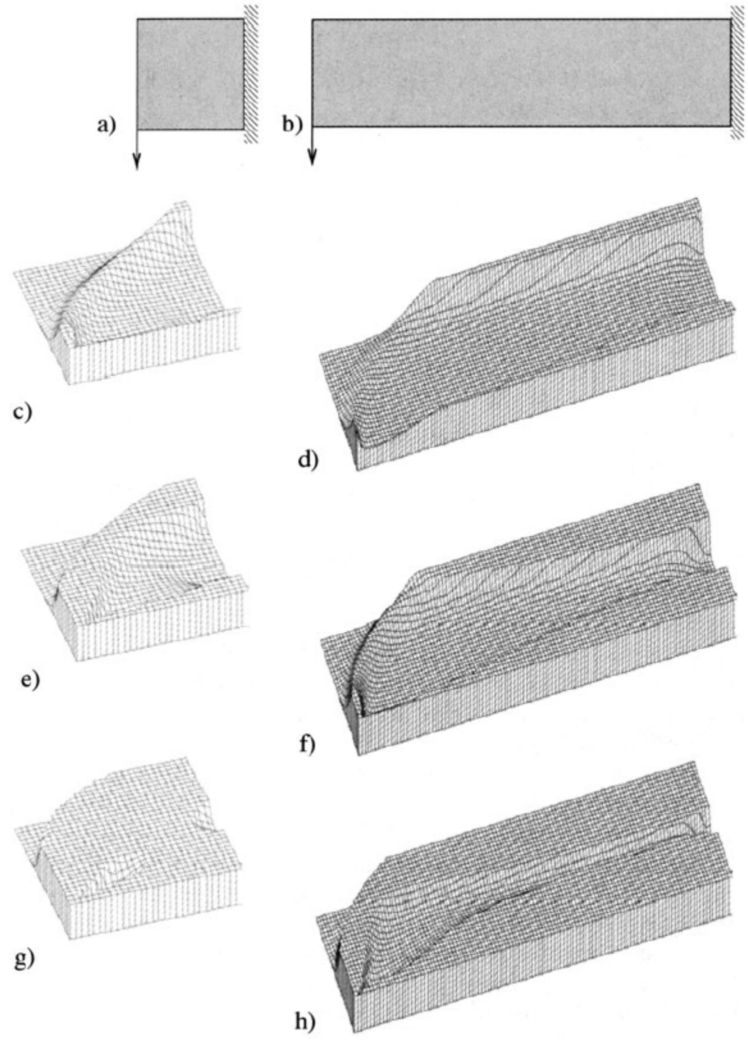

The design of a variable thickness sheet for two cantilever-like ground structures with aspect ratios a) 1:1 and b) 1:4. c) - h) : The optimal designs for a volume constraint c) and d) 30%, e) and f) 60% and g) and h) 90%, respectively, of the volume of a design with uniform thickness h_{max} (cf. constraints on thickness). Notice that the areas of intermediate thickness is considerable, especially for low amounts of available material. Thus the variable thickness design does not predict the topology of the structure as a true 2-dimensional object, but utilizes that the structure is in effect a 3-dimensional object.

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Explicit penalization of thickness The variable thickness design problem has been used as the inspiration for topology design methods where one seeks the optimum over all isotropic materials with given Poisson ratio and linearly-varying Young's modulus. This formulation results in designs with large domains of "grey" and modifications are necessary to obtain 0-1 designs. This can be accomplished by *adding to the objective an explicit penalty of intermediate densities*, for example in the form of functionals (we revert to using a density ρ as the design variable):

$$\mathcal{W}(\rho) = \int_\Omega \rho(x)(1-\rho(x))\mathrm{d}\Omega,\ \hat{\mathcal{W}}(\rho) = \int_\Omega \rho(x)^2(1-\rho(x))^2\mathrm{d}\Omega\ . \qquad (1.38)$$

Alternatively, the penalty function can be used as a constraint $\mathcal{W}(\rho) \leq \delta$ for some small $\delta \geq 0$.

The use of a penalty function such as $\mathcal{W}$ (or $\hat{\mathcal{W}}$) has a detrimental effect on the very nice mathematical properties of the original variable thickness sheet problem. For one, existence of solutions is no longer true. However, existence of solutions can be recovered (Borrvall & Petersson 2001b) by modifying the penalty function (1.38) to the form

$$\widetilde{\mathcal{W}}(\rho) = \int_\Omega (\rho * K)(x)(1-(\rho * K)(x))\mathrm{d}\Omega\ ,$$

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where one evaluates the original penalty function on a *filtered* version of the density p (we use here the notation introduced in section 1.3.1). The filter smoothes the density before penalization and as such provides for a more severe penalization than does W. Thus using W, the designs become almost entirely black and white (a 0-1 design) if the penalty factor is large enough. The penalty approach maintains the existence of solutions for the problem of minimum compliance and the maximization of the fundamental frequency. If a broader range of problems is to be considered, the restriction techniques should be applied. Note also that the use of the penalty W makes it impossible to use the efficient truss-type algorithm mentioned above. In order to maintain the structure of the original computational problem, it has been suggested instead to consider a sequence of problems where the volume constraint in each step K of the sequence is modified as

$$\int_{\Omega} w_K(x)\rho(x)\mathrm{d}\Omega \leq V$$

where the weight function W_K is fixed and determined from the optimal solution p_{K-1} to the prior step so as to penalize low density regions:

$$w_K(x) = \begin{cases} T_K & \text{if} \rho_{K-1} \leq \delta \\ 0 & \text{otherwise} . \end{cases}$$

With suitable big values of T_k and a small value of δ this scheme generates 0-1 designs and each step is computationally equivalent to the original variable thickness sheet problem. In implementation, the tuning of the penalization becomes an issue. Also note that as above, the advantages of this idea is closely linked to the properties of the minimum compliance problem.

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3. Plate design

Studies of the problem of variable thickness plate design and the appearance of stiffeners in such design problems have played a crucial role in the developments in optimal structural design. The design of variable thickness Kirchhoff plates or Mindlin plates is at first glance just another sizing problem of finding the optimal continuously varying thickness of the plate. The close connection with the 0-1 topology design problems is not entirely evident, but the cubic dependence of plate bending stiffness on the thickness of the plate implies that the optimal design prefers to achieve either of the bounds on the thickness, in essence a plate with integral stiffeners. This in turn implies non-existence of solutions unless the gradient of the thickness function is constrained or the problem is extended to include fields of infinitely many stiffeners.

Variable thickness design of Kirchhoff plates The minimum potential energy statement for a Kirchhoff plate is of the form

$$\min_{w} \left\{ \frac{1}{2} \int_{\Omega} \frac{h^3}{12} E^0_{ijkl} \kappa_{ij}(w) \kappa_{kl}(w) \mathrm{d}\Omega - \int_{\Omega} f w \mathrm{d}\Omega \right\}$$

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where f is the transverse load. The thickness of the plate is denoted by h and we assume that the mid-plane is a plane of symmetry. The deformation of the plate is described by the transverse displacement of the mid-plane w, with associated (linearized) curvature tensor $\kappa_{ij} = \frac{\partial^2 w}{\partial x_i \partial x_j}$, and the relationship between the curvature tensor and moment tensor M is given as

$$M_{ij} = D_{ijkl}\kappa_{kl} , \quad \text{with} \quad D_{ijkl} = \frac{h^3}{12}E^0_{ijkl} \ ,$$

where E^0_{ijkl} is the plane stress elasticity tensor of the given material. The similarity between the curvature-moment relation for plates and the strain-stress relation in elasticity hides the fundamental difference that the Kirchhoff plate is governed by a fourth order scalar equation, while standard linear elasticity is governed by a system of second order equations. As for the variable thickness sheet problem, the thickness of the plate also here automatically provides the plate design problem with a continuous design variable. Considering the minimization of compliance the most natural problem to consider is thus

$$\max_{D \in \mathrm{PE}_{ad}} \min_{w} \left\{ \frac{1}{2}\int_\Omega D_{ijkl}\kappa_{ij}(w)\kappa_{kl}(w)\mathrm{d}\Omega - \int_\Omega f w \mathrm{d}\Omega \right\}$$

with the set $\mathsf{PE}_{\mathrm{ad}}$ of bending stiffnesses given as

$$D_{ijkl} = \tfrac{h^3}{12}E^0_{ijkl}, \quad h \in L^\infty(\Omega) \ ,$$
$$0 \le h_{\min} \le h \le h_{\max} < \infty, \quad \textstyle\int_\Omega h \mathrm{d}\Omega \le V \ ,$$

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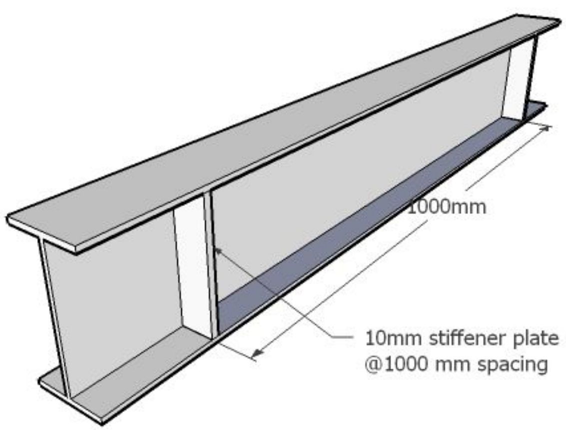
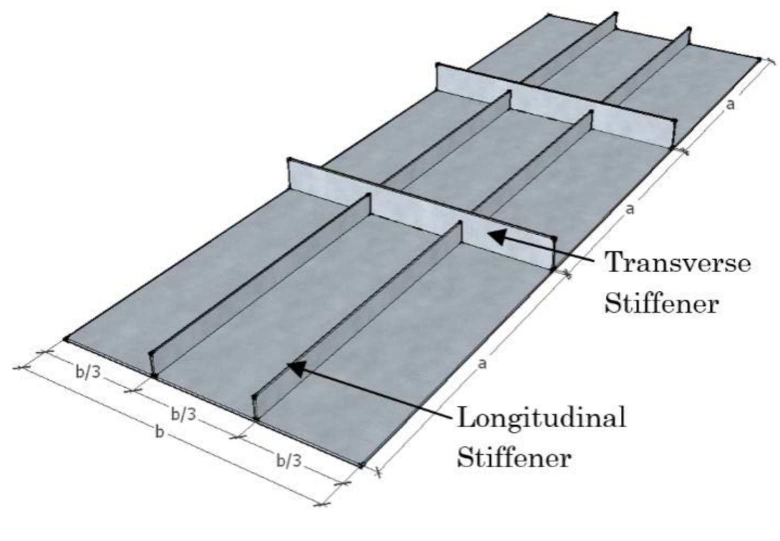
The number of stiffeners increase when the discretization of design is refined, with a resulting (substantial) decrease in compliance, a situation completely similar to the behaviour of the 0-1 topology design setting. Compared to the variable thickness design problem for sheets, this is caused by the cubic dependence of the stiffness of the plate on the thickness. Physically, this dependence makes it advantageous to move as much material as possible away from the mid-plane of the plate, for example in the form of integral stiffeners. A method to obtain mesh-independence and existence of solutions is analogous to what has been described previously, by restricting the variation of the thickness function, for example in the form of a constraint on the slope (gradient) of the thickness function. The computational procedure for computing optimal plate designs is completely analogous to the procedure described earlier and the optimality criteria and sensitivity calculations carry over, with strains and stresses interpreted as curvatures and moments, respectively.





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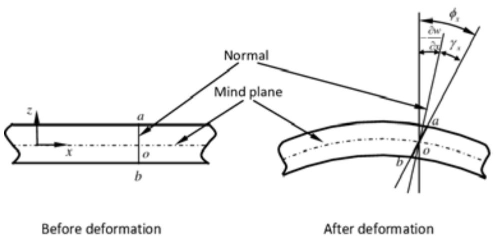
Topology design for Mindlin plates

The minimum potential energy statement for a *constant thickness* Mindlin plate constructed from one material is of the form

$$\min_u \left\{ \begin{array}{c} \frac{1}{2}\int_\Omega hE^0_{ijkl}\varepsilon_{ij}(u)\varepsilon_{kl}(u)\mathrm{d}\Omega + \frac{1}{2}\int_\Omega \frac{h^3}{12}E^0_{ijkl}\kappa_{ij}(u)\kappa_{kl}(u)\mathrm{d}\Omega \\ +\frac{1}{2}\int_\Omega hD^S_{ij}\gamma_i(u)\gamma_j(u)\mathrm{d}\Omega - \left(\int_\Omega fu\mathrm{d}\Omega + \int_{\Gamma_i} tu\mathrm{d}\Gamma\right) \end{array} \right\} ,$$

where f is the transverse and t the in-plane load. The thickness of the plate is denoted by h and we assume that the mid-plane is a plane of symmetry. Also, E^0_{ijkl} is the plane stress elasticity tensor and D^S is the transverse shear stiffness matrix. In Mindlin plate theory generalized displacements of the plate $u = (u_1, u_2, w, \theta_1, \theta_2)$ consist of the in-plane displacements (u_1, u_2), the fibre rotations (θ_1, θ_2) and the transverse displacement of the mid-plane w. The associated membrane, bending, and transverse shear strains are, respectively,



$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad \kappa_{ij} = \frac{1}{2}\left(\frac{\partial \theta_i}{\partial x_j} + \frac{\partial \theta_j}{\partial x_i}\right), \quad \text{and} \quad \gamma_i = \frac{\partial w}{\partial x_i} - \theta_i \ .$$

For plates there are several options for performing topology design, connected to the possibility to also consider out-of-plane variations of the build-up of the plate. For the design of a perforated plate one would thus use thickness functions that attain values 0 or $\bar{h}$, for example implemented with the help of a density function ρ and a SIMP interpolation:

$$h = \rho^p \bar{h} \ , \quad \mathrm{Vol} = \int_\Omega \rho\bar{h}\mathrm{d}\Omega \ .$$

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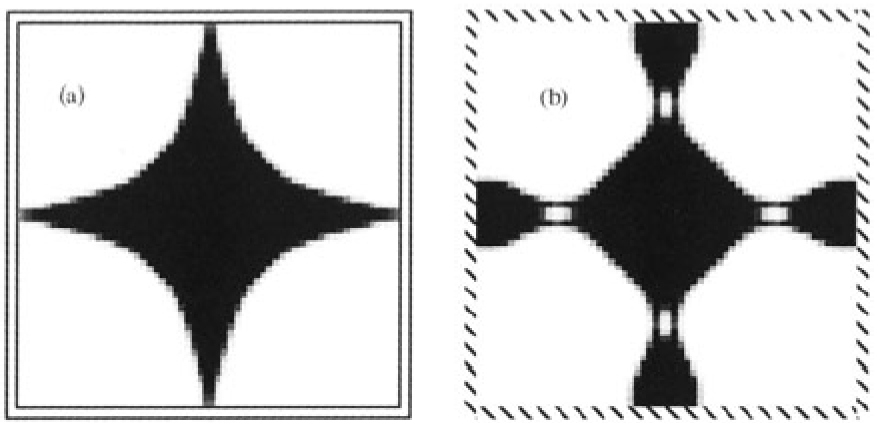
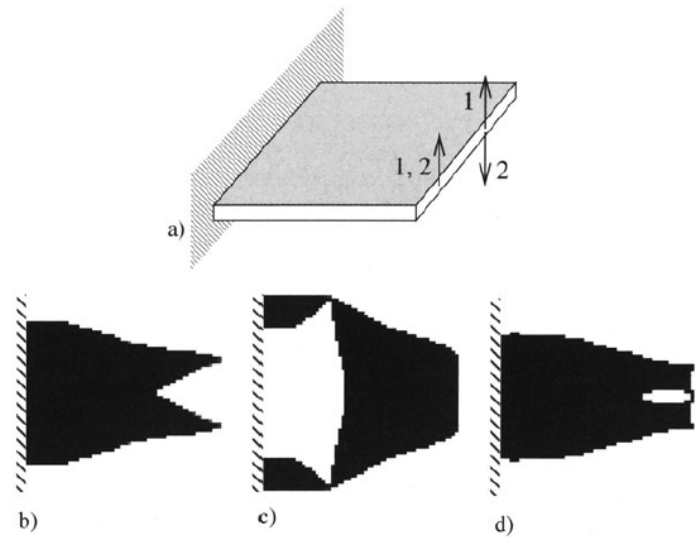
Other possibilities is to consider *reinforcement* of a given plate or to consider the design of a sandwich structure, where two outer skins are given and the topology design deals with the topology design of the inner core



Resulting topologies for compliance minimization of square Mindlin plates. The material volumes are restricted to 25% of the filled plates and the plates are loaded with a force at the center. a) Simply supported and b) clamped plate



Resulting topologies for compliance minimization of square Mindlin cantilever plates. a) Design problem with loads for the first and the second load case. b) Solution to one-load case problem with two upward oriented forces, c) solution to one-load case problem with one force upwards and one downwards and d) solution to two load case problem. The volume fraction is 50%

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4. Other interpolation schemes with isotropic materials

The use of SIMP or the penalized variable thickness formulation have in the last few years been supplemented by some alternative interpolation schemes that have certain theoretical or computationally advantageous features for specific problems. As they fall within the class of interpolation models with isotropic materials we briefly discuss them here.

Hashin-Shtrikman bounds The so-called Hashin-Shtrikman bounds for two-phase materials express the limits of *isotropic* material properties that one can possibly achieve by constructing composites (materials with microstructure) from two (or more) given, linearly elastic, isotropic materials.

These bounds give expressions of material parameters as functions of volume fraction, or for our purposes as functions of density ρ of material, and can thus be employed as interpolation schemes (all material laws involved will be isotropic). For our purposes we work with two materials, one with a low stiffness E^{min} and one with high stiffness E^{O}. The corresponding values of the Poisson ratios are ν^{min} and ν^{O}.

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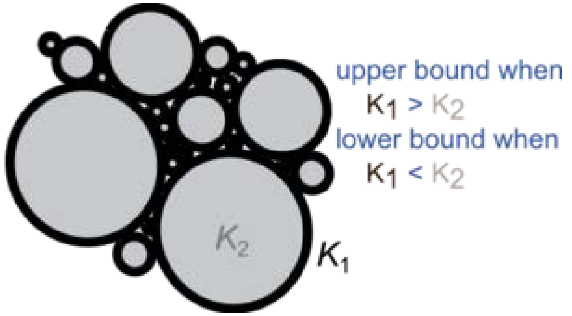
Hashin–Shtrikman bounds

The Hashin-Shtrikman bounds are the tightest bounds possible from range of composite moduli for a two-phase material. Specifying the volume fraction of the constituent moduli allows the calculation of rigorous upper and lower bounds for the elastic moduli of any composite material. The so-called Hashin-Shtrikman bounds for the bulk, K, and shear moduli μ is given by:

$$K_{\mathrm{HS}}^{\pm} = K_2 + \frac{\phi}{(K_1 - K_2)^{-1} + (1-\phi)(K_2 + \frac{4}{3}\mu_2)^{-1}}$$

$$\mu_{\mathrm{HS}}^{\pm} = \mu_2 + \frac{\phi}{(\mu_1 - \mu_2)^{-1} + \frac{2(1-\phi)(K_2+2\mu_2)}{5\mu_2(K_2+\frac{4}{3}\mu_2)}}$$



The upper bound is computed when $K_2 > K_1$. The lower bound is computed by interchanging the indices in the equations.
For the case of a solid-fluid mixture, K_2 is K_S, the bulk modulus of the solid component, and and K_1 is K_f, the bulk modulus of the fluid component.

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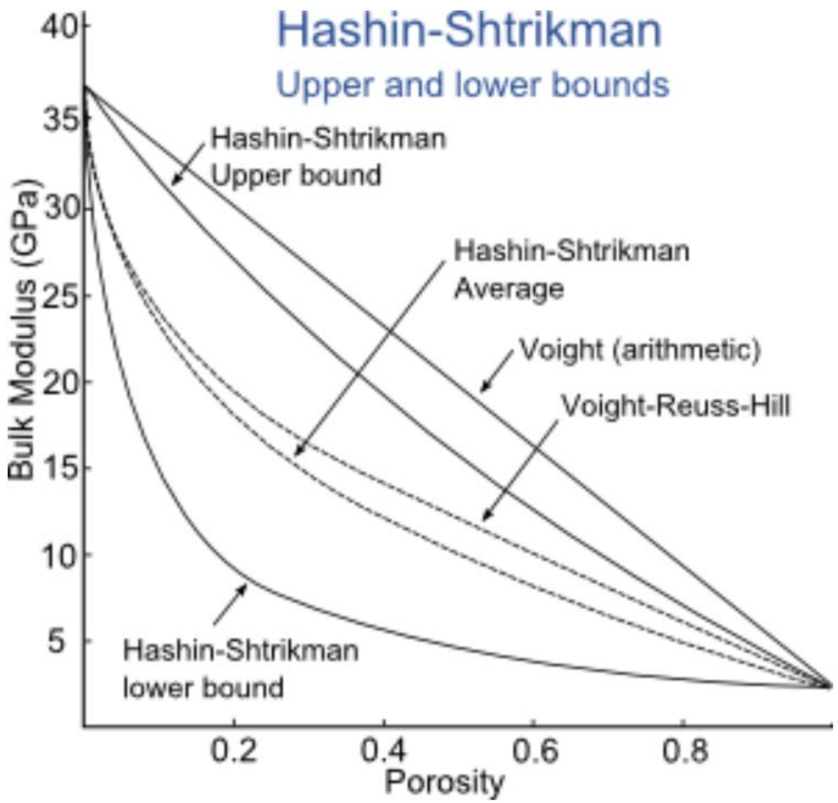
Quartz-Brine mixture: Quartz with solid mineral modulus, K_S = 36.6 GPa, and K_f = 2.2 GPa.



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The Hashin-Shtrikman bounds are typically expressed in terms of the bulk and shear moduli of the materials , κ and μ (corresponding to the eigenvalues of the stiffness tensor). Restricting ourselves here to 2-D plane elasticity, we have for isotropic materials that

$$\kappa = \frac{E}{2(1-\nu)}; \quad \mu = \frac{E}{2(1+\nu)} \qquad (\text{in } 2-D) \ .$$

The bounds are then in terms of these parameters (in 2-D) (we assume here that $\kappa^0 \geq \kappa^{\min}$ and $\mu^0 \geq \mu^{\min}$):

$$\kappa^{HS}_{\text{upper}} = (1-\rho)\kappa^{\min} + \rho\kappa^0 - \frac{(1-\rho)\rho(\kappa^{\min}-\kappa^0)^2}{(1-\rho)\kappa^0 + \rho\kappa^{\min} + \mu^0} \ ,$$

$$\mu^{HS}_{\text{upper}} = (1-\rho)\mu^{\min} + \rho\mu^0 - \frac{(1-\rho)\rho(\mu^{\min}-\mu^0)^2}{(1-\rho)\mu^0 + \rho\mu^{\min} + \frac{\kappa^0\mu^0}{\kappa^0+2\mu^0}} \ ,$$

$$\kappa^{HS}_{\text{lower}} = (1-\rho)\kappa^{\min} + \rho\kappa^0 - \frac{(1-\rho)\rho(\kappa^{\min}-\kappa^0)^2}{(1-\rho)\kappa^0 + \rho\kappa^{\min} + \mu^{\min}} \ ,$$

$$\mu^{HS}_{\text{lower}} = (1-\rho)\mu^{\min} + \rho\mu^0 - \frac{(1-\rho)\rho(\mu^{\min}-\mu^0)^2}{(1-\rho)\mu^0 + \rho\mu^{\min} + \frac{\kappa^{\min}\mu^{\min}}{\kappa^{\min}+2\mu^{\min}}} \ .$$

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Each combination of formulas $\kappa^{HS}_{\text{upper}}, \mu^{HS}_{\text{upper}}$ and $\kappa^{HS}_{\text{lower}}, \mu^{HS}_{\text{lower}}$ represents an interpolation of the material properties of the two materials, and any convex combination is also and interpolation scheme, which then satisfies the bounds. Thus a whole range of schemes can be generated. Here the *lower* bound interpolation penalizes intermediate densities most. Note that the Hashin-Shtrikman bounds represent materials that have both a Young's modulus *and* a Poisson ratio that vary with density (even if the two base materials have the same Poisson ratio).

If both materials have Poisson ratio equal to 1/3, then the upper and lower bounds (and all convex combinations) also represent a material law with Poisson ratio $\nu = 1/3$, and the bound can be expressed in terms of the Young's modulus only:

$$\text{for } \nu = 1/3: \begin{cases} E^{HS}_{\text{upper}} = E^0 \dfrac{\rho E^0 + (3-\rho)E^{\min}}{(3-2\rho)E^0 + 2\rho E^{\min}}, \\ E^{HS}_{\text{lower}} = E^{\min} \dfrac{(2+\rho)E^0 + (1-\rho)E^{\min}}{2(1-\rho)E^0 + (1+2\rho)E^{\min}} \end{cases}$$

This reduces further if the weak material is void ($E^{\min} = 0$):

$$E_{\text{upper}} = \frac{\rho E^0}{3-2\rho}, \qquad E_{\text{lower}} = \begin{cases} 0 & \text{for } \rho < 1, \\ E^0 & \text{for } \rho = 1, \end{cases}$$

which is then an interpolation with void and with a material with $\nu = 1/3$ and Young's modulus E^0. For many test cases in topology design one works within this framework of $\nu = 1/3$.

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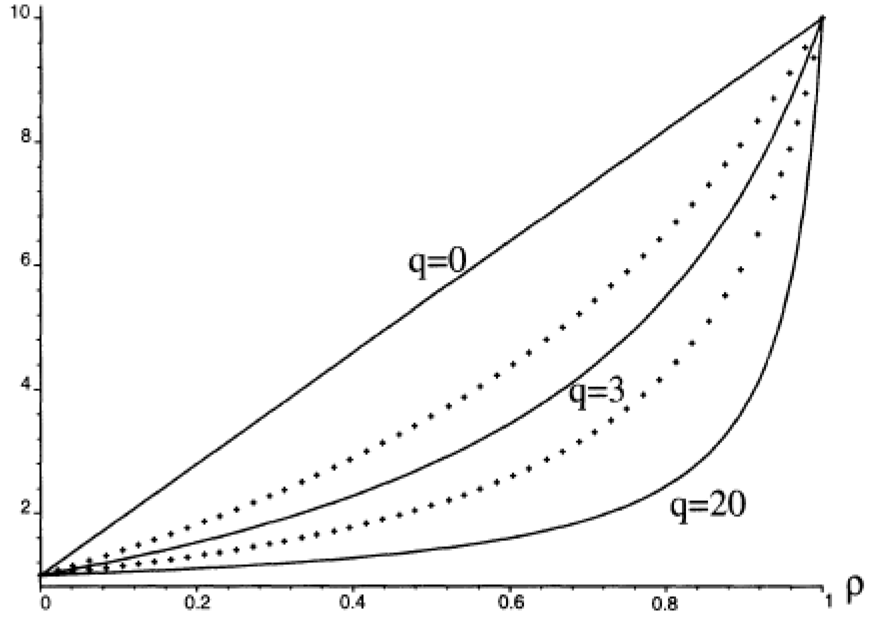

The interpolation using SIMP (solid curves) compared with the Hashin-Shtrikman bounds (dotted curves).

SIMP and the Hashin-Shtrikman bound If we require that an interpolation model in any sense can be related to a composite made of the given materials, then we should demand that the model satisfies the Hashin-Shtrikman bounds stated above. For SIMP one of the material phases is zero, i.e., $E_{min}=0$. Then the only relevant Hashin-Shtrikman bound simplify somewhat and it is possible to show that SIMP satisfies the bounds if the power of the model satisfies the inequalities. As already noted, this does not assure that a composite can actually be constructed.

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A Reuss-Voigt interpolation scheme The Voigt upper bound for the effective properties of a mixture of two materials state that for any strain field ε we have that the strain energy W of the composite is bounded from above by the expression

$$W \leq \left[\rho E^0_{ijkl} + (1-\rho)E^{\min}_{ijkl}\right] \varepsilon_{ij}\varepsilon_{kl} ,$$

where the two materials have elasticity tensors E^0 and $E^{\min}$, respectively, and where the volume fraction of the material E^0 is ρ. Likewise, the Reuss lower bound states that the energy W is bounded from below by the expression

$$W \geq \left[\rho C^0_{ijkl} + (1-\rho)C^{\min}_{ijkl}\right]^{-1} \varepsilon_{ij}\varepsilon_{kl} ,$$

where C denotes the compliance tensors of the materials.

These two bounds can be combined to a convex combination to what has been named a Reuss-Voigt interpolation scheme (Swan & Arora 1997, Swan & Kosaka 1997a)

$$E^{VR}_{ijkl}(\rho) = \alpha\left[\rho E^0_{ijkl} + (1-\rho)E^{\min}_{ijkl}\right] + (1-\alpha)\left[\rho C^0_{ijkl} + (1-\rho)C^{\min}_{ijkl}\right]^{-1} . \tag{1.50}$$

Here α is a parameter which weighs the contributions from the Voigt and Reuss bounds. If one of the materials is void, the interpolation introduces a jump (discontinuity) at $\rho = 1$ which is not present if both materials have some stiffness. In that case, for two materials that both have a Poisson's ratio of $\nu = 1/3$ we have that the Hashin-Shtrikman bounds are satisfied if and only if $\alpha = 1/3$ (Bendsøe & Sigmund 1999).

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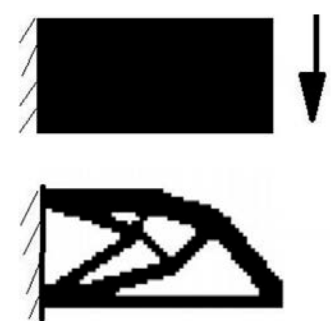
6. Alternative approaches

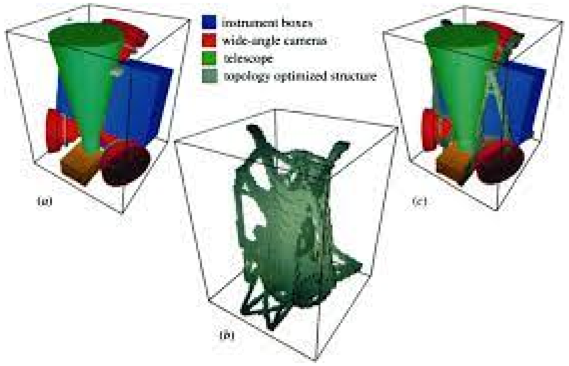
The technique for topology design of continuum structures that is described here is based on the concept of optimal distribution of material, using interpolation of material properties together with mathematical programming. As we shall see soon, this is a universally efficient approach for a broad range of problems in engineering design. In parallel with the development of this methodology, other schemes have also evolved. Some of these work within the same modelling framework using algorithms for *discrete* optimization or various types of growth/shrinking procedures, but a completely different modelling paradigm can also be found in for example the *bubble method*. We will here only briefly mention some of these concepts here.

Solving the discrete problem The introduction of the interpolation schemes for the 0-1 design problem is extremely useful as it allows for the use of mathematical programming methods for continuous (smooth) problems. However, it would be very useful if one could attack the original formulation directly. This has been done for the compliance design problem using dual methods, that have been shown to be effective in the absence of local constraints. Methods like simulated annealing or genetic algorithms have also been tested for more general settings, but their need for many function evaluations is computationally prohibitive, but for rather small scale examples (each call involves a costly finite element analysis on a grid at least as fine as the raster representation of the design). It has been shown that for a broad class of problems one can formulate the 0-1 topology design problem as a *linear* mixed continuous-integer programming problem and this will no doubt be useful for generating more efficient methods for treating the discrete format in the future.



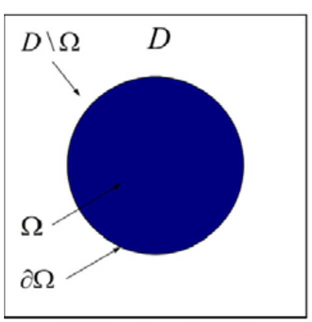
Topology Optimization (TO) by distribution of isotropic material

Growing and shrinking a structure; Bone remodeling Numerous methods have been proposed for dealing with topology design without the use of mathematical programming. They are typically named as "evolutionary“ methods, but they are not in any way connected to the use of genetic algorithms. On the contrary, these methods typically work with concepts that are similar to the idea of fully stressed design, i.e., material is added to highly stressed areas of a design and removed from understressed areas of the design, typically implemented by an addition or removal of elements from the FE model. Some implementations of such concepts are very similar to an optimality criteria type algorithm, but the removal and adding of elements can lead to erroneous results. This is basically because gradient information is used to perform changes of variables between zero and one. It is interesting to note that many models used in bio-mechanics for bone adaptation have a form which is similar to the optimality criteria algorithm. These models are usually based on energy arguments and are not derived from an optimization principle. This similarity in approach to material redistribution updates has also lead to bone adaptation models being proposed as topology redesign methods.

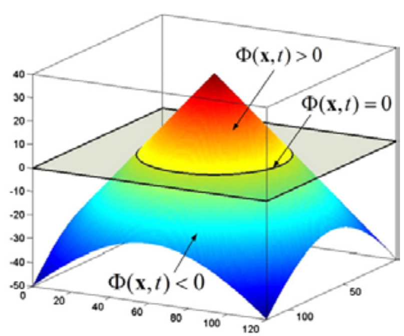


Topology Optimization (TO) by distribution of isotropic material

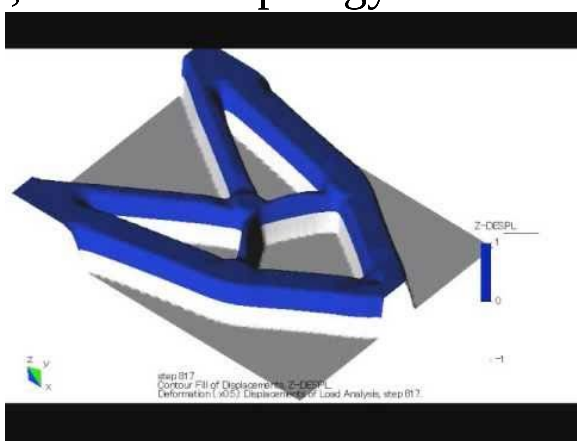
Topological variations and level sets The concepts of *topological derivatives* and the *bubble method* is based on utilizing ideas from the boundary variations technique for shape design as a basis for topology design. The topological derivative of a functional as compliance expresses the sensitivity with respect to the opening of a small (infinitesimal) hole at a certain position in the analysis domain. Likewise, in the bubble method a criterion is developed that allows for the prediction of the most effective location for creating a hole and this information is used to perform a boundary variations shape optimization of the resulting topology. The hole placement is then repeated in this shape optimized structure leading to good designs with smooth boundaries. A direct application of the topological derivative in a mathematical programming technique is presently not possible, as there is no evident underlying parametrization available; implementations have thus been based on techniques reminiscent of element removal techniques. The application of level-set techniques for topology design have also been proposed. The contours of a parametrized family of level-set functions are here used to generate the boundaries of a structure, and the topology can change with changes in the level-set function.



(a) Design domain



(b) level set model



Thank you for your attention