
# Design Optimization of Soft Robots

## *A Review of the State of the Art*

By Feifei Chen and Michael Yu Wang

Robotics has undergone a profound revolution in the past 50 years, moving from the laboratory and research institute to the factory and home. Kinematics and dynamics theories have been developed as the foundation for robot design and control, based on the conventional definition of robots: a kinematic chain of rigid links.

Currently, the boundaries among materials, structures, biology, intelligence, and robotics are blurring. We have a much wider interpretation of what a robot is. The past decade has seen the increasing use of soft materials (Young's modulus on the order of kilopascals to megapascals) to build robots, which are generally referred to as soft robots. This new generation of robots, originally inspired by natural lives, has grown rapidly and is enabling new robot abilities for applications ranging from wearable devices and biomedical engineering to search and rescue in unstructured environments [1]–[3].

Instead of relying on sliding or rolling motions as in conventional rigid robots, soft robots produce mobility based on the inherent compliance of soft materials. This fundamental change enables the integration of multiple functions into simple topologies by embedding actuators and sensors to build fully functional machines that can perform complex tasks. Here, the physical presence of soft robots plays a central role in generating adaptable behaviors. The body design

*Digital Object Identifier 10.1109/MRA.2020.3024280*
*Date of current version: 22 October 2020*

in free form is expected to imbue soft robots with programmable mechanical properties and desired responses to external stimuli, which unlocks new functionalities in the paradigm of so-called morphological computation and embodied intelligence [4], [5].

The transformative involvement of soft materials in robots also poses unprecedented challenges. The increased complexities of soft robotic systems, which may come from geometry, material, actuation, and their intricate coupling, are making conventional theories of robot design poorly applicable. The difficulties come not only from the lack of simulation and analysis tools to effectively and efficiently predict complex mechanical behaviors of soft robots but also from the lack of powerful optimization algorithms to automate the design process. One must often rely on intuitions, experiences, or bioinspiration for soft robot design, which can provide only limited scope. Research efforts have increasingly been made toward a comprehensive design paradigm to bridge the gap from theoretical and algorithmic perspectives.

In this article, instead of limiting the discussion to specific applications, we articulate the fundamental concepts of design optimization for soft robots. We exclude chemical- or material-level modifications but focus on mathematical design approaches to soft robots based on widely available materials. State-of-the-art progress is highlighted, with particular emphasis on the methods to approach design problems and their mathematical representation. The term *optimization* is not necessarily limited to algorithms to solve a formulated problem but more generally refers to innovations in any design aspect leading to better performance of soft robots. We conclude this review with a prospective look at future trends for design optimization in soft robotics.

## Design Architecture

The entire framework of design optimization for soft robots is generally hierarchical and iterative, as presented in Figure 1. A high-level task, such as locomotion and grasping, can be decomposed into a sequence of motion behaviors, including stretching, bending, twisting, or their combination. For example, bending motions typically dominate a grasping process, while alternating elongations and compressions may dominate locomotion. Once the desired mechanical behavior is determined, one may formulate it as an inverse design problem to be addressed by mathematical programming.

The translation of the physical problem as a mathematical optimization problem requires identifying and quantifying

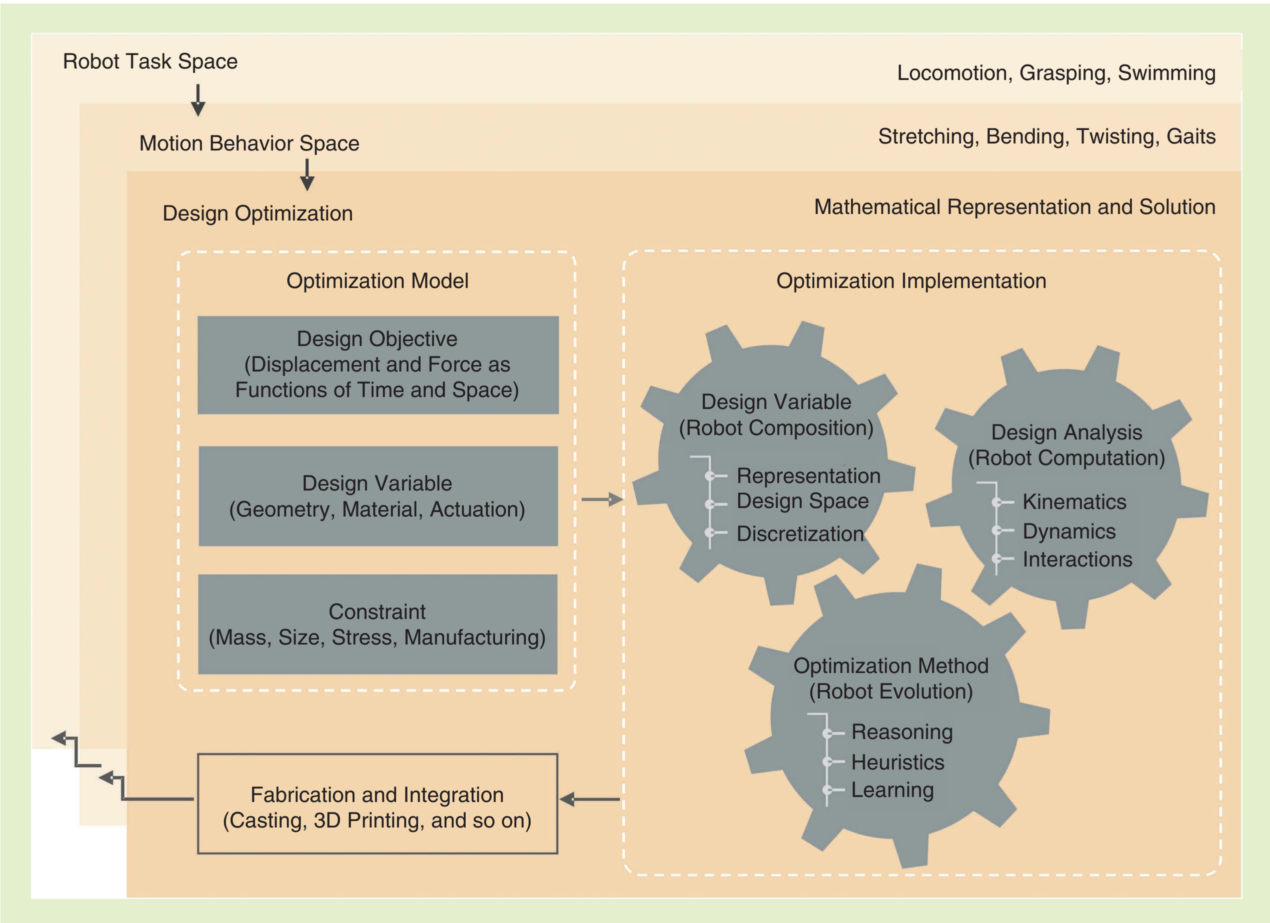


**Figure 1.** The architecture for design optimization of soft robots.

the objective, variables, and constraints. This process is also referred to as *modeling*. The appropriate model plays an essential role in the optimization process by bridging the robotic tasks and optimization algorithms. The optimization implementation necessitates an integration of the optimization model, analysis, and search algorithms. Here, we clarify several important concepts that will be mentioned throughout this review.

### *Design Objective (Robotic Behavior)*
The design objective of a soft robot is the mathematical abstraction of desired mechanical behaviors. In general, the behaviors can be described by a function of deformation and force of interest that may vary with time and space. For example, a soft actuator may be expected to replicate the motion of human fingers upon activation. The pursued robotic behaviors sometimes cannot readily be formulated in a mathematical sense. For example, one may expect a soft gripper to conform and adapt to unknown objects of different sizes and shapes. In this case, the challenge is to capture the "adaptability" in a rigorous mathematical language.

### *Design Variable (Robotic Composition)*
Design variables are concerned with how designers can approach the design problem and refer to the variables tunable to improve the design. The design variables of a soft robot translate into its composition, including the geometry, its material (and metamaterial), and the applied actuation field. These tunable variables shape the physical presence of soft robots and determine the mechanical behavior of robots in the environment under external stimuli. The mathematical representation of design variables has an important impact on the posedness and convexity of the optimization problem. All feasible design candidates create the design space to be explored.

### *Design Analysis (Robotic Computation)*
Simulation and analysis tools are required to evaluate the fitness of a design candidate. This prerequisite applies to all optimization algorithms. To shed light on practical robot design, it is necessary to involve physical properties and working conditions at the evaluation stage, which has been very challenging for soft robots. Due to the huge complexities of soft robotic systems, the evaluation is generally computationally difficult and expensive and usually takes the most time during an optimization process.

### *Optimization Method (Robotic Evolution)*
Optimization methods are required to guide designers to search for the optimal design candidate in the vast design space. In addition to knowledge of the fitness of design candidates, the optimization algorithm may call for further information, such as gradients and Hessians (usually not readily attainable), to accelerate convergence. Designers must always address the ubiquitous speed–accuracy tradeoff: more accurate information promises a better search direction, but it comes at a higher computational cost.

Design optimization is an iterative process. For a design candidate described by a set of design variables, one needs to evaluate its mechanical behavior by computation. If the design objective is not fulfilled in the numerical process, optimization algorithms are implemented for the robot evolution. The evolution process translates into changes in design variables, i.e., a new design is generated. Finally, the optimized design is prototyped, e.g., with advanced manufacturing technologies, and tested to see whether the design achieves the desired performance. If not, a redesign process is required. Observations based on optimization solutions, the manufacturing process, and experimental results may inspire further refinement of the optimization model in terms of its objective or constraints.

**The optimization model can be characterized by its design variables, i.e., the designable ingredients of robots.**

## Optimization Model
The optimization model can be characterized by its design variables, i.e., the designable ingredients of robots. Virtually all combinations of design variables are possible implementations for the design optimization of soft robots, but only some design candidates have been investigated. In this section, we review how researchers have approached design optimization problems from the perspectives of geometry, material, metamaterial, and actuation.

### *Design Variable: Geometry*
The geometry of a robot concerns how the robot should be shaped, which generally includes its lengths, areas, and volumes, and undoubtedly plays an essential role in defining the robotic behavior. In terms of the complexity and generality of the geometric representation, researchers have conducted geometry optimization of soft robots on the level of size, shape, and topology.

Size optimization represents the initial step and typically addresses regular shapes that can be explicitly described by parameters such as length, width, height, and angle. The exploration of soft pneumatic actuators has offered notable examples. The design objective is generally concerned with elongation, contraction, bending, and twisting motions, and the geometric parameters are related to the shape and arrangement of inner chambers. Dämmer et al. [6] described the cross section of a linear bellows-type pneumatic actuator with a set of parameters [Figure 2(a)] and implemented a gradient-based optimization to decrease the induced maximal principal strain for the given output deflections and forces.

The chamber cross section has also been optimized at the size level for the 1D bending actuator by Elsayed et al. [11], where they aimed to minimize the ballooning effect.

To incorporate the twisting motion in addition to bending, Wang et al. [13] tailored the widely adopted pneumatic networks (often referred to as PneuNets [12]) by shifting the chamber arrangement from vertical to oblique and investigated how the oblique angle programmed the combined bending and twisting motion. PneuNets were also modified in terms of chamber size to handle deformable objects using hybrid optimization algorithms, where the design objective was to match the deformed shape of objects such as a Dixie cup [14].

Shape optimization makes a further step toward design space exploration. The space of allowable shapes within which designers search does not admit a vector space structure, which causes an infinite-dimensional problem. Shape optimization problems are usually iteratively solved. In other words, one starts with an initial guess about a shape and gradually evolves it until convergence. This is the case reported in [7], where researchers developed a computational model for the inverse design of custom-shaped rubber balloons [Figure 2(b)]. They aimed to find the optimal balloon that approximates the target shape as closely as possible upon inflation. This inverse design problem was recast as a constrained optimization problem and solved by augmented Lagrangian methods. The same group further incorporated seams into the computational model to reproduce complex shapes with sharp creases [15].

Another excellent example of shape-matching design is provided by Siéfert et al. in [8], where, instead of iteratively solving the optimization problem, the authors developed a direct geometric solution based on an analytical model to program arbitrary 3D shapes. The key idea is to precisely control the spatially varying expansions of soft materials by a well-designed airway network embedded inside the matrix [Figure 2(c)]. This work offers a powerful tool to transform soft rubbery plates into desired 3D structures.

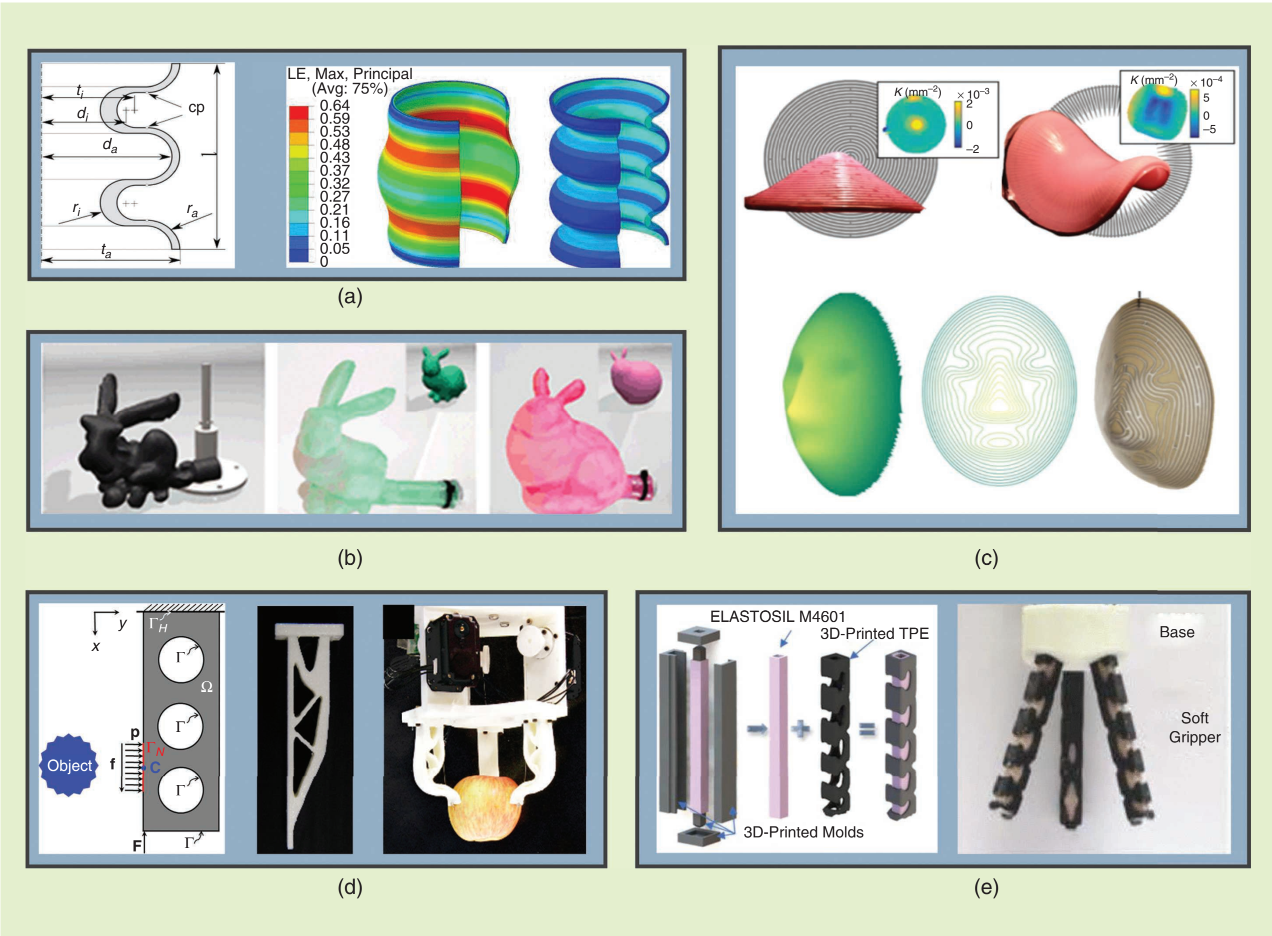


**Figure 2.** The geometry optimization. (a) A linear bellows-type pneumatic actuator described with a set of parameters (left) is optimized to minimize the induced maximal (Max.) principal strain (right) [6]. (b) The inverse design of the rubber balloons enables them to expand to the desired shapes upon pressurization [7]. (c) By precisely controlling the spatially varying expansions of soft materials with a well-designed airway network embedded inside the materials, one can program arbitrary 3D shapes [8]. (d) A cable-driven soft finger is modeled as a beam, subject to topology optimization (left), and the optimized fingers (middle) are assembled to make a soft gripper (right) [9]. (e) A pneumatic soft gripper is composed of an inner chamber made of rubber and an outer layer made of Tango (left), and the outer layer is topologically optimized to deliver maximum bending motion for conformal grasping (right) [10]. LE: logarithmic strain; TPE: thermoplastic elastomer.

To enable generally free-form evolution of shapes, e.g., changes in the connectivity of inner cavities of a deformable body, insight at the topology level is required. In comparison with size or shape optimization, topology optimization depends less on the initial design and works well when intuitive designs fail. Although topology optimization has been widely used in traditional computer-aided design as a versatile tool, few attempts have been made toward the automatic design of soft robots. The main challenge lies in integrating complicated soft material properties and actuation fields into the optimization framework, which causes difficulties in both theory and computation.

As initial attempts, researchers have applied topology optimization methods to design soft bending actuators for use in grippers driven by cables [9] [Figure 2(d)], [16], [17] or pneumatic actuators [10] [Figure 2(e)], [18], [19]. In these works, the gripper design problems were simply translated into the design of fingers modeled by cantilever beams, and their topological shapes were optimized by gradient-based algorithms. The optimization results typically have irregular structural forms and, thus, usually are difficult to manufacture using traditional methods such as molding and casting. Instead, they can be directly prototyped using additive manufacturing technologies [9], [10], [17], [18].

These initial attempts at topology optimization for soft robots did not fully capture the physics in their optimization models, such as the material nonlinearity, interactions, and frictions. In this sense, these works in principle still fall into the framework of traditional compliant mechanism design with topology optimization approaches by Sigmund [20] and Wang et al. [21], [22]. The incorporation of nonlinearities of soft materials may further unlock the potential of topology optimization for the generative design of soft robots [23].

## Design Variable: Material

Material represents another dimension in the design space to be explored and plays another key role in determining the behaviors of soft robots. A straightforward example is that, in addition to the geometry-based design, the directionality of motion can be programmed by combining different materials. From a mechanics perspective, multiple materials may imbue a structure with complex deformation modes under a given load, which enables novel functionalities that are hard, if not impossible, to access using a single material. Compared to the geometry approach, the material optimization approach may result in functional soft robots with geometrically simple and compact-sized bodies.

An intuitive case is that one may use fibers to passively constrain the deformation of flexible, fluid actuators along the user-defined directions to generate differential motions. This design concept is akin to the actuation principle of muscular hydrostats. Elastomers to fabricate fluid actuators are typically isotropic, while the fibers can be treated as an anisotropic material. Thus, the fibers can tailor the deformation of the actuator through their layout, leading to various motions, including extension, contraction, bending, and twisting. In [24], Connolly et al. presented a design strategy to track a predefined kinematic trajectory and developed an analytical model to identify the optimal fiber layout [Figure 3(a)]. In fact, this inherent mechanism has been well explored since the preliminary prototypes of McKibben artificial muscles [25], [26] and is still being employed in recent soft actuators and robots [27]–[31]. The focus of these examples is on the fiber layouts, while the fluid channels remain unchanged as the nondesign domain.

Among the feasible multimaterial approaches to design optimization of soft robots, functionally graded materials (FGMs) are opening up new possibilities. Despite being well investigated in material science, FGMs recently attracted the interest of roboticists because they can combine soft and rigid materials empowered by multimaterial printing technologies. One of the first efforts to exploit FGMs to design soft robots can be found in [32], where Bartlett et al. developed a robot powered by combustion and the bodies consisted of nine types of materials with gradient Young's modulus spanning three orders of magnitude. The smooth stiffness gradient avoids the stress concentration that typically occurs on the interface of two significantly different materials [Figure 3(b)]. However, the material gradient in the FGMs has yet to be optimized with fewer human interventions to fully unlock their potential to produce spatially varying stiffness and motions upon actuation.

To leverage the full design space spanned by multimaterials, free-form distributions of multimaterials and associated optimization algorithms are in high demand. Hiller and Lipson [33] demonstrated cantilever beams that deflected in predefined profiles by virtue of spatially varying soft and rigid materials, and the optimization was implemented with evolutionary search algorithms [Figure 3(c)]. Ma et al. [34] presented a systematic design and fabrication framework for soft pneumatic machines, with the desired motions of a heart beating upon pressurization, by first optimizing the chamber arrangement and subsequently optimizing the multimaterial distributions [Figure 3(d)].

The incorporation of active materials may further equip soft robots with new functions, although design optimization is in its earliest infancy. For example, inspired by bones and joints in human fingers, Yang et al. [35] incorporated shape memory polymers (SMPs) into the finger design as stiffness-tunable joints. The rational design of active materials such as SMPs in soft robots has not been explored much and represents a research direction to enrich the functionalities of soft robots.

## Design Variable: Metamaterial

A metamaterial is generally defined as a material encoded at microscopic or mesoscopic scales to exhibit physical properties, rarely found in natural materials, with various engineering applications. Metamaterials derive their properties not only from the base material but, more importantly, also from the featured structures at the microscopic scale in terms of shape, orientation, and arrangement.

Recently, metamaterials have been receiving increasing attention in soft robotics. The involvement of metamaterials may lead to paradigm shifts in the design of soft robots by directly encoding the desired complex motion within the material architectures, leading to conformable monolithic systems [42]. Currently, the rational design optimization of metamaterials remains in its early infancy, and we focus here on how various metamaterials provide new insight into the design of soft robots.

As mentioned, fibers have features of anisotropy and can be exploited to program motions of soft robots. By assembling fibers with user-defined patterns, general anisotropies that span more directions can be achieved. Textile fabrics are such examples whose anisotropies are directly encoded by the weaving or knitting paths at the stage of fabrication [Figure 4(a)]; they have been widely used in wearable robotic devices for hand, ankle, and foot rehabilitation [36], [43], [44]. Inspired by layered human muscles, such as the transverse abdominis, Zhu et al. [45] recently presented a new family of fluidic fabric muscle sheets based on composite fabric structures that admitted design options at multiple scales, adaptability to curved structures, and large work densities. They also investigated how combinations of the fabric type and stitch design modulate the patterns of stretchability.

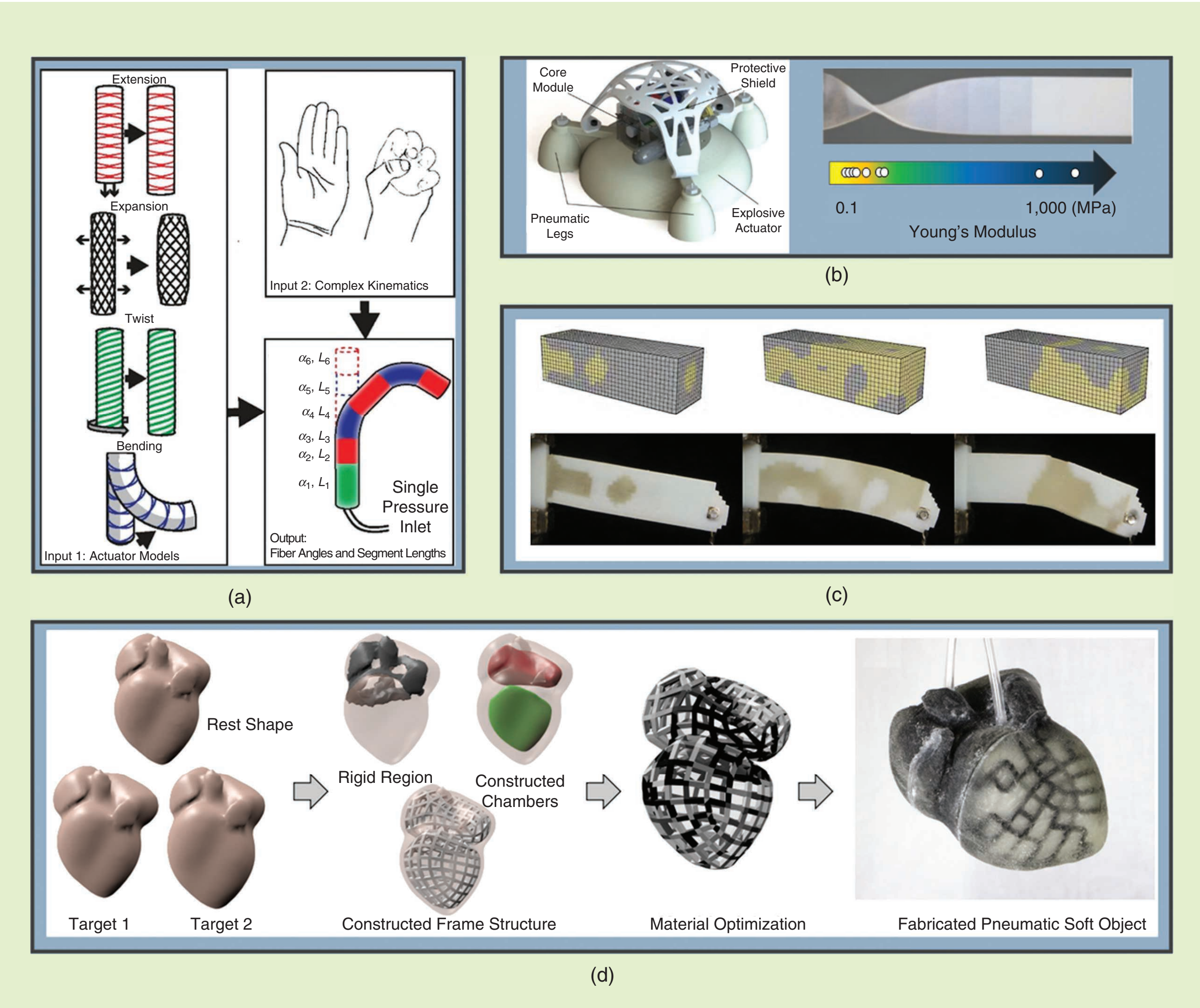


**Figure 3.** The material optimization. (a) The fibers can passively constrain the deformation of pneumatic tubes along user-defined directions to program the output motions (left); one may take a prescribed motion (top right) as the input and optimize the design parameters for actuators to replicate the desired motion upon pressurization (bottom right) [24]. (b) A combustion-powered robot made of nine different materials in terms of modulus attains a smooth stiffness gradient [32]. (c) Soft and hard materials in a cantilever beam are automatically designed using evolutionary algorithms to enable the beam to deflect in a user-defined profile [33]. (d) A systematic design and fabrication framework for soft pneumatic matters with desired shapes upon pressurization. Taking a rest shape and target shapes of a heart model as the input (left), the method first optimizes the chamber arrangement and frame structures (middle left) and subsequently optimizes the material distribution (middle right) of the frames to reproduce the motion behavior of a beating heart (right) [34].

Origami and kirigami represent a special category of metamaterials that lend themselves to programmable morphing of robots. Origami-based metamaterials are usually made by folding thin-walled sheets along predesigned creases to form ridges and valleys [46]. Driving rigid origami by vacuuming, artificial muscles were developed in [47] for use in soft grippers with excellent load capability. Jeong and Lee employed an origami twisted tower to fabricate the fingers of a robotic manipulator [Figure 4(b)], which potentially can be used to manipulate fragile objects [37]. Rafsanjani et al. [38] harnessed kirigami principles to remarkably improve the crawling speed of a soft actuator [Figure 4(c)]. The deformable kirigami surfaces buckle and induce remarkable directional frictional properties. Readers may refer to [48] for a comprehensive review of soft origami robots.

Slender beams are widely used as basic units in flexible metamaterials. The designable arrangements of elastic beam elements may lead to desired mechanical behaviors that are otherwise difficult to achieve, such as negative Poisson's ratio (auxetics). Mark et al. [39] provided an excellent example using auxetic and nonauxetic clutches to simplify the locomotion of a soft robot with only one actuator instead of three [Figure 4(d)]. More generally, auxetic metamaterials may endow a soft robot with the capability of shape matching upon actuation using cellular structures, which consist of auxetic and nonauxetic units [49].

Elastic beam elements may buckle when subjected to axial compressions. This simple phenomenon opens up a new avenue for reversible pattern transformations in metamaterials that consist of networks of elastic beams. This mechanism was exploited by Yang et al. [40] in soft grippers to produce several classical motions driven by a single negative pressure [Figure 4(e)]. To improve structural stiffness and enhance grasping force, the design was further improved in [50], where the output work was taken as the objective function.

More generally, a metamaterial derives its properties by encoding its constituent microstructures [5]. Schumacher et al.

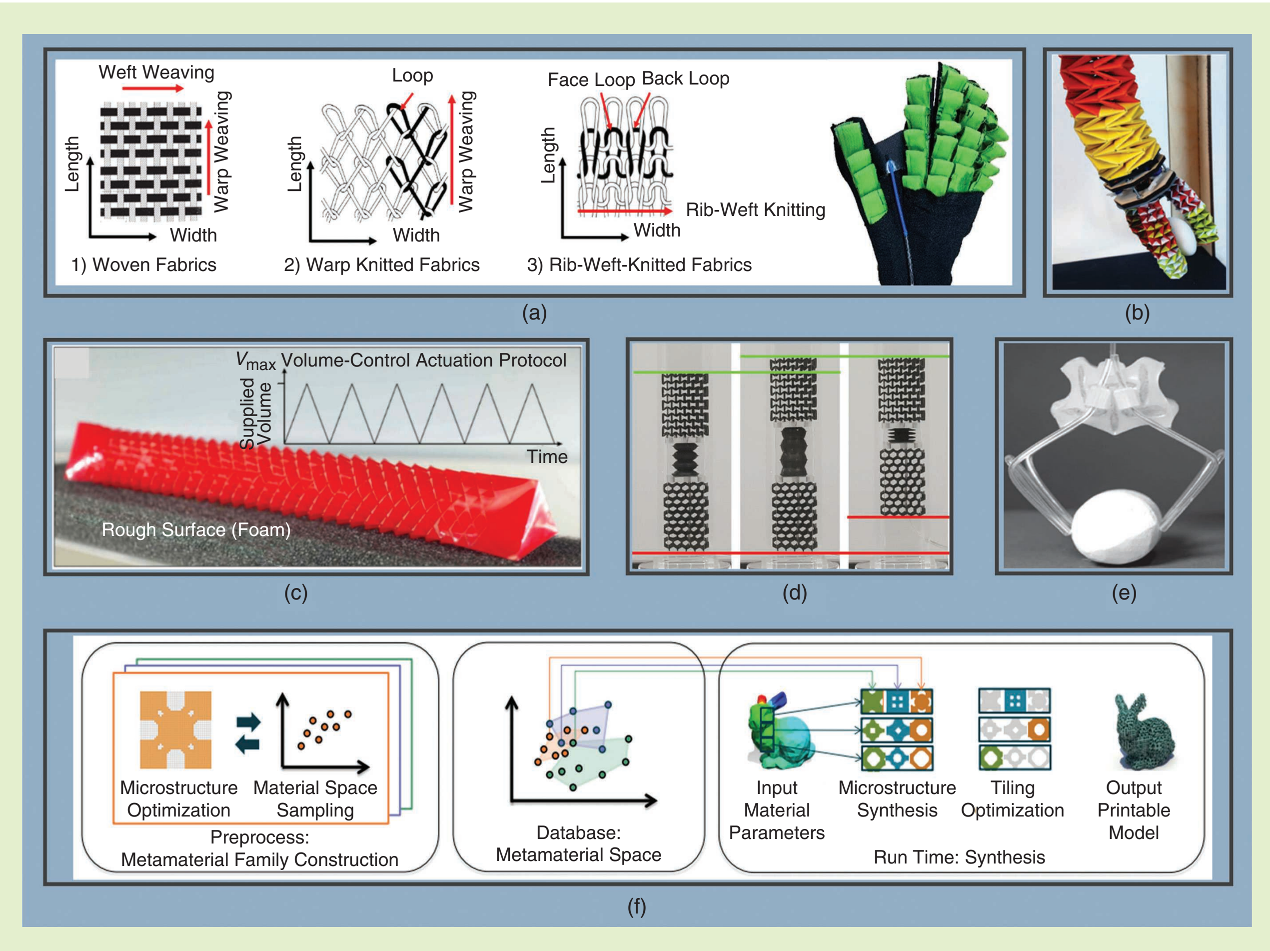


**Figure 4.** The metamaterial optimization. (a) Textile fabrics produced by weaving and knitting methods (left) are used to simultaneously program the motions and increase the load capabilities of a soft wearable assistive glove (right) [36]. (b) An origami twisted tower to fabricate the fingers of a robotic manipulator [37]. (c) An efficient crawling robot benefits from kirigami surfaces wrapped around an extending soft actuator [38]. (d) Auxetic and nonauxetic clutches simplify the locomotion of a soft robot with only one actuator [39]. (e) Elastic beams buckle under negative pressure and enable a soft gripper [40]. (f) An automatic design strategy of microstructures for physical prototypes with desired mechanical properties. Taking specified material parameters as the input, the method automatically optimizes the local microstructures that generate the target deformation behavior [41].

[41] proposed a method to design deformable objects with spatially varying microstructures using 3D printing. Optimization was conducted to design tiled microstructures by interpolating families of related structures to smoothly vary the material properties over a wide range [Figure 4(f)]. However, the microstructure configurations were limited by the prescribed family. The microstructure design may further benefit from unconstrained topology optimization with natural interconnections [51] and consideration of large deformation [52] and buckling phenomena [53].

### *Design Variable: Actuation*

Actuation plays an equally important role in design approaches for soft robots by directly determining the external stimuli. From the perspective of mechanics, actuators define the form, magnitude, and direction of the input loads applied to a soft robot. Various actuation technologies in soft robotics introduce new opportunities and challenges. Unlike rigid robots, where the input force and torques are applied only at the joints, soft robots can be driven by mechanical loads and more often by active materials that are responsive to multiple physical fields, which offers designers more freedom to modulate the actuation fields.

**Soft robots provide excellent examples of design-dependent problems, where the actuation is typically coupled with soft bodies.**

Cable tension and pneumatics represent the traditional actuation technologies in soft robots. Skouras et al. [54] developed a method to automate the design of cable-driven deformable characters that exhibit the desired deformation behaviors. The locations of cables on the character and material distribution were simultaneously optimized, which made the character deform to the target shape [Figure 5(a)]. Hiller and Lipson [33] proposed the concept of volumetric actuation materials for a pneumatic locomotive soft robot. Evolutionary algorithms were used, the fitness function taken to be the moving distance of the center of mass.

Dielectric elastomer actuators (DEAs), which form a classic category of electric active polymers, can generate large deformation when subjected to external high voltages [59]. Due to their advantages of large deformability and rapid response, DEAs have been widely used in soft robotic systems [60], [61]. However, current DEA design paradigms are mostly based on people's intuition or experiences, and a systematic mathematical modeling and optimization methodology is still lacking to exploit their actuation potentials for the desired motion tasks. Hajiesmaili and Clarke [55] made a first attempt by applying gradient electric fields to DEAs along the thickness direction through a layer-by-layer fabrication, and voltage-tunable negative and positive Gaussian curvatures were produced [Figure 5(b)].

More generally, Chen et al. [56] recently developed an automatic design methodology to maximize the displacements of interest of DEAs by topology optimization of the spatially varying electric fields. The optimized design remarkably improved the output displacements by up to 75% compared to their intuitive counterparts, with applications in triggering planar sheets to shape-morph into the desired 3D configurations [Figure 5(c)]. A density-based topology optimization method was applied to the automatic design of DEAs by Wang et al. [62]. In addition, metastructures encoded with designable anisotropies can be combined with DEAs to produce programmable deformations, as demonstrated by a unidirectional actuator in [63].

Magnetic fields are also widely used to drive soft robots by providing a far-field actuation controlled in an untethered manner, and their advantages are long-range, dexterous, precise, fast, and robust characteristics. The magnetically responsive materials are expected to largely deform, navigate in complex workspaces, and perform specific tasks. To program their deformations, a popular avenue is to embed magnetic particles into a soft matrix to create spatially varying magnetic actuations and lead to the desired motions. Kim et al. [64] offered a delicate fabrication solution by directly encoding the layout of ferromagnetic particles in the printing process. Lum et al. [57] developed a design methodology to automatically generate the required magnetization profile and actuating fields, so that a soft cantilever deformed to the desired shapes [Figure 5(d)]. Recently, Tian et al. [58] employed a topology optimization approach to automate the layout design of the ferromagnetic domain. The objective function consists of subobjective functions for kinematics and stiffness requirements. The optimization method was verified on a gripper [Figure 5(e)].

### *Integrated Design*

Although we have classified the optimization model of soft robots in terms of design variables, including geometry, material, metamaterial, and actuation, the boundaries among these variables are not clear. A metamaterial deals with both geometry and material at a small scale. A spatially varying actuation field is usually embodied within the geometry or material. This is the case for pneumatic actuation, where the pressurization is closely associated with the chamber geometry, and magnetic actuation, where the external magnetic field is distributed on the 3D distributed ferromagnetic domain. Thus, an integrated strategy for design optimization is necessary, and consideration of a single design variable generally does not suffice.

In the context of topology optimization, the applied loads may depend on the specific design candidates, which are generally referred to as *design-dependent problems*. The load dependency has been difficult to address, even in cases of linear elasticity [65]. In general, all structures involving solid and fluid interactions carry such design-dependent loads. Soft robots provide excellent examples of design-dependent problems, where the actuation is typically coupled with soft bodies. Thus, designers must cooptimize the actuation fields with the structural features, and powerful algorithms are in high demand.

Furthermore, the morphology of a soft robot can be codesigned with the control strategies, since its performance is concurrently determined by the soft bodies,

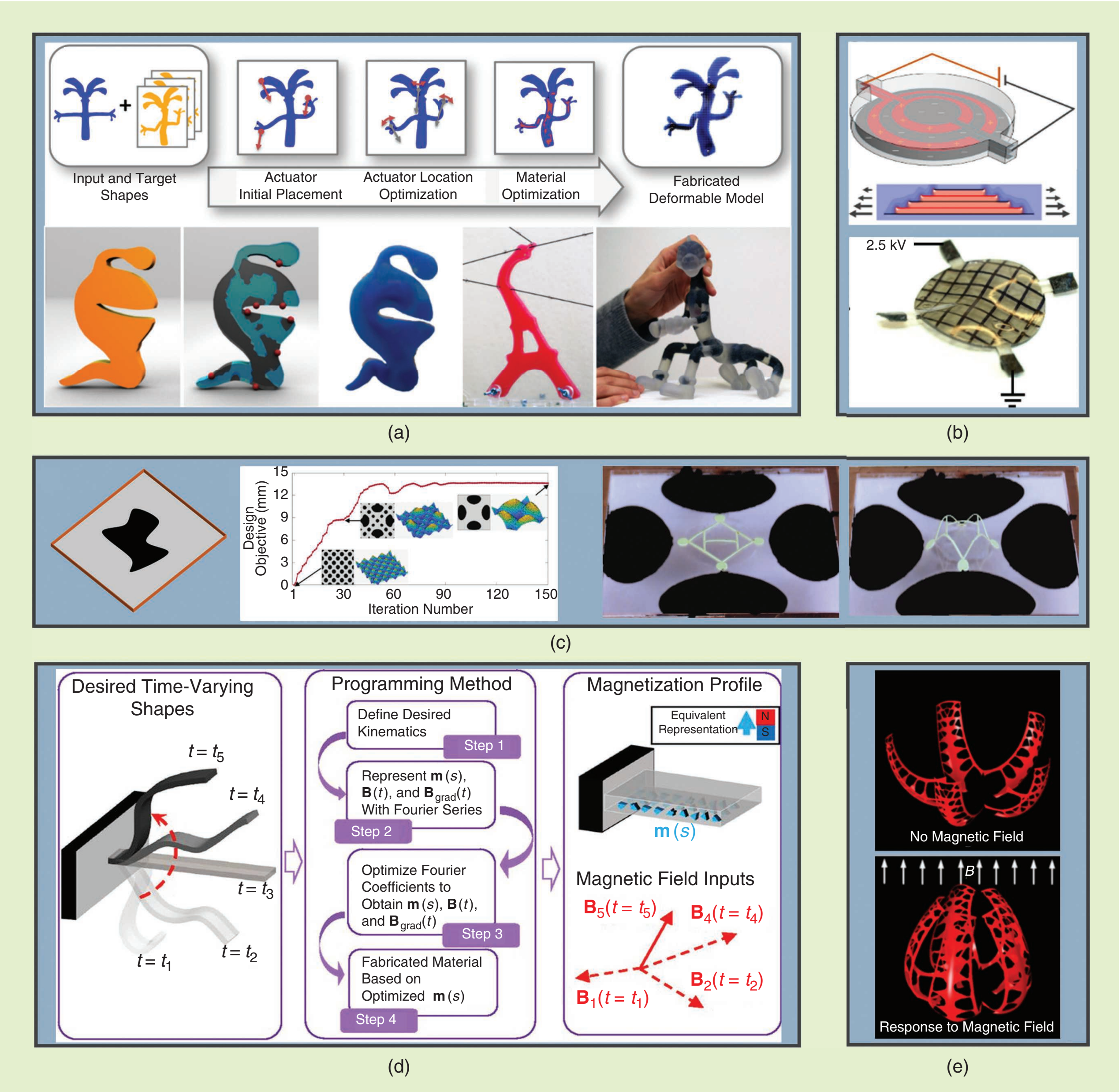


**Figure 5.** The actuation optimization. (a) The automatic design of cable-driven objects to produce physical replicas of the designed characters [54]. (b) Shape-morphing with dielectric elastomers is achieved by gradient electric fields along the thickness direction (top), which results in tunable Gaussian curvatures (bottom) [55]. (c) An automatic design methodology to maximize the displacements of interest of DEAs is enabled by topology optimization of the external spatially varying electric fields (left). The optimization process (middle) creates a design that triggers a planar sheet to buckle into 3D shapes upon electric activation (right) [56]. (d) A programming methodology automatically generates the required magnetization profile and actuating fields, so that soft magnetic materials deform to the time-varying shapes [57]. (e) A soft gripper made of ferromagnetic soft elastomers performs the desired motions from the reference state (top) to a deformed state (bottom) by optimizing the ferromagnetic domain using a topology optimization approach [58].

interactions, and control signals. Deimel et al. [66] investigated the feasibility of codesigning the morphology of soft hands and their control strategies for grasping and found that the codesign always outperformed the counterpart optimization limited to only one design domain. Spielberg et al. [67] proposed a "learning-in-the-loop optimization" design method that allows for the cooptimization of the controller and material parameters, using differentiable simulation techniques. These works represent initial attempts to create an end-to-end design paradigm for soft robots and should be further generalized to more complicated scenarios and validated through physical experiments.

### *Design Space Representation*

To incorporate the physical design variables into an optimization model, a first prerequisite is the mathematical representation of the design space, which is spanned by the aforementioned design variables. The representation should, in general, be able to describe all candidates in the full design space in a unified mathematical framework.

**In the framework of topology optimization, to describe an arbitrary topological shape, two classes of representation methods have been widely used.**

In the framework of topology optimization, to describe an arbitrary topological shape, two classes of representation methods have been widely used. The first category is density-based methods, where the design variables are represented by the continuous "artificial density" of 0–1 [68], [69]. Depending on the physical problem, the spatially varying density may describe the existence or removal of a material or an actuation field, and its distribution is typically discretized by finite elements and interpolated using shape functions. The other representation approach uses an implicit description of boundaries to parameterize the geometry, i.e., level-set-based methods that implicitly define the interfaces among material phases or actuation fields by iso-contours of a level-set function [70]–[72]. This implicit function enables a crisp description of the free-form boundaries. In comparison with explicit boundary descriptions, level-set functions enable the much more convenient tracking of topological changes.

When dealing with structural shape and topology optimization on free-form surfaces, the conformal mapping theory originating from differential geometry on the Riemannian manifold can be combined with topology optimization theories to recast the manifold embedded in the 3D space as a 2D topology optimization problem in the Euclidean space. Ye et al. [73] provided a unified level-set-based computational framework for the generative design of free-form structures by conformally mapping the manifolds onto a 2D rectangle domain where the level-set function is defined, which allows for the convenient use of conventional computational schemes for level set methods.

### Optimization Implementation

To explore the vast design space spanned by the geometry, material, and actuation fields, optimization tools that can automatically search for the optimal design candidates are essential. Powerful optimization algorithms are expected to refine the existing designs and, more importantly, create novel free-form designs that are otherwise hardly attainable by human intuitions or experiences. As summarized in Table 1, we organize the referred works in the "Optimization Model" section in terms of the design variable, report the employed optimization methods, and briefly comment on their generality and applications.

### *Simulation and Analysis*

An important prerequisite to the implementation of optimization algorithms is the simulation and analysis tool that enables designers to evaluate the performance of the current robot design. This prerequisite has been very challenging for soft robots, mainly due to the nonlinearity, multiphysical coupling, and complex interplay between multiple bodies and the environments. In general, one can hardly derive analytical (or semianalytical) solutions for the kinematics of a soft robot but must resort to numerical computation. The analytical solutions listed in Table 1 are case specific or simplified by assumptions such as linearity. Nonlinear finite element analysis has dominated because it can accurately capture complex mechanical behaviors. However, nonlinear solvers tend to suffer convergence issues and are usually limited to relatively small deformations. In addition, the computational cost is very high, which hinders efficient evaluations of designs.

Many attempts have been made to perform fast and robust simulation. Instead of computing the continuous deformation fields, Hiller and Lipson [77] applied nonlinear relaxation where the structure was represented as a network of basic elements, including springs, beams, and masses. However, the parameter identification of the elements is a great challenge, and various actuation technologies can hardly be incorporated into the framework.

Based on the finite element method (FEM), Duriez and colleagues [78] have developed a well-known physics-based simulation engine, SOFA, which simulates the deformation of soft robots by progressively solving a quasi-static equilibrium function for each sample time. The method was recently further improved to achieve real-time computation with a reduced model [74] and has been verified on real soft robots [Figure 6(a)]. The computation efficiency requires further improvement, and the complicated interactions of soft robots with the environment need to be captured.

An alternative approach is to transform the actuation of soft robots into a geometric change. Recently, Fang et al. [75] attempted to solve the kinematics of soft robots from a geometry approach, and their framework incorporated cables, DEAs, and pneumatic actuations using varying line, area, and volume elements, respectively [Figure 6(b)]. The properties of multiple materials are geometrically modeled by calibration, and the kinematics solving is recast as a

**Table 1. A summary of some representative works on soft robot design optimization.**

| Design variable | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| □ | ■ | ★ | ⇑ | **Objective** | **Actuation** | **Design Space** | **Analysis** | **Optimization Method** | **Generality** | **Application** | **References** |
| □ | | | | Stress | Pneumatic | Size | N-FEM | Gradient | + | Linear actuator | [6] |
| □ | | | | Bending | Pneumatic | Size | N-FEM | Enumerative | + | Bending actuator | [11] |
| □ | | | | Twisting | Pneumatic | Size | N-FEM | Enumerative | + | Combined bending and twisting | [13] |
| □ | ■ | | | Bending | Pneumatic | Size | N-FEM | Hybrid | + | Gripper | [14] |
| □ | | | | Shape matching | Pneumatic | Shape | N-FEM | Gradient | ++ | Inverse shape design | [7] |
| □ | | | ⇑ | Shape matching | Pneumatic | Shape | Analytical | N/A | +++ | Inverse shape design | [8] |
| □ | | | | Bending | Cable | Topology | L-FEM | Gradient | +++ | Gripper | [9], [16], [17] |
| □ | | | | Bending | Pneumatic | Topology | L-FEM | Gradient | +++ | Gripper | [10], [18], [19] |
| | ■ | | | Extension, bending, twisting | Pneumatic | Size | Analytical | Nonlinear least square | +++ | Finger | [24] |
| | ■ | | | Gradient stiffness | Combustion | Size | N-FEM | N/A | + | Conceptual | [32] |
| | ■ | | ⇑ | Bending, extension | Cable, pneumatic | Topology | N-FEM | Heuristics | ++ | Actuator | [33] |
| | ■ | | | Shape matching | Pneumatic | Topology | N-FEM | Gradient | +++ | Artificial heart | [34] |
| | | ★ | | Shape matching | Compression | Size | N-FEM | N/A | ++ | Not specific | [49] |
| | | ★ | | Output work | Pneumatic | Size | N-FEM | Gradient | + | Gripper | [50] |
| | | ★ | | Shape and force | Pneumatic | Size | Analytical | Inspiration | ++ | Artificial muscle | [45] |
| | | ★ | | Microstructure material property | N/A | Topology | L-FEM | Gradient | +++ | General | [41] |
| | ■ | | ⇑ | Desired motion | Cable | Topology | L-FEM | Gradient | +++ | General | [54] |
| | | | ⇑ | Gaussian curvature | DEA | Size | N-FEM | N/A | ++ | General | [55] |
| | | | ⇑ | Point displacement | DEA | Topology | N-FEM | Gradient | +++ | Not specific | [56], [62] |
| | | | ⇑ | Shape matching | Magnetic | Shape | N-FEM | Gradient | ++ | General | [57] |
| □ | | | ⇑ | Bending | Magnetic | Topology | L-FEM | Gradient | +++ | Gripper | [58] |

□: geometry; ■: material; ★: metamaterial; ⇑: actuation; L-FEM: linear FEM; N-FEM: nonlinear FEM; N/A: not applicable.
Generality is described by the qualitative evaluation scale, +++, ++, +, from highest to lowest values.

constrained optimization problem. This geometry approach suffers from the limitation that it is based on a linear blending method; thus, it cannot well capture deformation with large strains.

To predict the dynamic behaviors of multiple bodies, Macklin et al. [76] further developed a simulation framework for hybrid rigid and soft bodies, in consideration of contacts and frictions, using a nonsmooth Newton method to address the underlying nonlinear complementarity problems. The nonlinear dynamics models of different bodies were coupled through a smooth isotropic friction model, and a complementarity preconditioner was applied to improve the convergence. To model the pressure loading, the authors adopted an activation function that applies a uniform internal volumetric stress to the domain of interest [Figure 6(c)]. However, more physical experiments need to be conducted to validate the computation framework.

Despite these attempts, there is no fast and effective simulation tool for the computation of soft robots, and this has been a major barrier to their design optimization. Some portable solutions have been developed, but they do not represent a universal solution. The ideal simulation tool is expected to have high efficiency and acceptable accuracy, robustness in a wide variety of problems, numerical stability, and the capability of addressing various actuation fields, multiphysics, nonlinearities, and interactions with the environment.

### Optimization Method

The vast design space of soft robots is extremely difficult for designers to manage. To explore the space and identify the (locally) optimal design, one usually starts from an initial guess and updates the existing design with a better one until the algorithm converges to a feasible solution or the prescribed objective is fulfilled. The search direction is crucial, and designers typically must conduct a sensitivity analysis. In the strategy of line search or trust region, one may define the direction of search based on the information of the objective function and its gradients or Hessians, which require that the problem is differentiable and twice differentiable, respectively. The gradient-based methods include the steepest descent and conjugate gradient as employed in the works [9], [10], [16], [17], [56], while Newton's method requires information of the Hessian matrix.

Although gradient-based optimization algorithms tend to get stuck in local optima, they have better scalability to the number of variables, which is particularly advantageous for handling large-scale problems. This is generally the case for topology optimization of soft robots with an extremely large number of design variables. There are numerous methods to

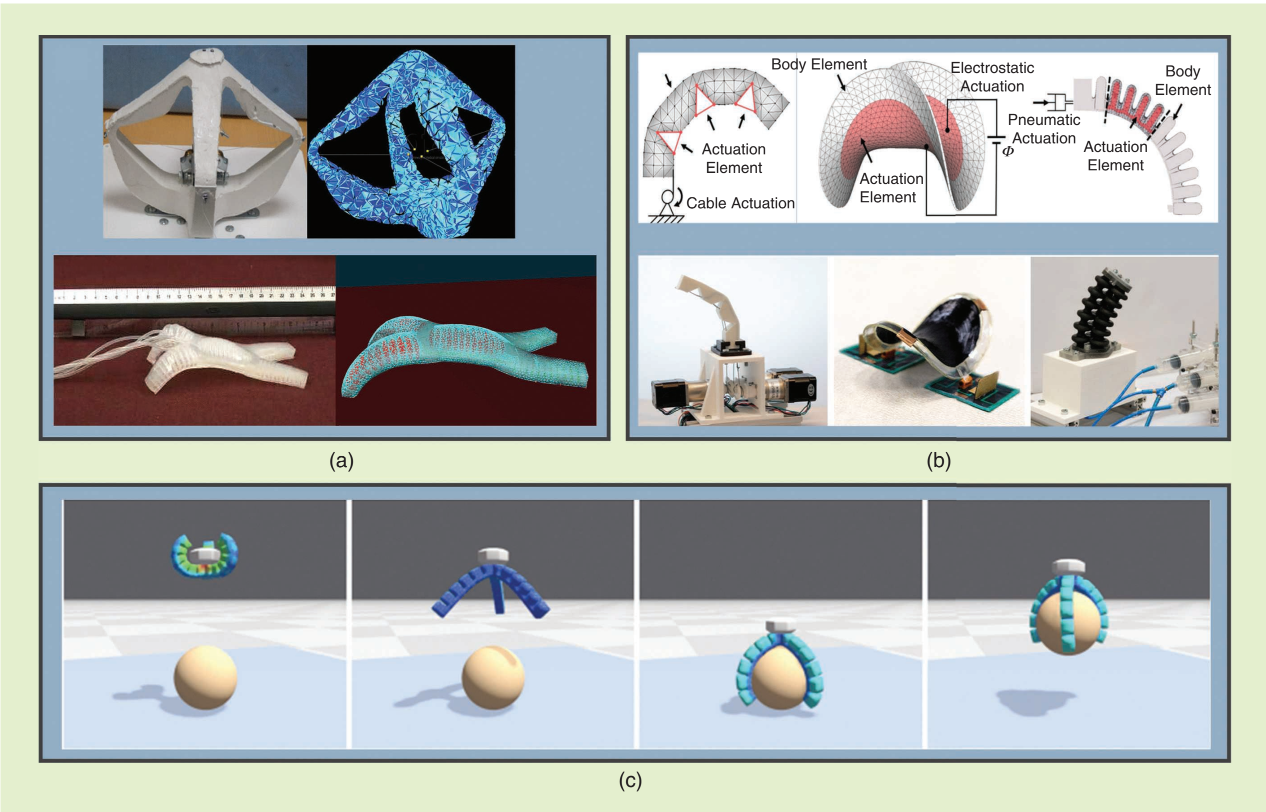


**Figure 6.** The simulation of kinematics and dynamics of soft robots. (a) A physics-based simulation engine, SOFA, realizes real-time simulation with a reduced model [74]. (b) A geometry approach to transforming the kinematics of soft robots into geometric change, which incorporates cables, DEAs, and pneumatic actuations using varying line, area, and volume elements respectively [75]. (c) The dynamics simulation of a soft gripper with robust contact coupling between the gripper and the object [76].

obtain the gradient information, including the simplest finite difference method in a numerical manner and the direct method and adjoint method in an analytical manner. Readers may refer to [79] and [80] for a comprehensive review on methods of sensitivity analysis of computational models in a unified mathematical framework.

Since soft robots have high nonlinearities in terms of geometry, material, and multiphysical fields, the gradients and Hessians of structural responses with respect to the associated design variables sometimes can be elusive to derive (if they exist). In addition, for discrete systems in which the derivative of the objective with respect to the design variables does not exist, gradient-based methods do not apply, e.g., for jamming-driven soft robots, where the behavior of jamming particles can hardly be captured by continuum mechanics.

There may be constraints imposed on the design optimization problems of soft robots, related to stress, mass, volume, and manufacturing. In the optimization model, one may combine the constraints as penalty functions with the objective function to translate the problem into an unconstrained one and solve it using the Lagrangian method. The augmented Lagrangian method is more widely used, especially in the field of topology optimization, since it helps suppress the ill conditioning by incorporating explicit Lagrange multipliers into the objective function.

Recently, evolutionary algorithms, e.g., genetic algorithms, have become a popular alternative approach in the design optimization of soft robots. This approach is essentially heuristic and does not require knowledge of gradients. Researchers have provided examples of evolutionary algorithms employed to design soft manipulators [81], joints [82], locomotion robots [33], [83], [84], and DEAs [85], [86] by stochastically exploring the design space [87]. With a representation, e.g., the Gaussian mixtures, genetic algorithms typically produce new designs by mutating or combining existing ones. The key step is the selection of existing designs to combine, e.g., the deterministic crowding selection method in [33].

The use of evolutionary algorithms does not exclude the computation of soft robots. In other words, when evaluating the fitness of each new design, one still must perform nonlinear simulation and analysis of the physical model to guide the search process. Without knowledge of the gradients, evolutionary algorithms usually suffer from poor scalability, premature convergence, and slow convergence speed, which may result in extremely high computational costs for large-scale design variables of soft robots.

Learning from nature is another alternative approach when rational design optimization is difficult, i.e., the so-called bioinspired design. Natural intelligence is the delicate result of the long-term evolution of a body and a brain together. Many examples of bioinspired soft robots have been offered based on learning from octopuses [88], [89] and elephant trunks [90], [91], but many of them are limited to a copy of natural organisms instead of engineering replicates created based on the inherent physical principles [92]. This phenomenon is mainly caused by the lack of powerful and reliable actuators and multifunctional materials on par with natural counterparts. Nevertheless, designers can greatly benefit from bioinspiration by identifying key principles and transforming them into the design of soft robots.

## Discussion and Future Outlook

Despite progress on the design optimization of soft robots, significant research gaps need to be filled to create new robots that can perform complex tasks in practical applications. Here, we list some main limitations.

1) The optimization model is usually simple and restrictive, based on either the geometry space, material properties, or simplified robotic behaviors. The soft robot design involves strong interplay among geometry, material, structural conditions, and actuation paths to achieve motion and power performance customized for a particular class of tasks the robots are expected to perform. Current optimization models can provide only limited insights into the design problems.
2) There is no effective, efficient, and robust simulation tool to rapidly evaluate the performance of a design candidate. The large deformation in the material of soft robots typically induces nonlinearities, multiphysical coupling, and stability issues. Their full kinematics and dynamics are complex mechanics problems that are difficult to tackle.
3) Reliable and robust optimization algorithms have not been developed. The conventional optimization methods based on gradients require a great deal of mathematical reasoning. Heuristic algorithms promise a feasible solution, but their poor scalability cannot handle large-scale optimization problems. The potential of optimization to create new designs for soft robots remains to be explored.

These limitations also point to potential future research efforts and prospects, as these problems may be addressed from the following perspectives.

### Modeling of Soft Active Materials

Soft active materials in response to external stimuli are increasingly used in the construction of soft robots, as summarized in recent articles [93]–[95]. Structural engineering of these materials is enabling novel controllable mechanical responses and extending the functionalities of soft robots. However, many material systems have no mechanical models to mathematically describe their properties. To be readily encoded in optimization, the mechanical model is expected to be simple and sufficient to capture the key material properties.

### Modeling of Robotic Behaviors

Performing a given task generally requires a sequence of movements. An analog can be found in the rigid robot design. Rodriguez and Mason translated the desired mechanical function of an end effector for manipulation into a sequence of contact constraints in a geometry sense, and they subsequently derived the shape of the effector by synthesizing these constraints in a field vector [96]. The key insight was to create an extended space spanned by the Cartesian product of the configuration space of the mechanism and its workspace. In the future, the workspace of a soft robot when performing a given task, instead of only one working state as in most existing studies, may be incorporated into the optimization model.

In some scenarios, the physical model of soft robots may not be fully specified, e.g., when it depends on interactions that are unknown at the stage of formulation. Instead of guessing about the uncertain quantities, designers may expect more robust solutions by introducing extra knowledge of the quantities into the model. For example, the interactions between a soft gripper and objects may be characterized by a number of possible scenarios with different contact conditions [9], and the probabilities of each scenario can be estimated through experimental tests. Therefore, one may employ stochastic optimization algorithms to quantify these uncertainties so that the model can be optimized to generate the desired performance.

### Efficient Simulation Tools

The major concerns for the simulation of soft robots are computational cost, convergence, and stability. In addition to large-scale computation, simulation may suffer from convergence issues. Since soft robots typically experience large deformation, extensive distortions may occur locally, thin members may easily buckle, and there may be multiple stable solutions. These phenomena usually need to be addressed case by case, which hinders the automatic evaluation in an optimization loop.

In the field of computer graphics, people usually need to produce physically plausible solutions. In particular, simulation tools developed to animate deformable bodies are promising to lend themselves to simulations of soft robots [97], [98]. In their simulation framework, analysis problems are commonly modeled as constrained optimization problems, refined by preconditioning treatments, decomposed into a set of subproblems, and iteratively solved by various algorithms, such as sequential quadratic programming. Recently, Hu et al. [99] developed ChainQueen, a real-time differentiable physical simulator for soft robots  based on the moving least-squares material point method. The differentiable simulator can be naturally incorporated into gradient-based optimization algorithms to allow for the codesign of soft robots. However, these promising simulation frameworks have not been well verified in physical scenarios. In addition, commonly used computational tricks in the context of computer graphics for numerical convergence and speed must be carefully addressed according to physical laws, and more actuation technologies should be encoded into the simulation framework.

### Powerful Optimization Algorithms

Mathematical formulations of design optimization for soft robots can be addressed in the framework of current optimization theory [100], except they are generally complicated and characterized by large scale, nonlinearity, nonconvexity, and possibly discontinuity and uncertainty. The optimization model may be ill conditioned, and the sensitivity of the objective to the design variables can be elusive to obtain. The theory of topology optimization from a structure or mechanism perspective can help lay a foundation for design optimization of soft robots by further incorporating the nonlinearities and various actuation technologies into the framework.

Selection and development of optimization algorithms largely depend on the optimization model. There is no universal solution; instead, people need to employ appropriate algorithms for the formulated problem. For continuous and smooth design objective and variables, gradient- and Hessian-based algorithms are preferred to find (locally) optimal solutions. For discrete problems, the design space is not continuous, and heuristic algorithms may apply. When the optimization problem is not deterministic, stochastic optimization techniques should be developed. Tradeoffs between convergence and storage and between robustness and speed are always important numerical issues in the optimization implementation.

## Conclusions

Soft robot design rests on the twin pillars of material–structure and properties–performance relations, in an analogy to the well-known Olson's linear concept of "materials by design." The process of relating materials to structure is essentially a modeling simulation task, while the process of relating properties to performance is effectively a synthesis optimization exercise. Establishing optimization-based design methods is an important step toward enabling the rapid and concurrent design for both materials and machines with the potential for significant advancement.

The performance of soft robots can be enhanced by exploring the ample design space offered by geometry, material, metamaterial, and actuation. With the support of high-performance computing, robust and efficient simulation tools and optimization algorithms are essential. Once promising designs are identified, their practical implementation may require advanced technologies for fabrication and manufacturing. Advances in multimaterial 3D printing are promising to address the fabrication challenges.

In the long term, an end-to-end design framework will incorporate robot morphologies, interactions with the environment, and control signals. Optimization-based design methods will encompass a unified mathematical representation of the state variables and physical laws of soft materials, powerful simulators, and optimization algorithms, which open up new possibilities of encoding complex behaviors of a soft robot within its physical body. We have proposed possible research prospects with an expectation that design optimization tools will empower soft robots with currently unforeseen functionalities.

## Acknowledgment

Feifei Chen would like to acknowledge support from the National Natural Science Foundation of China under Grant No. 51905340 and from the Shanghai Sailing Program under Project 19YF1422900.

## References

[1] C. Majidi, "Soft robotics: A perspective-current trends and prospects for the future," *Soft Robot.*, vol. 1, no. 1, pp. 5–11, 2014. doi: 10.1089/soro.2013.0001.

[2] D. Rus and M. T. Tolley, "Design, fabrication and control of soft robots," *Nature*, vol. 521, no. 7553, pp. 467–475, 2015. doi: 10.1038/nature14543.

[3] C. Laschi, B. Mazzolai, and M. Cianchetti, "Soft robotics: Technologies and systems pushing the boundaries of robot abilities," *Sci. Robot.*, vol. 1, no. 1, p. eaah3690, 2016. doi: 10.1126/scirobotics.aah3690.

[4] C. Laschi and B. Mazzolai, "Lessons from animals and plants: The symbiosis of morphological computation and soft robotics," *IEEE Robot. Autom. Mag.*, vol. 23, no. 3, pp. 107–114, 2016. doi: 10.1109/MRA.2016.2582726.

[5] D. Howard, A. E. Eiben, D. F. Kennedy, J.-B. Mouret, P. Valencia, and D. Winkler, "Evolving embodied intelligence from materials to machines," *Nature Mach. Intell.*, vol. 1, no. 1, pp. 12–19, 2019. doi: 10.1038/s42256-018-0009-9.

[6] G. Dämmer, S. Gablenz, A. Hildebrandt, and Z. Major, "Design and shape optimization of PolyJet bellows actuators," in *Proc. 2018 IEEE Int. Conf. Soft Robot. (RoboSoft)*, pp. 282–287. doi: 10.1109/ROBOSOFT.2018.8404933.

[7] M. Skouras, B. Thomaszewski, B. Bickel, and M. Gross, "Computational design of rubber balloons," *Comput. Graph. Forum*, vol. 31, no. 2, pp. 835–844, 2012. doi: 10.1111/j.1467-8659.2012.03064.x.

[8] E. Siéfert, E. Reyssat, J. Bico, and B. Roman, "Bio-inspired pneumatic shape-morphing elastomers," *Nature Mater.*, vol. 18, no. 1, pp. 24–28, 2019. doi: 10.1038/s41563-018-0219-x.

[9] F. Chen et al., "Topology optimized design, fabrication, and characterization of a soft cable-driven gripper," *IEEE Robot. Autom. Lett.*, vol. 3, no. 3, pp. 2463–2470, 2018. doi: 10.1109/LRA.2018.2800115.

[10] H. Zhang, A. S. Kumar, F. Chen, J. Y. Fuh, and M. Y. Wang, "Topology optimized multimaterial soft fingers for applications on grippers, rehabilitation, and artificial hands," *IEEE/ASME Trans. Mechatronics*, vol. 24, no. 1, pp. 120–131, 2019. doi: 10.1109/TMECH.2018.2874067.

[11] Y. Elsayed et al., "Finite element analysis and design optimization of a pneumatically actuating silicone module for robotic surgery applications," *Soft Robot.*, vol. 1, no. 4, pp. 255–262, 2014. doi: 10.1089/soro.2014.0016.

[12] B. Mosadegh et al., "Pneumatic networks for soft robotics that actuate rapidly," *Adv. Functional Mater.*, vol. 24, no. 15, pp. 2163–2170, 2014. doi: 10.1002/adfm.201303288.

[13] T. Wang, L. Ge, and G. Gu, "Programmable design of soft pneu-net actuators with oblique chambers can generate coupled bending and twisting motions," *Sensors Actuators A, Phys.*, vol. 271, pp. 131–138, Mar. 2018. doi: 10.1016/j.sna.2018.01.018.

[14] Z. Wang and S. Hirai, "Geometry and material optimization of a soft pneumatic gripper for handling deformable object," in *Proc. 2018 IEEE Int. Conf. Robot. Biomimetics (ROBIO)*, pp. 612–617. doi: 10.1109/ROBIO.2018.8665234.

[15] M. Skouras et al., "Designing inflatable structures," *ACM Trans. Graph.*, vol. 33, no. 4, pp. 1–10, 2014. doi: 10.1145/2601097.2601166.

[16] Y. Liu and M. Y. Wang, "Topology design of a conforming gripper with distributed compliance via a level set method," in *Proc. 2014 IEEE Int. Conf. Robot. Biomimetics (ROBIO 2014)*, pp. 2191–2196. doi: 10.1109/ROBIO.2014.7090662.

[17] C.-H. Liu et al., "Optimal design of a soft robotic gripper for grasping unknown objects," *Soft Robot.*, vol. 5, no. 4, pp. 452–465, 2018. doi: 10.1089/soro.2017.0121.

[18] H. Zhang, M. Y. Wang, F. Chen, Y. Wang, A. S. Kumar, and J. Y. Fuh, "Design and development of a soft gripper with topology optimization," in *Proc. 2017 IEEE/RSJ Int. Conf. Intell. Robot. Syst. (IROS)*, pp. 6239–6244. doi: 10.1109/IROS.2017.8206527.

[19] Y. Chen, Z. Xia, and Q. Zhao, "Optimal design of soft pneumatic bending actuators subjected to design-dependent pressure loads," *IEEE/ASME Trans. Mechatronics*, vol. 24, no. 6, pp. 2873–2884, 2019. doi: 10.1109/TMECH.2019.2943418.

[20] O. Sigmund, "On the design of compliant mechanisms using topology optimization," *J. Struct. Mech.*, vol. 25, no. 4, pp. 493–524, 1997. doi: 10.1080/08905459708945415.

[21] M. Y. Wang, S. Chen, X. Wang, and Y. Mei, "Design of multimaterial compliant mechanisms using level-set methods," *J. Mech. Design*, vol. 127, no. 5, pp. 941–956, 2005. doi: 10.1115/1.1909206.

[22] M. Y. Wang, "A kinetoelastic formulation of compliant mechanism optimization," *J. Mechanisms Robot.*, vol. 1, no. 2, p. 021011, 2009. doi: 10.1115/1.3056476.

[23] F. Chen, Y. Wang, M. Y. Wang, and Y. Zhang, "Topology optimization of hyperelastic structures using a level set method," *J. Comput. Phys.*, vol. 351, pp. 437–454, Dec. 2017. doi: 10.1016/j.jcp.2017.09.040.

[24] F. Connolly, C. J. Walsh, and K. Bertoldi, "Automatic design of fiber-reinforced soft actuators for trajectory matching," *Proc. Nat. Acad. Sci.*, vol. 114, no. 1, pp. 51–56, 2017. doi: 10.1073/pnas.1615140114.

[25] M. Gavrilović and M. Marić, "Positional servo-mechanism activated by artificial muscles," *Med. Biol. Eng.*, vol. 7, no. 1, pp. 77–82, 1969. doi: 10.1007/BF02474672.

[26] C.-P. Chou and B. Hannaford, "Measurement and modeling of McKibben pneumatic artificial muscles," *IEEE Trans. Robot. Autom.*, vol. 12, no. 1, pp. 90–102, 1996. doi: 10.1109/70.481753.

[27] G. Singh and G. Krishnan, "An isoperimetric formulation to predict deformation behavior of pneumatic fiber reinforced elastomeric actuators," in *Proc. 2015 IEEE/RSJ Int. Conf. Intell. Robot. Syst. (IROS)*, pp. 1738–1743. doi: 10.1109/IROS.2015.7353602.

[28] J. Bishop-Moser and S. Kota, "Design and modeling of generalized fiber-reinforced pneumatic soft actuators," *IEEE Trans. Robot.*, vol. 31, no. 3, pp. 536–545, 2015. doi: 10.1109/TRO.2015.2409452.

[29] P. Polygerinos et al., "Modeling of soft fiber-reinforced bending actuators," *IEEE Trans. Robot.*, vol. 31, no. 3, pp. 778–789, 2015. doi: 10.1109/TRO.2015.2428504.

[30] S. Y. Kim, R. Baines, J. Booth, N. Vasios, K. Bertoldi, and R. Kramer-Bottiglio, "Reconfigurable soft body trajectories using unidirectionally stretchable composite laminae," *Nature Commun.*, vol. 10, no. 1, p. 3464, 2019. doi: 10.1038/s41467-019-11294-7.

[31] J. Zhang et al., "Geometric confined pneumatic soft-rigid hybrid actuators," *Soft Robot.*, early access, Feb. 21, 2020. doi: 10.1089/soro.2018.0157.

[32] N. W. Bartlett et al., "A 3D-printed, functionally graded soft robot powered by combustion," *Science*, vol. 349, no. 6244, pp. 161–165, 2015. doi: 10.1126/science.aab0129.

[33] J. Hiller and H. Lipson, "Automatic design and manufacture of soft robots," *IEEE Trans. Robot.*, vol. 28, no. 2, pp. 457–466, 2012. doi: 10.1109/TRO.2011.2172702.

[34] L.-K. Ma, Y. Zhang, Y. Liu, K. Zhou, and X. Tong, "Computational design and fabrication of soft pneumatic objects with desired deformations," *ACM Trans. Graph.*, vol. 36, no. 6, pp. 1–12, 2017. doi: 10.1145/3130800.3130850.

[35] Y. Yang, Y. Chen, Y. Li, M. Z. Chen, and Y. Wei, "Bioinspired robotic fingers based on pneumatic actuator and 3d printing of smart material," *Soft Robot.*, vol. 4, no. 2, pp. 147–162, 2017. doi: 10.1089/soro.2016.0034.

[36] L. Ge et al., "Design, modeling, and evaluation of fabric-based pneumatic actuators for soft wearable assistive gloves," *Soft Robot.*, early access, Jan. 29, 2020. doi: 10.1089/soro.2019.0105.

[37] D. Jeong and K. Lee, "Design and analysis of an origami-based three-finger manipulator," *Robotica*, vol. 36, no. 2, pp. 261–274, 2018. doi: 10.1017/S0263574717000340.

[38] A. Rafsanjani, Y. Zhang, B. Liu, S. M. Rubinstein, and K. Bertoldi, "Kirigami skins make a simple soft actuator crawl," *Sci. Robot.*, vol. 3, no. 15, p. eaar7555, 2018. doi: 10.1126/scirobotics.aar7555.

[39] A. G. Mark, S. Palagi, T. Qiu, and P. Fischer, "Auxetic metamaterial simplifies soft robot design," in *Proc. 2016 IEEE Int. Conf. Robot. Automation (ICRA)*, pp. 4951–4956. doi: 10.1109/ICRA.2016.7487701.

[40] D. Yang et al., "Buckling of elastomeric beams enables actuation of soft machines," *Adv. Mater.*, vol. 27, no. 41, pp. 6323–6327, 2015. doi: 10.1002/adma.201503188.

[41] C. Schumacher, B. Bickel, J. Rys, S. Marschner, C. Daraio, and M. Gross, "Microstructures to control elasticity in 3D printing," *ACM Trans. Graph.*, vol. 34, no. 4, pp. 1–13, 2015. doi: 10.1145/2766926.

[42] A. Rafsanjani, K. Bertoldi, and A. R. Studart, "Programming soft robots with flexible mechanical metamaterials," *Sci. Robot.*, vol. 4, no. 29, p. eaav7874, 2019. doi: 10.1126/scirobotics.aav7874.

[43] P. Polygerinos, Z. Wang, K. C. Galloway, R. J. Wood, and C. J. Walsh, "Soft robotic glove for combined assistance and at-home rehabilitation," *Robot. Auton. Syst.*, vol. 73, pp. 135–143, Nov. 2015. doi: 10.1016/j.robot.2014.08.014.

[44] M. Cianchetti, C. Laschi, A. Menciassi, and P. Dario, "Biomedical applications of soft robotics," *Nature Rev. Mater.*, vol. 3, no. 6, pp. 143–153, 2018. doi: 10.1038/s41578-018-0022-y.

[45] M. Zhu, T. N. Do, E. Hawkes, and Y. Visell, "Fluidic fabric muscle sheets for wearable and soft robotics," *Soft Robot.*, vol. 7, no. 2, pp. 179–197, 2020. doi: 10.1089/soro.2019.0033.

[46] Z. Zhakypov and J. Paik, "Design methodology for constructing multimaterial origami robots and machines," *IEEE Trans. Robot.*, vol. 34, no. 1, pp. 151–165, 2018. doi: 10.1109/TRO.2017.2775655.

[47] S. Li, D. M. Vogt, D. Rus, and R. J. Wood, "Fluid-driven origami-inspired artificial muscles," *Proc. Nat. Acad. Sci.*, vol. 114, no. 50, pp. 13,132–13,137, 2017. doi: 10.1073/pnas.1713450114.

[48] D. Rus and M. T. Tolley, "Design, fabrication and control of origami robots," *Nature Rev. Mater.*, vol. 3, no. 6, pp. 101–112, 2018. doi: 10.1038/s41578-018-0009-8.

[49] M. Mirzaali, S. Janbaz, M. Strano, L. Vergani, and A. A. Zadpoor, "Shape-matching soft mechanical metamaterials," *Sci. Rep.*, vol. 8, no. 1, p. 965, 2018. doi: 10.1038/s41598-018-19381-3.

[50] D. Guo and Z. Kang, "Chamber layout design optimization of soft pneumatic robots," *Smart Mater. Struct.*, vol. 29, no. 2, p. 025017, 2020.

[51] Y. Wang, F. Chen, and M. Y. Wang, "Concurrent design with connectable graded microstructures," *Comput. Method Appl. Mech. Eng.*, vol. 317, pp. 84–101, Apr. 2017. doi: 10.1016/j.cma.2016.12.007.

[52] A. Clausen, F. Wang, J. S. Jensen, O. Sigmund, and J. A. Lewis, "Topology optimized architectures with programmable Poisson's ratio over large deformations," *Adv. Mater.*, vol. 27, no. 37, pp. 5523–5527, 2015. doi: 10.1002/adma.201502485.

[53] G. L. Bluhm, O. Sigmund, F. Wang, and K. Poulios, "Nonlinear compressive stability of hyperelastic 2d lattices at finite volume fractions," *J. Mech. Phys. Solid*, vol. 137, p. 103,851, Apr. 2020. doi: 10.1016/j.jmps.2019.103851.

[54] M. Skouras, B. Thomaszewski, S. Coros, B. Bickel, and M. Gross, "Computational design of actuated deformable characters," *ACM Trans. Graph.*, vol. 32, no. 4, pp. 1–10, 2013. doi: 10.1145/2461912.2461979.

[55] E. Hajiesmaili and D. R. Clarke, "Reconfigurable shape-morphing dielectric elastomers using spatially varying electric fields," *Nature Commun.*, vol. 10, no. 1, p. 183, 2019. doi: 10.1038/s41467-018-08094-w.

[56] F. Chen, K. Liu, Y. Wang, J. Zou, G. Gu, and X. Zhu, "Automatic design of soft dielectric elastomer actuators with optimal spatial electric fields," *IEEE Trans. Robot.*, vol. 35, no. 5, pp. 1150–1165, 2019. doi: 10.1109/TRO.2019.2920108.

[57] G. Z. Lum et al., "Shape-programmable magnetic soft matter," *Proc. Nat. Acad. Sci.*, vol. 113, no. 41, pp. E6007–E6015, 2016. doi: 10.1073/pnas.1608193113.

[58] J. Tian, X. Zhao, X. Gu, and S. Chen, "Designing ferromagnetic soft robots (FerroSoRo) with level-set-based multiphysics topology optimization," in *Proc. 2020 Int. Conf. Robot. Automation (ICRA)*, pp. 10,067–10,074. doi: 10.1109/ICRA40945.2020.9197457.

[59] R. Pelrine, R. Kornbluh, Q. Pei, and J. Joseph, "High-speed electrically actuated elastomers with strain greater than 100%," *Science*, vol. 287, no. 5454, pp. 836–839, 2000. doi: 10.1126/science.287.5454.836.

[60] G.-Y. Gu, J. Zhu, L.-M. Zhu, and X. Zhu, "A survey on dielectric elastomer actuators for soft robots," *Bioinspiration Biomimetics*, vol. 12, no. 1, p. 011003, 2017. doi: 10.1088/1748-3190/12/1/011003.

[61] U. Gupta, L. Qin, Y. Wang, H. Godaba, and J. Zhu, "Soft robots based on dielectric elastomer actuators: A review," *Smart Mater. Struct.*, vol. 28, no. 10, p. 103,002, 2019.

[62] B. Chen, N. Wang, X. Zhang, and Y. Kuang, "Design of dielectric elastomer actuators using topology optimization on electrodes," *Smart Mater. Struct.*, vol. 29, no. 7, p. 075029, 2020. doi: 10.1088/1361-665X/ab8b2d.

[63] K. Liu, S. Chen, F. Chen, and X. Zhu, "A unidirectional soft dielectric elastomer actuator enabled by built-in honeycomb metastructures," *Polymers*, vol. 12, no. 3, p. 619, 2020. doi: 10.3390/polym12030619.

[64] Y. Kim, H. Yuk, R. Zhao, S. A. Chester, and X. Zhao, "Printing ferromagnetic domains for untethered fast-transforming soft materials," *Nature*, vol. 558, no. 7709, pp. 274–279, 2018. doi: 10.1038/s41586-018-0185-0.

[65] B.-C. Chen and N. Kikuchi, "Topology optimization with design-dependent loads," *Finite Element Anal. Design*, vol. 37, no. 1, pp. 57–70, 2001. doi: 10.1016/S0168-874X(00)00021-4.

[66] R. Deimel, P. Irmisch, V. Wall, and O. Brock, "Automated co-design of soft hand morphology and control strategy for grasping," in *Proc. 2017 IEEE/RSJ Int. Conf. Intell. Robot. Syst. (IROS)*, pp. 1213–1218. doi: 10.1109/IROS.2017.8202294.

[67] A. Spielberg, A. Zhao, Y. Hu, T. Du, W. Matusik, and D. Rus, "Learning-in-the-loop optimization: End-to-end control and co-design of soft robots through learned deep latent representations," in *Proc. Adv. Neural Inf. Process. Syst.*, 2019, pp. 8284–8294.

[68] M. P. Bendsoe and N. Kikuchi, "Generating optimal topologies in structural design using a homogenization method," *Comput. Methods Appl. Mech. Eng.*, vol. 71, no. 2, pp. 197–224, 1988. doi: 10.1016/0045-7825(88)90086-2.

[69] O. Sigmund, "A 99 line topology optimization code written in Matlab," *Struct. Multidisciplinary Optim.*, vol. 21, no. 2, pp. 120–127, 2001. doi: 10.1007/s001580050176.
[70] J. A. Sethian, *Level Set Methods and Fast Marching Methods: Evolving Interfaces in Computational Geometry, Fluid Mechanics, Computer Vision, and Materials Science*, vol. 3. Cambridge, U.K.: Cambridge Univ. Press, 1999.
[71] M. Y. Wang, X. Wang, and D. Guo, "A level set method for structural topology optimization," *Comput. Methods Appl. Mech. Eng.*, vol. 192, nos. 1–2, pp. 227–246, 2003. doi: 10.1016/S0045-7825(02)00559-5.
[72] G. Allaire, F. Jouve, and A.-M. Toader, "Structural optimization using sensitivity analysis and a level-set method," *J. Comput. Phys.*, vol. 194, no. 1, pp. 363–393, 2004. doi: 10.1016/j.jcp.2003.09.032.
[73] Q. Ye, Y. Guo, S. Chen, N. Lei, and X. D. Gu, "Topology optimization of conformal structures on manifolds using extended level set methods (X-LSM) and conformal geometry theory," *Comput. Method Appl. Mech. Eng.*, vol. 344, pp. 164–185, Feb. 2019. doi: 10.1016/j.cma.2018.08.045.
[74] O. Goury and C. Duriez, "Fast, generic, and reliable control and simulation of soft robots using model order reduction," *IEEE Trans. Robot.*, vol. 34, no. 6, pp. 1565–1576, 2018. doi: 10.1109/TRO.2018.2861900.
[75] G. Fang, C.-D. Matte, R. B. Scharff, T.-H. Kwok, and C. C. Wang, "Kinematics of soft robots by geometric computing," *IEEE Trans. Robot.*, vol. 36, no. 4, pp. 1272–1286, 2020. doi: 10.1109/TRO.2020.2985583.
[76] M. Macklin, K. Erleben, M. Müller, N. Chentanez, S. Jeschke, and V. Makoviychuk, "Non-smooth newton methods for deformable multibody dynamics," *ACM Trans. Graph.*, vol. 38, no. 5, pp. 1–20, 2019. doi: 10.1145/3338695.
[77] J. Hiller and H. Lipson, "Dynamic simulation of soft multimaterial 3d-printed objects," *Soft Robot.*, vol. 1, no. 1, pp. 88–101, 2014. doi: 10.1089/soro.2013.0010.
[78] C. Duriez et al., "Framework for online simulation of soft robots with optimization-based inverse model," in *Proc. 2016 IEEE Int. Conf. Simul., Model., Program. Auton. Robot. (SIMPAR)*, pp. 111–118. doi: 10.1109/SIMPAR.2016.7862384.
[79] K. K. Choi and N.-H. Kim, *Structural Sensitivity Analysis and Optimization 1: Linear Systems*. New York: Springer-Verlag, 2006.
[80] J. R. Martins and J. T. Hwang, "Review and unification of methods for computing derivatives of multidisciplinary computational models," *AIAA J.*, vol. 51, no. 11, pp. 2582–2599, 2013. doi: 10.2514/1.J052184.
[81] D. M. Bodily, T. F. Allen, and M. D. Killpack, "Multi-objective design optimization of a soft, pneumatic robot," in *Proc. 2017 IEEE Int. Conf. Robot. Autom. (ICRA)*, pp. 1864–1871. doi: 10.1109/ICRA.2017.7989218.
[82] X. Li, K. Sun, C. Guo, T. Liu, and H. Liu, "Design, modeling and characterization of a joint for inflatable robotic arms," *Mechatronics*, vol. 65, p. 102,311, Feb. 2020. doi: 10.1016/j.mechatronics.2019.102311.
[83] J. Rieffel, D. Knox, S. Smith, and B. Trimmer, "Growing and evolving soft robots," *Artif. Life*, vol. 20, no. 1, pp. 143–162, 2014. doi: 10.1162/ARTL_a_00101.
[84] H. Lipson, "Challenges and opportunities for design, simulation, and fabrication of soft robots," *Soft Robot.*, vol. 1, no. 1, pp. 21–27, 2014. doi: 10.1089/soro.2013.0007.
[85] Y. Wang, L. Zhang, S. Daynes, H. Zhang, S. Feih, and M. Y. Wang, "Design of graded lattice structure with optimized mesostructures for additive manufacturing," *Mater. Design*, vol. 142, pp. 114–123, Mar. 2018. doi: 10.1016/j.matdes.2018.01.011.
[86] E. Bortot, O. Amir, and G. Shmuel, "Topology optimization of dielectric elastomers for wide tunable band gaps," *Int. J. Solid Struct.*, vol. 143, pp. 262–273, June 2018. doi: 10.1016/j.ijsolstr.2018.03.014.
[87] G. Zames et al., "Genetic algorithms in search, optimization and machine learning," *Inform. Technol. J.*, vol. 3, no. 1, pp. 301–302, 1981.
[88] C. Laschi, M. Cianchetti, B. Mazzolai, L. Margheri, M. Follador, and P. Dario, "Soft robot arm inspired by the octopus," *Adv. Robot.*, vol. 26, no. 7, pp. 709–727, 2012. doi: 10.1163/156855312X626343.
[89] M. Wehner et al., "An integrated design and fabrication strategy for entirely soft, autonomous robots," *Nature*, vol. 536, no. 7617, pp. 451–455, 2016. doi: 10.1038/nature19100.
[90] X. Wang, H. Faraji, and Y. Mengüç, "Incorporate oblique muscle contractions to strengthen soft robots," in *Proc. 2018 IEEE Int. Conf. Robot. Autom. (ICRA)*, pp. 4200–4205. doi: 10.1109/ICRA.2018.8461139.
[91] M. Schaffner, J. A. Faber, L. Pianegonda, P. A. Rühs, F. Coulter, and A. R. Studart, "3' printing of robotic soft actuators with programmable bioinspired architectures," *Nature Commun.*, vol. 9, no. 1, pp. 1–9, 2018. doi: 10.1038/s41467-018-03216-w.
[92] B. Mazzolai and C. Laschi, "A vision for future bioinspired and biohybrid robots," *Sci. Robot.*, vol. 5, no. 38, p. eaba6893, 2020. doi: 10.1126/scirobotics.aba6893.
[93] J. M. McCracken, B. R. Donovan, and T. J. White, "Materials as machines," *Adv. Mater.*, vol. 32, no. 20, p. 1,906,564, 2020. doi: 10.1002/adma.201906564.
[94] J. Zhang et al., "Robotic artificial muscles: Current progress and future perspectives," *IEEE Trans. Robot.*, vol. 35, no. 3, pp. 761–781, 2019. doi: 10.1109/TRO.2019.2894371.
[95] Z. Shen, F. Chen, X. Zhu, K.-T. Yong, and G. Gu, "Stimuli-responsive functional materials for soft robotics," *J. Mater. Chem. B*, early access, Aug. 18, 2020. doi: 10.1039/D0TB01585G.
[96] A. Rodriguez and M. T. Mason, "Effector form design for 1DOF planar actuation," in *Proc. 2013 IEEE Int. Conf. Robot. Autom.*, pp. 349–356. doi: 10.1109/ICRA.2013.6630599.
[97] J. Kim and N. S. Pollard, "Fast simulation of skeleton-driven deformable body characters," *ACM Trans. Graph.*, vol. 30, no. 5, pp. 1–19, 2011. doi: 10.1145/2019627.2019640.
[98] J. Tan, G. Turk, and C. K. Liu, "Soft body locomotion," *ACM Trans. Graph.*, vol. 31, no. 4, pp. 1–11, 2012. doi: 10.1145/2185520.2185522.
[99] Y. Hu et al., "Chainqueen: A real-time differentiable physical simulator for soft robotics," in *Proc. 2019 Int. Conf. Robot. Autom. (ICRA)*, pp. 6265–6271. doi: 10.1109/ICRA.2019.8794333.
[100] J. Nocedal and S. Wright, *Numerical Optimization*. New York: Springer-Verlag, 2006.

***Feifei Chen***, State Key Laboratory of Mechanical System and Vibration, Shanghai Jiao Tong University, and Robotics Institute, School of Mechanical Engineering, Shanghai Jiao Tong University. Email: ffchen@sjtu.edu.cn.

***Michael Yu Wang***, Robotics Institute, Department of Mechanical and Aerospace Engineering, and Department of Electronic and Computer Engineering of Hong Kong University of Science and Technology, Clear Water Bay, Kowloon. Email: mywang@ust.hk.